

MATH 545 DR. MCLOUGHLIN’S CLASS SOME SAMPLING DISTRIBUTION RESULTS HANDOUT VII

Let $n, p, \alpha, \beta, \sigma, \mu, \gamma, \theta, \lambda, c$ be constants.

Theorem 1 (DeMoivre - Laplace): Let $X \sim \text{Bin}(x, n, p)$. Let $Y = \frac{X - np}{\sqrt{np(1-p)}}$.

As $n \longrightarrow \infty$, it is the case that $Y \longrightarrow Z$ where $Z \sim \text{Nor}(z, 0, 1)$.

Theorem 2: Let $X \sim \text{Nor}(x, \mu, \sigma)$. Let $Z = \frac{X - \mu}{\sigma}$ it is the case that $Z \sim \text{Nor}(z, 0, 1)$.

Theorem 3: Let $X \sim \text{Nor}(x, \mu_X, \sigma_X)$. Let X_1, X_2, \dots, X_n be independent, identically distributed random variables from $X \sim \text{Nor}(x, \mu_X, \sigma_X)$. Let $Y = \bar{X}$ it is the case that $E[Y] = E[\bar{X}] = \mu_X$. So, $\mu_{\bar{X}} = \mu_X$.

Theorem 4: Let $X \sim \text{Nor}(x, \mu_X, \sigma_X)$. Let X_1, X_2, \dots, X_n be independent, identically distributed random variables from $X \sim \text{Nor}(x, \mu_X, \sigma_X)$. Let $Y = \bar{X}$ it is the case that

$$\text{Var}[Y] = \frac{\sigma_X^2}{n}. \text{ So, } \sigma_{\bar{X}}^2 = \frac{\sigma_X^2}{n}.$$

Theorem 5: Let $X \sim \text{Nor}(x, \mu_X, \sigma_X)$. Let X_1, X_2, \dots, X_n be independent, identically distributed random variables from $X \sim \text{Nor}(x, \mu_X, \sigma_X)$. Let $Y = s^2 = \sum_{i=1}^n \frac{(X - \bar{X})^2}{n-1} = \frac{1}{n-1} \sum_{i=1}^n (X - \bar{X})^2$.

$$E[Y] = \sigma_X^2. \text{ So, } E[s^2] = \sigma_X^2.$$

Theorem 6: Let $X \sim \text{Nor}(x, \mu, \sigma)$. Let X_1, X_2, \dots, X_n be independent, identically distributed random variables from $X \sim \text{Nor}(x, \mu, \sigma)$ Let $Y = \bar{X}$

it is the case that $Y \sim \text{Nor}(y, \mu, \frac{\sigma}{\sqrt{n}})$.

Theorem 7: Let X_1, X_2, \dots, X_n be independent, identically distributed random variables from $X \sim \text{Nor}(x, \mu, \sigma)$. Let $Y = \frac{(n-1)s^2}{\sigma^2}$ it is the case that $Y \sim \chi_{(n-1)}^2$.

Theorem 8 (Gossett): Let $Z \sim \text{Nor}(z, 0, 1)$, $U \sim \chi_m^2$, and Z and U be independent.

Let $W = \frac{Z}{\sqrt{U/m}}$ it is the case that $W \sim t_m$ (t with m degrees of freedom or t with $df = m$).

Theorem 9: Let $X \sim \text{Nor}(x, \mu_X, \sigma_X)$. Let X_1, X_2, \dots, X_n be independent, identically distributed random variables from $X \sim \text{Nor}(x, \mu_X, \sigma_X)$

it is the case that $\frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} \sim \text{Nor}(z, 0, 1)$.

Theorem 10 (Gossett): Let $X \sim \text{Nor}(x, \mu, \sigma)$. Let X_1, X_2, \dots, X_n be independent, identically distributed random variables from $X \sim \text{Nor}(x, \mu, \sigma)$

it is the case that $\frac{\bar{X} - \mu}{s/\sqrt{n}} \sim t_{(n-1)}$.

Theorem 11 (Fisher – Snedecor): Let $U \sim \chi_n^2$ and $V \sim \chi_m^2$ and U and V be independent. Let $Y =$

$\frac{U/n}{V/m}$ it is the case that $Y \sim F_{n, m}$.

Theorem 12: Let $U \sim \chi_n^2$ and $V \sim \chi_m^2$ and U and V be independent. Let $Y = \frac{U/n}{V/m}$.

Let $W = \frac{1}{Y}$ it is the case that $W \sim F_{m, n}$.

Theorem 13: Let $X_{11}, X_{12}, \dots, X_{1n_1}$ be independent, identically distributed random variables from $X_1 \sim \text{Nor}(x_1, \mu_1, \sigma_1)$. Let $X_{21}, X_{22}, \dots, X_{2n_2}$ be independent, identically distributed random variables from $X_2 \sim \text{Nor}(x_2, \mu_2, \sigma_2)$.

Let $Y = \frac{s_1^2 \sigma_2^2}{s_2^2 \sigma_1^2}$ it is the case that $Y \sim F_{n_1-1, n_2-1}$.

Theorem 14 (Central Limit Theorem / Law of Large Numbers) :

Let $X \sim f_X(x)$. Let $E[X] = \mu_x$ and $\text{Var}[X] = \sigma_x^2$ be constant.

(1) As $n \longrightarrow \infty$, it is the case that $X \longrightarrow Y_1$ where $Y_1 \sim \text{Nor}(y_1, n \cdot E[X], \sqrt{n} \cdot \text{SD}[X])$.

(2) Let $Y_2 = \bar{X}$

As $n \longrightarrow \infty$, it is the case that $Y_2 \longrightarrow Y_3$ where $Y_3 \sim \text{Nor}(y_3, E[X], \frac{\text{SD}[X]}{\sqrt{n}})$.

Definition 1: Let $X \sim f_X(x)$. Let $\alpha, \beta, \sigma, \mu, \gamma, \lambda, \dots, \theta$ be (possible) parameters for $f_X(x)$.

We say $\hat{\alpha}$ is an estimator of α , $\hat{\beta}$ is an estimator for β , \dots , $\hat{\theta}$ is an estimator for θ .