Worksheet III

On The Topology of $\mathbb{R}$

More Exercises Related to the Definitions, Lemmas, Theorems, and Corollaries

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Let $U = \mathbb{R}$. Let a, b, c, d, x, y, or z be reals.

1. Claim: | a - b | = | c - d | if and only if (a + d) = (b + c).
2. Claim: | a - d | = | b - d | if and only if | a | = | b |.
3. Construct a set A $\subseteq$ $\mathbb{R}$ that has an interior that is non-empty.
4. Construct a set A $\subseteq$ $\mathbb{R}$ that has a boundary that is non-empty.
5. Construct a set A $\subseteq$ $\mathbb{R}$ that has int(A) = A.
6. Construct a set A $\subseteq$ $\mathbb{R}$ that has bd(A) = A.
7. Construct a set A $\subseteq$ $\mathbb{R}$ where $A' \neq \emptyset$.
8. Construct a set A $\subseteq$ $\mathbb{R}$ where $A' = A$.
9. Construct a set A $\subseteq$ $\mathbb{R}$ where $A' \subset A$.
10. Construct a set A $\subseteq$ $\mathbb{R}$ where $A' \not\subset A$.
11. Construct a set A $\subseteq$ $\mathbb{R}$ where $A' \subseteq A$.
12. Construct a set A $\subseteq$ $\mathbb{R}$ where A' is a degenerate set.
13. Construct a set A $\subseteq$ $\mathbb{R}$ where A' is a non-empty degenerate set.
14. Construct a set A $\subseteq$ $\mathbb{R}$ that has a closure that is non-empty.
15. Construct a set A $\subseteq$ $\mathbb{R}$ that has cl(A) = A.
16. Prove Theorem 14.04 of handout III.
17. Prove Theorem 14.05 of handout III.
18. Prove $\mathbb{Q} \neq \emptyset$.
19. Prove $\mathbb{I} \neq \emptyset$.
20. Convince yourself that

Theorem W-1 : $\mathbb{Q}$ is dense in $\mathbb{R}$

Theorem W-2 : $\mathbb{I}$ is dense in $\mathbb{R}$

are true claims but don't bother proving them yet.

21. Construct a set A $\subseteq$ $\mathbb{R}$ that is closed.
22. Construct a set A $\subseteq$ $\mathbb{R}$ that is open.
23. Construct a set A $\subseteq$ $\mathbb{R}$ that is closed and is not open.
24. Construct a set A $\subseteq$ $\mathbb{R}$ that is open and is not closed.
25. Construct a set A $\subseteq$ $\mathbb{R}$ that is perfect.
26. Construct a set A $\subseteq$ $\mathbb{R}$ that is neither open nor closed.
27. Construct a set A $\subseteq$ $\mathbb{R}$ that is open and closed.
28. Construct a set A $\subseteq$ $\mathbb{R}$ that is open, closed, and perfect.
29. Construct a set A $\subseteq$ $\mathbb{R}$ that is connected.
30. Construct a set A $\subseteq$ $\mathbb{R}$ that is disconnected.

For all of the construction exercises, try to also draw a picture for the sets on a line. Try to visualise the property that you are attempting to exemplify.