

Worksheet IV
MORE ON THE TOPOLOGY OF \mathbb{R}
YET MORE DEFINITIONS, LEMMAS, THEOREMS, AND
COROLLARIES
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Let $U = \mathbb{R}$. Let $a, b, c, d, x, y,$ or z be reals.

1. Claim: \exists a set $S \subseteq \mathbb{R}$ that is dense in \mathbb{R} .
2. Claim: \exists a set $S \subseteq \mathbb{R}$ that is disconnected.
3. Claim: \exists a set $S \subseteq \mathbb{R}$ that is totally disconnected.

- 4A. Claim: $a \in \mathbb{Q} \wedge b \in \mathbb{Q} \implies (a + b) \in \mathbb{Q}$.
- 4B. Claim: $a \in \mathbb{Q} \wedge b \in \mathbb{Q} \implies (a \cdot b) \in \mathbb{Q}$.
- 4C. Claim: $a \in \mathbb{Q} \wedge b \in \mathbb{Q} \wedge b \neq 0 \implies (a \div b) \in \mathbb{Q}$.

- 5A. Claim: $a \in \mathbb{I} \wedge b \in \mathbb{I} \implies (a + b) \in \mathbb{I}$.
- 5B. Claim: $a \in \mathbb{I} \wedge b \in \mathbb{I} \implies (a \cdot b) \in \mathbb{I}$.
- 5C. Claim: $a \in \mathbb{I} \wedge b \in \mathbb{I} \implies (a \div b) \in \mathbb{I}$.

6. Claim: $a \in \mathbb{Q} \wedge b \in \mathbb{I} \implies (a + b) \in \mathbb{I}$.

- 7A. Claim: \mathbb{Q} is dense in \mathbb{Q} .
- 7B. Claim: \mathbb{Q} is dense in \mathbb{I} .
- 7C. Claim: \mathbb{Q} is dense in \mathbb{R} .

- 8A. Claim: \mathbb{I} is dense in \mathbb{Q} .
- 8B. Claim: \mathbb{I} is dense in \mathbb{I} .
- 8C. Claim: \mathbb{I} is dense in \mathbb{R} .

- 9A. Claim: \mathbb{N} is perfect.
- 9B. Claim: \mathbb{Z} is perfect.
- 9C. Claim: \mathbb{Q} is perfect.
- 9D. Claim: \mathbb{I} is perfect.
- 9E. Claim: \mathbb{R} is perfect.

- 10A. Claim: \mathbb{N} is nowhere dense in \mathbb{R} .
- 10B. Claim: \mathbb{Z} is nowhere dense in \mathbb{R} .
- 10A. Claim: \mathbb{Q} is nowhere dense in \mathbb{R} .