MAT 260 LINEAR ALGEBRA
LECTURE 1

WING HONG TONY WONG

Brief review — set notations

Most of modern mathematics are built upon set theory. Hence, to learn modern subjects such as linear algebra, we must first understand the basic set language.

Question 1. What is a **set**?

Question 2.
(a) How do we describe a set?
(b) How do we verify an element is in the set or not?

Exercise 3.
(a) Write the set notation for all rational numbers less than 4 and greater than 1.
(b) Write a set of your own using defining property. Be creative!

Question 4.
(a) What is a **subset** of a set?
(b) If A and B are both sets, how do you verify whether A is a subset of B?
(c) What is the formal definition of $A \subseteq B$?
(d) What is the formal definition of $A = B$?

Exercise 5.
(a) Is it true that $\emptyset \subseteq \{3, 5, 8\}$?
(b) Is it true that $\emptyset \in \{3, 5, 8\}$?
(c) Is it true that $\{3, 5\} \in \{\{3\}, \emptyset, \{8, 5\}, \{3, 8\}, 3, 8, 5\}$?
(d) Is it true that $\{3, 5\} \subseteq \{\{3\}, \emptyset, \{8, 5\}, \{3, 5\}, \{3, 8, 5\}\}$?
(e) Is it true that $\{3, 5, 8\} \subseteq \{8, 3, 5\}$?

Problem 6.
Is it true that $\{3, 5, 8\} = \{5, 8, 3, 5, 3\}$? Use the definition derived from Question 4(d) to prove your assertion.

Date: Wednesday, January 22, 2020.

Question 7. Let X be a set, and let $A, B \subseteq X$.
(a) What is the formal definition of the **union** of A and B, i.e. $A \cup B$?
(b) What is the formal definition of the **intersection** of A and B, i.e. $A \cap B$?
Let I be a nonempty set, and let $A_i \subseteq X$ for all $i \in I$.
(c) What is the formal definition of $\bigcup_{i\in I} A_i$?
(d) What is the formal definition of $\bigcap_{i\in I} A_i$?

Notations in this course

- $\forall$: for all.
- $\exists$: there exists.
- s.t.: such that.
- $\rightarrow\leftarrow$: contradiction.
- WTS: want to show.
- WLOG: Without loss of generality.

Steps for problem solving

(1) Understand the question
- Many students fail to solve a problem because they do not even understand the question.
- It is critical to be able to understand the terminology and distinguish between the conditions and the conclusions.

(2) Collect suitable tools
- The most important tool in mathematics is the **definitions**.
- Other tools include theorems, examples, and previous homework problems.

(3) Build the logical bridge
- Apart from trying to start from the conditions, we can also try to start from the conclusions and ask what could be the most likely step that leads to the final conclusion. Then we replace the final conclusion with the new goal and repeat the process.

(4) Present your solution
- The way we present is often drastically different from the way we think when coming up with the solution. Our presentation should be as concise as possible. Mathematicians and magicians are very similar in this aspect — we both hide our thinking process and only present the essentials.
- We must present our solutions in a logical order, and we must clearly define every single symbol **before** we use it. We write in paragraphs, occasionally with the aid of symbols and equations.
- A rule of thumb in presentation is: present in a way that you can understand your own solutions three years later.

(5) Evaluate your solution
- What was the most difficult part of this solution? What were the key steps? What are the major take-aways from this solution?
- How does this result fit in the big picture? Can you extend your results? Can you weaken the conditions or strengthen the conditions? Can you apply your results or lines of thoughts to another situation?