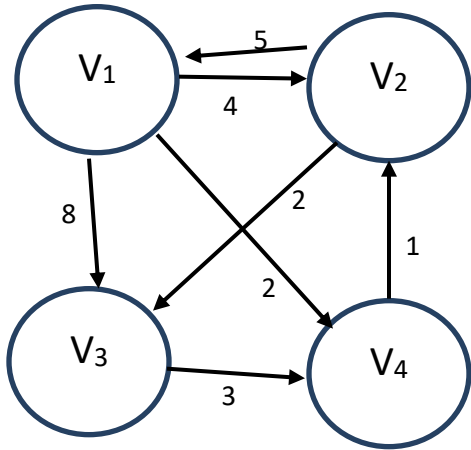


# All Pairs Shortest Path Example

## Finding Shortest Paths Between All Vertices in a Graph

### Warshall-Floyd Algorithm



An adjacency matrix is used to represent the graph, and analyzed to find what is effectively a transitive closure.

The adjacency matrix is at right.

The idea behind Warshall-Floyd is that there is a path from vertex  $i$  to vertex  $j$  if there is a

path from  $i$  to  $k$  and a path from  $k$  to  $j$ .

We want to find minimum paths by cost. We will examine existing costs and compare with cost via vertex  $k$ .

	V <sub>1</sub>	V <sub>2</sub>	V <sub>3</sub>	V <sub>4</sub>
V <sub>1</sub>	0	4	8	2
V <sub>2</sub>	5	0	2	∞
V <sub>3</sub>	∞	∞	0	3
V <sub>4</sub>	∞	1	∞	0

- Step 1:** Transfer all of the costs in the original matrix to the corresponding positions of a new adjacency matrix. The diagonal costs remain 0.
- Step 2.** List separately the rows that have values in column  $k$  ( $=1$  this time) and the columns that have 1's in row  $k$ .  
 Rows: 2      Columns: 2, 3, 4
- Step 3.** Pair each of the row numbers with each of the column numbers, and put the cost in the corresponding position of the matrix, if there isn't one with lower cost there already. Any remaining empty positions are left blank.

	V <sub>1</sub>	V <sub>2</sub>	V <sub>3</sub>	V <sub>4</sub>
V <sub>1</sub>	0	4	8	2
V <sub>2</sub>	5	0	2	7
V <sub>3</sub>			0	3
V <sub>4</sub>		1		0

(2,2), (2,3), (2,4) -

(2,2) has a 0; we can't do better. (2,3) has a 2. The alternative is  $v_2$  to  $v_1$  and  $v_1$  to  $v_3$ . But, that costs 13, and the current cost is 2. We stick with the 2. But, for (2,4), there is no value (it was ∞); all the blanks are available to be filled in.

Using  $v_1$  as the intermediary, we can go from  $v_2$  to  $v_1$  and  $v_1$  to  $v_4$ . That cost is  $5+2=7$ .

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## Finding Shortest Paths Between All Vertices in a Graph

### Warshall-Floyd Algorithm

A graphic way of accomplishing this is by drawing boxes around row and column  $k$ .

We look at intersections, as depicted in the matrix, where the row and column both have a concrete value.

The 0 will remain. The 2 is less than the sum of the values in the row and column (5 and 8) and therefore remains.  $(v_2, v_4)$  is blank and therefore the sum will be placed,  $5+2 = 7$ .

	$V_1$	$V_2$	$V_3$	$V_4$
$V_1$	0	4	8	2
$V_2$	5	0	2	7
$V_3$			0	3
$V_4$		1		0

We now work off the updated matrix.

Rows: 1,4 Columns: 1,3,4

$(1,1)$  is 0;  $(1,3)$  is 8, but the two values that intersect are 4 and 2. 8 should become 6. Note that the 8 is at one of the intersections.  $(1,4)$  has a 2, which is less than 11.

Now, Row 4.  $(v_4, v_2)$  has cost 1, and  $(v_2, v_1)$  has cost 5. Therefore,  $v_4$  to  $v_1$  has cost 6.

Using the graphic method,  $(v_4, v_3)$  has cost 3. Confirm this on the graph.

Complete the rest of this algorithm as an exercise.

	$V_1$	$V_2$	$V_3$	$V_4$
$V_1$	0	4	<del>8</del> 6	2
$V_2$	5	0	2	7
$V_3$			0	3
$V_4$	6	1	3	0