Research Basis for Catchup Math

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Preface

Kutztown University is a 4 year undergraduate university that is one of 14 such universities that comprise the Pennsylvania State System of Higher Education. Historically, they grew out of the Pennsylvania Normal Schools that were established shortly after the Civil War to prepare teachers for the growing U.S. population. They are now full service universities that still emphasize teacher preparation, in addition to offering a range of higher education opportunities to Pennsylvania's youth.

Dr. Ryan is an experimental psychologist (not a practitioner) who received his Ph.D. from the University of Pittsburgh in 1997. His doctoral dissertation examined the effects of comparing training examples on people's ability to induce an equation. In Dr. Ryan's experiments, college students worked with some relatively easier and some relatively harder word problems involving the weighted averaging of ratios. The students were trained either to only solve or to both solve and compare some of the easier problems. They were trained to solve the problems only by being shown a set of steps, not an equation. The dissertation experiments showed that comparing the training examples enabled people to induce the equation in which they had not been trained, and to use it to solve the relatively harder problems.

Catchup Math (catchupmath.com) is a web-based resource that is based on several principles of learning that have been uncovered through experimental and non-experimental studies from educational research and from behavioral and cognitive psychology research. It is used for remediating students' difficulties in learning topics in mathematics, ranging from Pre-Algebra through Intermediate Algebra. Students may either be assigned to review specific topics by their teacher, or they may be given a pre-test to diagnose their level of proficiency and then be assigned to review topics based on their performance on the test. As Catchup Math presents the topics to the student, it makes use of features that capitalize on the evidence based principles of learning that have been uncovered by educational and cognitive psychology researchers.

The Principles of Learning

1. Mastery Learning

In a mastery learning technique, lessons are based on a task analysis that breaks larger units of knowledge into a sequence of sub-units, each of which supports the learning of those that follow. Students must reach a criterion set by their teacher on one sub-unit before going on to the next. Thus, this technique is based on the well established behavioral technique of *shaping*, that is, the rewarding of gradually and successively closer approximations to a desired final behavior.

2. Multiple Examples

Conventional classroom teaching has included explanation of the subject matter accompanied by a limited number of examples, after which students are assigned problems to work out on their own. However, a multiple examples approach increases the use of "worked out" examples as opposed to the use of unsolved problems. This technique is based on two cognitive principles. The first is the principle of reducing cognitive load by having the examples already worked out for the student. The second is the principle of varying the types of examples presented. Presenting a larger number of examples affords the use of a wider variety of types of problems. This works hand in hand with the first principle. The reduction in cognitive load by not requiring the student to solve the problems enables the student to use cognitive resources for important learning tasks such as processing the similarities and differences among the variety of types of problems.

3. Scaffolding

Although students can sometimes learn through their own exploration, it has been found that they benefit from hints and other aids, called scaffolding, during their attempts at discovery learning. In a natural setting these hints could come from a more competent peer, but they can also be provided by a teacher or expert in an educational setting.

4. Zone of Proximal Development

Students need to attempt problems that are in their “zone of proximal development”. That is, the problems should be challenging enough so as to motivate their curiosity and so that there is a potential to learn something new, but not so challenging as to invite failure. This principle works hand in hand with scaffolding. The zone of proximal development is defined by a set of tasks that are just challenging enough so that students would not be able to do them on their own, but would be able to do them with the help of some scaffolding.

5. Desirable Difficulties

The classical definition of learning is that it is a relatively permanent change in behavior due to experience. The modern cognitive approach to learning involves studying the component processes, such as attention and perception, that support the acquisition of knowledge and conceptual change. Nevertheless, the ultimate goal of education is to produce learning that is more than just initial acquisition. In addition to acquisition, more like the older behavioral definition of learning, the goal includes both retention and ability to transfer, that is, the ability to use the knowledge in different situations from that in which it was learned.

However, instruction can be done in many different ways, and cognitive psychologists have found that methods of instruction that lead to fast and easy acquisition, perversely, lead to poor retention and transfer. Fortunately, although ironically, methods of instruction that make it difficult for the learner to achieve fast and easy acquisition have been shown to enhance retention and transfer. Thus, such difficulties have been dubbed “desirable difficulties”.

The Evidence Supporting the Principles

1. Mastery Learning

Mastery learning has been shown to lead to higher performance than other methods as well as better retention. For example, Semb, Ellis, and Araujo (1993) found that students who had used mastery learning retained more than students who had not used mastery learning both at a 4 month retention interval and an 11 month retention interval.

2. Multiple Examples

Providing students with worked out examples of math problems has been found to be more effective than simply assigning the same problems for the students to work out on their own. In one experiment (Carroll, 1994), 40 high school students were instructed in how to solve linear equations. In an "acquisition phase" the students were divided into two groups and their instruction differed in the following way: in the "conventional learning" group, students were assigned 44 unsolved problems to work out (in the classroom and at home homework), and in the "worked examples" group students were provided with the same problems, but half of the problems were accompanied by correct solutions. After completion of the assigned problems, both groups were tested on 12 related problems, 10 of which were very similar to the linear equations presented in the acquisition phase, and 2 of which were word problems, used to test whether students could transfer and extend their knowledge to a new context. No worked out examples were available during the test. The test results revealed that students in the "worked examples" group outperformed students in the "conventional learning" group on both types of the test problems. A second experiment, employed a similar methodology but focused on "low achieving" students (students with a history of failure in mathematics, and students identified as learning disabled). Here, the data revealed that students in the "worked examples" group required less acquisition time, needed less direct instruction, made fewer errors, and made fewer types of errors than students in the "conventional learning" group.

