## Fundamentals of Machine Learning for Predictive Data Analytics <br> Chapter 3: Data Exploration Sections 3.5, 3.6, 3.7

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(1) Advanced Data Exploration

- Visualizing Relationships Between Features
- Measuring Covariance \& Correlation
(2) Data Preparation
- Normalization
- Binning
- Sampling
(3) Summary


## Advanced Data Exploration

| ID | Position | HEIGHT | WEIGHT | CAREER <br> STAGE | AGE | SPONSORSHIP <br> EARNINGS | SHOE <br> SPONSOR |
| :---: | :---: | :---: | :---: | :---: | :---: | ---: | :---: |
| 1 | forward | 192 | 218 | veteran | 29 | 561 | yes |
| 2 | center | 218 | 251 | mid-career | 35 | 60 | no |
| 3 | forward | 197 | 221 | rookie | 22 | 1,312 | no |
| 4 | forward | 192 | 219 | rookie | 22 | 1,359 | no |
| 5 | forward | 198 | 223 | veteran | 29 | 362 | yes |
| 6 | guard | 166 | 188 | rookie | 21 | 1,536 | yes |
| 7 | forward | 195 | 221 | veteran | 25 | 694 | no |
| 8 | guard | 182 | 199 | rookie | 21 | 1,678 | yes |
| 9 | guard | 189 | 199 | mid-career | 27 | 385 | yes |
| 10 | forward | 205 | 232 | rookie | 24 | 1,416 | no |
| 11 | center | 206 | 246 | mid-career | 29 | 314 | no |
| 12 | guard | 185 | 207 | rookie | 23 | 1,497 | yes |
| 13 | guard | 172 | 183 | rookie | 24 | 1,383 | yes |
| 14 | guard | 169 | 183 | rookie | 24 | 1,034 | yes |
| 15 | guard | 185 | 197 | mid-career | 29 | 178 | yes |
| 16 | forward | 215 | 232 | mid-career | 30 | 434 | no |
| 17 | guard | 158 | 184 | veteran | 29 | 162 | yes |
| 18 | guard | 190 | 207 | mid-career | 27 | 648 | yes |
| 19 | center | 195 | 235 | mid-career | 28 | 481 | no |
| 20 | guard | 192 | 200 | mid-career | 32 | 427 | yes |
| 21 | forward | 202 | 220 | mid-career | 31 | 542 | no |
| 22 | forward | 184 | 213 | mid-career | 32 | 12 | no |
| 23 | forward | 190 | 215 | rookie | 22 | 1,179 | no |
| 24 | guard | 178 | 193 | rookie | 21 | 1,078 | no |
| 25 | guard | 185 | 200 | mid-career | 31 | 213 | yes |
| 26 | forward | 191 | 218 | rookie | 19 | 1,855 | no |
| 27 | center | 196 | 235 | veteran | 32 | 47 | no |
| 28 | forward | 198 | 221 | rookie | 22 | 1,409 | no |
| 29 | center | 207 | 247 | veteran | 27 | 1,065 | no |
| 30 | center | 201 | 244 | mid-career | 25 | 1,111 | yes |

- A scatter plot is based on two axes: the horizontal axis represents one feature and the vertical axis represents a second.
- Each instance in a dataset is represented by a point on the plot determined by the values for that instance of the two features involved.


Figure: An example scatter plot showing the relationship between the Height and Weight features from the professional basketball squad dataset in Table $4{ }^{[4]}$.

(a)

(b)

Figure: Example scatter plots showing (a) the strong negative covariance between the Sponsorship Earnings and Age features and (b) the HEight and Age features from the dataset in Table $4{ }^{[4]}$.

- A scatter plot matrix (SPLOM) shows scatter plots for a whole collection of features arranged into a matrix.
- This is useful for exploring the relationships between groups of features - for example all of the continuous features in an ABT.


Figure: A scatter plot matrix showing scatter plots of the continuous features from the professional basketball squad dataset.

- The simplest way to visualize the relationship between two categorical variables is to use a collection of small multiple bar plots.


Figure: Using small multiple bar plot visualizations to illustrate the relationship between the Career Stage and Shoe Sponsor features.

Shoe Sponsor = no



Shoe Sponsor = yes


Figure: Using small multiple bar plot visualizations to illustrate the relationship between the Position and Shoe Sponsor features.

- If the number of levels of one of the features being compared is no more than three we can use stacked bar plots as an alternative to the small multiples approach.

(a) Career Stage and Shoe Sponsor

Figure: Stacked bar plot visualizations.

- To visualize the relationship between a continuous feature and a categorical feature a small multiples approach that draws a histogram of the values of the continuous feature for each level of the categorical feature is useful.


