

MATH 351
ADVANCED CALCULUS (REAL ANALYSIS) I
DR. MCLOUGHLIN
WORKSHEET 7 SEQUENCES 2

Let $U = \mathbb{R}$

Part A Prove each:

Theorem 18.01: The sequence $\{s_n\}_{n=1}^{\infty}$ is constant *iff* it is both non-increasing and non-decreasing.

Theorem 18.02: Let $\{s_n\}_{n=1}^{\infty}$ be a constant sequence. It converges.

Theorem 18.03A: The increasing sequence $\{s_n\}_{n=1}^{\infty}$ is non-decreasing.

Theorem 18.03B: The decreasing sequence $\{s_n\}_{n=1}^{\infty}$ is non-increasing.

Part B Consider each of the following

Claim 18.12: There exists a sequence $f(n), f: \mathbb{N} \longrightarrow \mathbb{R}$, that has the 'shrinking difference' property but is NOT a Cauchy sequence. Prove or disprove the claim.

Claim 18.13: There exists sequences $\{f_n\}_{n=1}^{\infty}$, that is a well defined sequence that does not converge in the 'blow up' sense,' and $\{g_n\}_{n=1}^{\infty}$ that is a well defined sequence that does not converge in the 'blow down' sense,' such that the sequence defined as $\{h_n\}_{n=1}^{\infty} \ni h(n) = f(n) + g(n)$ does not converge in the 'blow down' sense.' Prove or disprove the claim.

Claim 18.14: There exists sequences $\{f_n\}_{n=1}^{\infty}$, that is a well defined sequence that does not converge in the 'blow up' sense,' and $\{g_n\}_{n=1}^{\infty}$ that is a well defined sequence that does not converge in the 'blow down' sense,' such that the sequence defined as $\{h_n\}_{n=1}^{\infty} \ni h(n) = f(n) + g(n)$ does not converge in the 'oscillating' or 'alternating' sense. Prove or disprove the claim.

Claim 18.14: There exists sequences $\{f_n\}_{n=1}^{\infty}$, that is a well defined sequence that does not converge in the 'blow up' sense,' and $\{g_n\}_{n=1}^{\infty}$ that is a well defined sequence that does not converge in the 'blow down' sense,' such that the sequence defined as $\{h_n\}_{n=1}^{\infty} \ni h(n) = f(n) + g(n)$ converges. Prove or disprove the claim.