Related research (Pass & Van Merrienboer, 1994) sheds light on the cognitive underpinnings of the effects described above. In this study, 60 college-aged students were instructed in geometry concepts. As in the Carroll (1994) experiments, students were assigned un-worked problems to solve or worked out examples to review (unlike the Carroll study, the "worked examples" group was assigned no un-worked problems to solve). In this study, the researchers manipulated the nature of the problems presented to the students: within each group, some students received problems which were all similar to each other, while others received a more varied problem set. Furthermore, the researchers measured the "cognitive load" experienced by the students. This research revealed that while students in the worked examples group completed their work more quickly, they perceived the work as less demanding and displayed better transfer performance at test. The effect was most pronounced for the students given highly-variable problems. The researchers suggest that the reduced cognitive load associated with the worked examples enabled students to "take advantage of" the variability in problems by using the available cognitive resources to process the underlying similarity in the problems (i.e., the mathematical concepts being taught), and to integrate the current problem with existing knowledge (Linn, 2000).

3. Scaffolding

Scaffolding is a concept that has been used in education with varying degrees of success, depending upon how it is used. For example, at times any kind of intervention that has the characteristic of providing support, the metaphorical idea underlying this technique, has been called scaffolding. However, the original theoretical idea of scaffolding involved more than just this metaphorical idea (Putambekar & Hubscher, 2005). For example, it should include the use of ongoing diagnosis (Lodewyk & Winne, 2005).

4. Zone of Proximal Development

Cognitive psychologists have recently empirically investigated how best to apply scaffolding using modern technological tools. For example a “difficulty factors analysis” in mathematics is a method used to determine the appropriate boundaries for the zone of proximal development. Rittle-Johnson and Koedinger (2005) have used difficulty factors analyses to develop tools to define the boundaries of a student's zone of proximal development specifically in computer based classroom instruction in mathematics.

5. Desirable Difficulties

Schmidt and Bjork (1992) found that there was a common theme in their research on motor learning and verbal learning. Various manipulations of instructional methods had differential effects on acquisition of learning versus its later retention and transfer. For example, they found these differential effects for blocked versus random practice, massed versus spaced practice, similar versus varied examples, and immediate versus delayed feedback. In every case, the former versus the latter method led to easier and faster acquisition, but poorer retention and transfer. Conversely, although the more difficult methods interfered with initial acquisition, the methods that led to better retention and transfer were random practice, spaced practice, varied examples, and delayed feedback.

How Catchup Math Uses the Principles to Help Students Learn

1. Mastery Learning

The principle underlying mastery learning is that, as the student is placed in gradually more challenging tasks, the student must not advance to the more challenging level until meeting a criterion that insures that the student has the prerequisite prior knowledge. This is accomplished in Catchup Math by establishing the appropriate requirements for practice and for quiz performance. Regarding practice, as stated in the Getting Started Guide, "At a minimum, the students must attempt the required practice problems and view the solutions all the way to the end in order to proceed to the next topic." Regarding quiz performance, again as stated in the Getting Started Guide, "Once the quiz has been passed and the topics reviewed, students are advanced to the next section of the program that covers new topics".

2. Multiple Examples

Catchup Math creates instruction that is effective for the students by providing accessible, worked solutions to problems. The goal is to provide instruction that reduces cognitive load and results in knowledge that is integrated with students' existing ideas such that it can be more effectively used in a wide range of contexts. Students using Catchup Math focus on a correct series of steps needed to reach the solution.

The theory of knowledge integration as described by Linn (2000) and others discuss four tenets in knowledge integration: make the material accessible, make thinking visible, promote autonomy, and encourage social supports. Catchup Math's instructional approach helps to make the material accessible by presenting relevant problems to students to promote integration of the ideas obtained while learning how to solve these problems with the students' existing knowledge and ideas. Catchup Math helps to make the expert's problem solving thought process more visible by clearly showing the sequence of steps taken and providing explanations, discussions of alternative paths and approaches, and showing a complete history of the steps taken to reach the solution. Catchup Math promotes autonomy by prompting students with questions that encourage them to reflect on the information presented. This promotes an inquiry process that will help students to learn autonomously.

3. Scaffolding

Catchup Math uses graphs, diagrams, conceptual activities, hints that the student can choose to look at during a quiz, printed lessons, and video lessons to explain the solutions when applicable. The use of multimedia to provide alternate forms of representation has been shown to be effective when properly presented (Mayer, 1997). Catchup Math provides supportive video, written, and programmed activities as companion review narrations to reinforce instruction to make it easier to follow.

5. Zone of Proximal Development

Assignments should not be too easy, or too hard, but just at the right level. The right level entails having the appropriate prior knowledge for a new task before attempting it. In Catchup Math, students can enroll in the appropriate remediation program by taking a quiz. The quiz determines their level of prior knowledge. Then, the software places them in the lowest level where they need help.

5. Desirable Difficulties

Catchup Math introduces desirable difficulties at various points in the student's learning. For example, it provides different difficulty levels in the games, such as identifying prime numbers or fractions in their lowest terms. At the highest difficulty levels, these games are challenging enough so that they make learning and retrieval much more effortful than at the lower levels. As another example, there is feedback on the quizzes, but the feedback is delayed, not immediate. Delaying the feedback makes it more effortful to use the feedback to learn from than if the feedback were immediate. Such increased effort may actually impede initial knowledge acquisition, but at the same time the research suggests that it should have positive effects on long term retention.

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