Figure: Using small multiple histograms to visualize the relationship between the Age feature and the Position feature.

(a) Height


Figure: Using small multiple histograms to visualize the relationship between the HEIGHT feature and the Position feature.

- A second approach to visualizing the relationship between a categorical feature and a continuous feature is to use a collection of box plots.
- For each level of the categorical feature a box plot of the corresponding values of the continuous feature is drawn.


Figure: Using box plots to visualize the relationship between the AGE and the Position feature.


Figure: Using box plots to visualize the relationship between the Height feature and the Position feature.

- As well as visually inspecting scatter plots, we can calculate formal measures of the relationship between two continuous features using covariance and correlation.
- For two features, $a$ and $b$, in a dataset of $n$ instances, the sample covariance between $a$ and $b$ is

$$
\begin{equation*}
\operatorname{cov}(a, b)=\frac{1}{n-1} \sum_{i=1}^{n}\left(\left(a_{i}-\bar{a}\right) \times\left(b_{i}-\bar{b}\right)\right) \tag{1}
\end{equation*}
$$

where $a_{i}$ and $b_{i}$ are values of features $a$ and $b$ for the $i^{t h}$ instance in a dataset, and $\bar{a}$ and $\bar{b}$ are the sample means of features $a$ and $b$.

- Covariance values fall into the range $[-\infty, \infty]$ where negative values indicate a negative relationship, positive values indicate a positive relationship, and values near zero indicate that there is little or no relationship between the features.

Calculating covariance between the HEIGHT feature and the Weight and Age features from the basketball players dataset.

|  | HEIGHT |  |  |  |  |  |  |  |
| ---: | :---: | ---: | :---: | ---: | ---: | ---: | ---: | ---: |
| $(h)$ | $h-\bar{h}$ | WEIGHT |  |  |  |  |  |  |
| $(w)$ | $w-\bar{w}$ | $(h-\bar{h}) \times$ <br> $(w-\bar{w})$ | AGE <br> $(a)$ | $a-\bar{a}$ | $(h-\bar{h}) \times$ <br> $(a-\bar{a})$ |  |  |  |
| 1 | 192 | 0.9 | 218 | 3.0 | 2.7 | 29 | 2.6 | 2.3 |
| 2 | 218 | 26.9 | 251 | 36.0 | 967.5 | 35 | 8.6 | 231.3 |
| 3 | 197 | 5.9 | 221 | 6.0 | 35.2 | 22 | -4.4 | -26.0 |
| 4 | 192 | 0.9 | 219 | 4.0 | 3.6 | 22 | -4.4 | -4.0 |
| 5 | 198 | 6.9 | 223 | 8.0 | 55.0 | 29 | 2.6 | 17.9 |
|  |  |  |  | $\ldots$ |  |  |  |  |
| 26 | 191 | -0.1 | 218 | 3.0 | -0.3 | 19 | -7.4 | 0.7 |
| 27 | 196 | 4.9 | 235 | 20.0 | 97.8 | 32 | 5.6 | 27.4 |
| 28 | 198 | 6.9 | 221 | 6.0 | 41.2 | 22 | -4.4 | -30.4 |
| 29 | 207 | 15.9 | 247 | 32.0 | 508.3 | 27 | 0.6 | 9.5 |
| 30 | 201 | 9.9 | 244 | 29.0 | 286.8 | 25 | -1.4 | -13.9 |
| Mean | 191.1 |  | 215.0 |  |  | 26.4 |  |  |
| Std Dev | 13.6 |  | 19.8 |  |  | 4.2 |  | 570.8 |
| Sum |  |  |  |  |  |  |  |  |

Calculating covariance between the HEIGHT feature and the Weight and Age features from the basketball players dataset.

$$
\begin{aligned}
\operatorname{cov}(\mathrm{HEIGHT}, \mathrm{WEIGHT}) & =\frac{7,009.9}{29}=241.72 \\
\operatorname{cov}(\mathrm{HEIGHT}, \mathrm{AGE}) & =\frac{570.8}{29}=19.7
\end{aligned}
$$

- Correlation is a normalized form of covariance that ranges between -1 and +1 .
- The correlation between two features, $a$ and $b$, can be calculated as

$$
\begin{equation*}
\operatorname{corr}(a, b)=\frac{\operatorname{cov}(a, b)}{s d(a) \times s d(b)} \tag{2}
\end{equation*}
$$

where $\operatorname{cov}(a, b)$ is the covariance between features $a$ and $b$ and $s d(a)$ and $s d(b)$ are the standard deviations of $a$ and $b$ respectively.

