The Left to Right Bias Affects Start Unknown Equations

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Ryan and Snoddy (2011) compared performance on a word problem to performance on an equation that could be used to solve the word problem. The equation was presented in both a start unknown and a result unknown format. The pattern of errors suggested that when students failed, it might have been due to both equations being set up so that working from left to right would be *incongruent* with the correct order of operations for that equation and the participants having a bias to work from left to right. In a second experiment, they confirmed the left to right bias by showing that performance was slightly, although barely significantly, impaired by presenting a result unknown equation in an incongruent rather than a congruent form. In the present study we demonstrated the left to right incongruency impairment more convincingly by using the usually more difficult start unknown form of an equation for both the incongruent and congruent conditions. However, a confound with another potential difficulty factor, having to work outside of parentheses before working within them, remains to be disentangled.

Koedinger, Alibali, and Nathan, (2008) showed that in some cases, unlike what most math teachers and students believe, a word problem can be easier to solve than its equivalent equation. However, the equation that was compared to the word problem was a start unknown equation, which is known to be more difficult than a result unknown equation (Koedinger & Nathan, 2004). Ryan and Snoddy (2011 - Experiment 1) compared performance on the same word problem used by Koedinger et al. to performance on both the start unknown equation that Koedinger et al. had used and the result unknown version of that equation. They found that the word problem was still easier than either form of the equation.

However, the pattern of errors suggested that when students failed, it might have been due to both equations being set up so that working from left to right would be *incongruent* with the correct order of operations for that equation and the participants having a bias to work from left to right. Ryan and Snoddy (2011 - Experiment 2) confirmed the left to right bias. When a result unknown equation was set up so that working from left to right would violate the correct order of operations (e.g., $64 + 3(20.50) = X$, which was called an *incongruent* equation) college students were slightly (but barely significantly) less likely to correctly solve the equation than when it was set up so that working from left to right would lead to correctly following the order of operations (e.g., $3(20.50) + 64 = X$, which was called a *congruent* equation). An examination of the types of errors responsible for the failures to solve suggested that the incongruence may indeed have been responsible for the difference in solution rates. They found that participants were significantly more likely to fail to solve an incongruent equation than a congruent equation due to violating the order of operations, rather than due to some other error.

However in Ryan and Snoddy (2011 - Experiment 2), both equations were result unknown equations. Because start unknown equations are known to be more difficult than result unknown equations (Koedinger & Nathan, 2004) we hypothesized that this left to right error might be even more pronounced in start unknown equations.

To test this hypothesis, in the study reported here, we gave college students equations that varied both in the position of the unknown (i.e., result unknown versus start unknown) and in whether or not working from left to right would follow or violate the correct order of operations (i.e., congruent versus incongruent). We also gave them the word problem for which the result unknown equations would be the symbolic representation. The word problem was used in this study just to confirm that our participants performed at a level similar to the level of the participants in the previous study (Koedinger et al., 2008) that had used an almost identical word problem, and upon which this study was based. Our problem used the value $63, whereas the previous study had used $64 .

Method

Participants

The participants were 573 college students who participated in partial fulfillment of a research participation requirement of their Introductory Psychology class at Kutztown University, an undergraduate teaching university that is one of 14 schools in the Pennsylvania State System of Higher Education.

Materials

The materials (see Table 1) included the equations suggested for this study by Ryan and Snoddy (2011). Those equations and the word problem were adapted from previous difficulties factors research in algebra (Koedinger et al., 2008; Koedinger & Nathan, 2004).

Table 1

Problems used to examine the difficulty of equations, relative to a word problem, as a function of the placement of the unknown and congruence with the order of operations

	Unknown	
Type of Problem	Result-Unknown	Start-Unknown
Congruent equation	$3(20.50) + 63 = X$	$-21 + \frac{X}{3} = 20.50$
Incongruent equation	$63 + 3(20.50) = X$	$(X - 63) \div 3 = 20.50$
Word problem	**Mom won some money in a lottery. She kept $63 for herself and gave each of her 3 sons an equal portion of the rest of it. If each son got $20.50, how much did Mom win?**	

Procedure

The word problem or equation was presented on an 8 1/2” by 11” sheet of paper. The instructions printed on the page for the word problem were “Please carefully read the problem below. Try to set up an equation that would enable you to solve it, if you can. In any event, try to solve problem. Regardless of whether you set up an equation and solve it, or you use some other method, please show all your work.” The instructions printed on the page for the equations were “Solve for X. Please show all your work.”

Each participant was run either individually or in a small enough group so that the participants could be separated far enough to minimize the possibility of seeing another participant's work. Each participant was randomly assigned to receive either the word problem (N = 114), the result-unknown congruent equation (N = 117), the result-unknown incongruent equation (N = 116), the start-unknown congruent equation (N = 115), or the start-unknown incongruent equation (N = 111). The participants were allowed to use a calculator and they were not timed.