- Correlation values fall into the range [ $-1,1$ ], where values close to -1 indicate a very strong negative correlation (or covariance), values close to 1 indicate a very strong positive correlation, and values around 0 indicate no correlation.
- Features that have no correlation are said to be independent.

Calculating correlation between the HEIGHT feature and the Weight and Age features from the basketball players dataset.

$$
\begin{aligned}
\operatorname{corr}(\text { Height }, \text { Weight }) & =\frac{241.72}{13.6 \times 19.8}=0.898 \\
\operatorname{corr}(\text { Height, Age }) & =\frac{19.7}{13.6 \times 4.2}=0.345
\end{aligned}
$$

- In the majority of ABTs there are multiple continuous features between which we would like to explore relationships.
- Two tools that can be useful for this are the covariance matrix and the correlation matrix.
- The covariance matrix, usually denoted as $\sum$, between a set of continuous features, $\{a, b, \ldots, z\}$, is given as

$$
\sum_{\{a, b, \ldots, z\}}=\left[\begin{array}{cccc}
\operatorname{var}(a) & \operatorname{cov}(a, b) & \cdots & \operatorname{cov}(a, z)  \tag{3}\\
\operatorname{cov}(b, a) & \operatorname{var}(b) & \cdots & \operatorname{cov}(b, z) \\
\vdots & \vdots & \ddots & \vdots \\
\operatorname{cov}(z, a) & \operatorname{cov}(z, b) & \cdots & \operatorname{var}(z)
\end{array}\right]
$$

- Similarly, the correlation matrix is just a normalized version of the covariance matrix and shows the correlation between each pair of features:

$$
\underset{\{a, b, \ldots, z\}}{\text { correlation matrix }}=\left[\begin{array}{cccc}
\operatorname{corr}(a, a) & \operatorname{corr}(a, b) & \cdots & \operatorname{corr}(a, z)  \tag{4}\\
\operatorname{corr}(b, a) & \operatorname{corr}(b, b) & \cdots & \operatorname{corr}(b, z) \\
\vdots & \vdots & \ddots & \vdots \\
\operatorname{corr}(z, a) & \operatorname{corr}(z, b) & \cdots & \operatorname{corr}(z, z)
\end{array}\right]
$$

- Calculating covariances matrix for the HEIGHT feature and the Weight and Age features from the basketball players dataset.

$$
\sum_{\text {<Height, Weight,Age> }}=\left[\begin{array}{ccc}
185.128 & 241.72 & 19.7 \\
241.72 & 392.102 & 24.469 \\
19.7 & 24.469 & 17.697
\end{array}\right]
$$

- Calculating correlation matrix for the HEIGHT feature and the Weight and Age features from the basketball players dataset.

$$
\underset{\text { <Height,Weight,Age> }}{\text { correlation matrix }}=\left[\begin{array}{ccc}
1.0 & 0.898 & 0.345 \\
0.898 & 1.0 & 0.294 \\
0.345 & 0.294 & 1.0
\end{array}\right]
$$

- The scatter plot matrix (SPLOM) is really a visualization of the correlation matrix.
- This can be made more obvious by including the correlation coefficients in SPLOMs in the cells above the diagonal.

- Correlation is a good measure of the relationship between two continuous features, but it is not by any means perfect.
- Some of the limitations of measuring correlation are illustrated very clearly in the famous example of Anscombe's quartet by Francis Anscombe.

- Perhaps the most important thing to remember in relation to correlation is that correlation does not necessarily imply causation.


## Data Preparation

- Some data preparation techniques change the way data is represented just to make it more compatible with certain machine learning algorithms.
- Normalization
- Binning
- Sampling
- Normalization techniques can be used to change a continuous feature to fall within a specified range while maintaining the relative differences between the values for the feature.
- We use range normalization to convert a feature value into the range [low, high] as follows:

$$
\begin{equation*}
a_{i}^{\prime}=\frac{a_{i}-\min (a)}{\max (a)-\min (a)} \times(h i g h-\text { low })+\text { low } \tag{5}
\end{equation*}
$$

- Another way to normalize data is to standardize it into standard scores.
- A standard score measures how many standard deviations a feature value is from the mean for that feature.
- We calculate a standard score as follows:

$$
\begin{equation*}
a_{i}^{\prime}=\frac{a_{i}-\bar{a}}{s d(a)} \tag{6}
\end{equation*}
$$

The result of normalising a small sample of the HEIGHT and Sponsorship Earnings features from the professional basketball squad dataset.

|  | HEIGHT |  |  | SPONSORSHIP EARNINGS |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  | Values | Range | Standard | Values | Range | Standard |
|  | 192 | 0.500 | -0.073 | 561 | 0.315 | -0.649 |
|  | 197 | 0.679 | 0.533 | 1,312 | 0.776 | 0.762 |
|  | 192 | 0.500 | -0.073 | 1,359 | 0.804 | 0.850 |
|  | 182 | 0.143 | -1.283 | 1,678 | 1.000 | 1.449 |
|  | 206 | 1.000 | 1.622 | 314 | 0.164 | -1.114 |
|  | 192 | 0.500 | -0.073 | 427 | 0.233 | -0.901 |
|  | 190 | 0.429 | -0.315 | 1,179 | 0.694 | 0.512 |
|  | 178 | 0.000 | -1.767 | 1,078 | 0.632 | 0.322 |
|  | 196 | 0.643 | 0.412 | 47 | 0.000 | -1.615 |
|  | 201 | 0.821 | 1.017 | 1111 | 0.652 | 0.384 |
| Max | 206 |  |  | 1,678 |  |  |
| Min | 178 |  |  | 47 |  |  |
| Mean | 193 |  |  | 907 |  |  |
| Std Dev | 8.26 |  |  | 532.18 |  |  |

- Binning involves converting a continuous feature into a categorical feature.
- To perform binning, we define a series of ranges (called bins) for the continuous feature that correspond to the levels of the new categorical feature we are creating.
- We will introduce two of the more popular ways of defining bins:
- equal-width binning
- equal-frequency binning
- Deciding on the number of bins can be difficult. The general trade-off is this:
- If we set the number of bins to a very low number we may lose a lot of information
- If we set the number of bins to a very high number then we might have very few instances in each bin or even end up with empty bins.


- The equal-width binning algorithm splits the range of the feature values into $b$ bins each of size $\frac{\text { range }}{b}$.


(j) 15 Equal-width bins
- Equal-frequency binning first sorts the continuous feature values into ascending order and then places an equal number of instances into each bin, starting with bin 1.
- The number of instances placed in each bin is simply the total number of instances divided by the number of bins, $b$.

(k) 5 Equal-frequency bins (I) 10 Equal-frequency bins

(m) 15 Equal-frequency bins
- Sometimes the dataset we have is so large that we do not use all the data available to us in an ABT and instead sample a smaller percentage from the larger dataset.
- We need to be careful when sampling, however, to ensure that the resulting datasets are still representative of the original data and that no unintended bias is introduced during this process.
- Common forms of sampling include:
- top sampling
- random sampling
- stratified sampling
- under-sampling
- over-sampling
- Top sampling simply selects the top $s \%$ of instances from a dataset to create a sample.
- Top sampling runs a serious risk of introducing bias, however, as the sample will be affected by any ordering of the original dataset.
- We recommend that top sampling be avoided.
- Our recommended default, random sampling randomly selects a proportion of $s \%$ of the instances from a large dataset to create a smaller set.
- Random sampling is a good choice in most cases as the random nature of the selection of instances should avoid introducing bias.
- Stratified sampling is a sampling method that ensures that the relative frequencies of the levels of a specific stratification feature are maintained in the sampled dataset.
- To perform stratified sampling:
- the instances in a dataset are divided into groups (or strata), where each group contains only instances that have a particular level for the stratification feature
- $s \%$ of the instances in each stratum are randomly selected
- these selections are combined to give an overall sample of $s \%$ of the original dataset.
- In contrast to stratified sampling, sometimes we would like a sample to contain different relative frequencies of the levels of a particular feature to the distribution in the original dataset.
- To do this, we can use under-sampling or over-sampling.
- Under-sampling begins by dividing a dataset into groups, where each group contains only instances that have a particular level for the feature to be under-sampled.
- The number of instances in the smallest group is the under-sampling target size.
- Each group containing more instances than the smallest one is then randomly sampled by the appropriate percentage to create a subset that is the under-sampling target size.
- These under-sampled groups are then combined to create the overall under-sampled dataset.
- Over-sampling addresses the same issue as under-sampling but in the opposite way around.
- After dividing the dataset into groups, the number of instances in the largest group becomes the over-sampling target size.
- From each smaller group, we then create a sample containing that number of instances using random sampling with replacement.
- These larger samples are combined to form the overall over-sampled dataset.


## Summary

- The key outcomes of the data exploration process are that the practitioner should
(1) Have gotten to know the features within the ABT, especially their central tendencies, variations, and distributionsprobability distribution.
(2) Have identified any data quality issues within the ABT, in particular missing values, irregular cardinality, and outliers.
(3) Have corrected any data quality issues due to invalid data.
(4) Have recorded any data quality issues due to valid data in a data quality plan along with potential handling strategies.
(5) Be confident that enough good quality data exists to continue with a project.
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