Results

As shown in Figure 1, performance on the word problem was almost perfect. Given that the performance on the almost identical word problem in Koedinger et al. (2008) was 83%, we concluded that our participants were not performing at a level any lower than in that previous study. For the result unknown equations we failed to confirm the earlier result. Performance on the incongruent equations was only very slightly, and not significantly, lower than for the congruent equations. For the start unknown equations, however, our hypothesis was supported. Performance on the incongruent equations was significantly lower than on the congruent equations, X^2 (1, N = 226) = 4.47, p = .035.

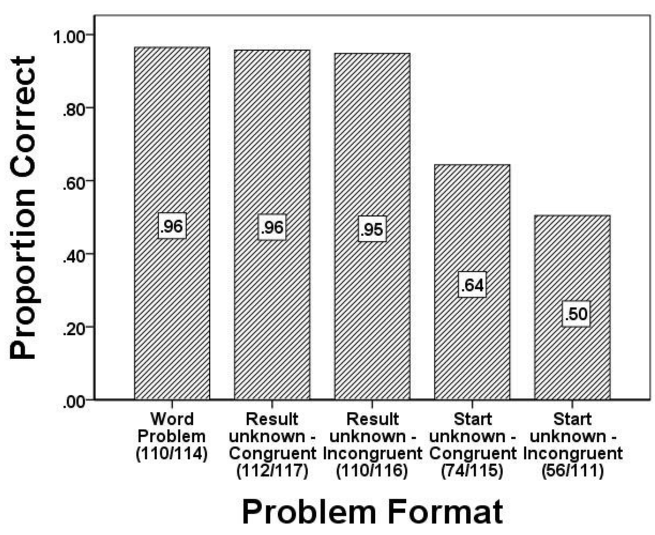

Figure 1. Proportion correct (and number correct out of sample size) for each problem format.

Discussion

Our main hypothesis was supported. Setting up an equation so that the correct order of operations to solve it was incongruent with working from left to right impaired the performance of our participants on the more difficult start unknown equations.

The fact that such incongruency did not produce the same effect for the relatively easier result unknown equation could have been a ceiling effect. This is suggested by the almost perfect performance on both the congruent and incongruent versions of those problems. This interpretation is further supported by the finding that our participants performed even better on the word problem than had the participants in Koedinger et al. (2008).

It is not clear, however, that a left to right bias alone accounts for these results. It should be noted that for the easier result unknown equations, the correct process is one of simplifying an expression that does not contain the unknown. Students are often taught an acronym (PEMDAS) to help them remember the correct order of operations (parentheses, exponents, multiplication or division, addition or subtraction) for the simplifying process. For the more difficult start unknown equations, the correct process is one of undoing the various operations surrounding the unknown, a process known as *unwinding* (Koedinger & Nathan, 2004). On the one hand, this unwinding process can sometimes be done in more than one order. For instance, following the PEMDAS order when unwinding operations would work for the start unknown congruent equation we used. However, doing so would require undoing a division with multiplication, and the multiplication would need to be distributed over two terms. On the other hand, unwinding the processes in the reverse of the order of operations for simplifying (i.e., SADMEP) will lead to a correct solution without having to do any distributing of one operation over another. Thus, start unknown problems entail the difficulty that the simplest way to solve them is to reverse the usual order of operations.

The above explanation of this important difference between start and result unknown equations can help explain why students find start unknown equations more difficult. It can help explain why we didn't find an incongruency effect for the result unknown equations. Even when a result unknown equation is incongruent, requiring a right to left rather than a left to right, solution procedure, as long as a student remembers PEMDAS it can help them avoid the left to right error. On the other hand, for the start unknown equations there is already the difficulty that the best and simplest order of operations is SADMEP. But a consideration of exactly how our students had to use SADMEP for our congruent and incongruent equations shows that there are two possible explanations for the exceptionally low performance on the incongruent start unknown equation.

On the one hand, for both the congruent and the incongruent start unknown equations the best and simplest solution procedure was to use SADMEP instead of PEMDAS. But for the congruent start unknown equation, not only was the correct direction of work left to right, but also, the use of SADMEP instead of PEMDAS required only undoing a subtraction with addition before undoing a division with multiplication. On the other hand, for the incongruent start unknown equation, not only was the correct direction of work right to left, but in addition, using SADMEP instead of PEMDAS required undoing a division with multiplication before working in a parentheses. Therefore, the reason for the exceptionally poor performance on the incongruent start unknown equation could have been not only because of a tendency to want to work left to right instead of the correct right to left, but also could have been additionally due to an especially strong reluctance to perform any other operation before an operation within a parentheses.

To untangle these two potential explanations for our result will require comparing performance on equations for which we cross congruency versus incongruency with the need to put some operation ahead of doing what is in parentheses versus not needing to do that.

References

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