ADVANCED CALCULUS (REAL ANALYSIS) I
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MORE DEFINITIONS, AXIOMS, AND THEOREMS § 12
WORKSHEET 2

Let $U = \mathbb{R}$ for all definitions and $A \subseteq \mathbb{R}$ (of course).

Definition 1: Let A = [a, b] such that $a \le b$. then A is called an interval.
Definition 2: Let A = (a, b) such that $a \le b$. then A is called a segment.
Definition 3: Let A = [a, b) such that $a \le b$. then A is called a half segment (or a half interval).
Definition 4: Let A = (a, b] such that $a \le b$. then A is called a half segment (or a half interval).

Definition 5: $\mathbb{R}$ can be denoted as $(-\infty, \infty)$ where $-\infty$ and ∞ are just symbols to mean on to the left ad infinitum and on to the right ad infinitum (meaning $-\infty$ and ∞ are *not* numbers).

Let $U = \mathbb{R}$ for all claims. Let a, b, c, d, x, y, and z denote real numbers. Let p, q, r, and s denote rational numbers. . Let j, k, m, and n denote natural numbers.

Prove or disprove:

1. Claim: Let $A \subseteq \mathbb{R}$, A is an interval. Then, $A \neq \emptyset$.

2. Claim: Let $A \subseteq \mathbb{R}$, A is a segment. Then, $A \neq \emptyset$.

3. Claim: Let $A \subseteq \mathbb{R}$, A is a half segment. Then, $A \neq \emptyset$.

4. Claim: $\exists\, A \subseteq \mathbb{R}, \ni$ A is both a segment and an interval.

5. Claim: Let $p \in \mathbb{Q} \wedge q \in \mathbb{Q} \ni p < q$. $\exists\, r \in \mathbb{Q} \ni p < r < q$.

6. Claim: Let a, b, $\wedge$ c be real. $a < b \Rightarrow a \cdot c < b \cdot c$.
7. Claim: If $b \neq 0$, then $\frac{a}{b} = \frac{ad}{bd}$.
8. Claim: $(a - b) - (c - d) = (a + d) - (b + c)$
9. Claim: $|a - b| = |c - d|$ if and only if $(a + d) = (b + c)$.
10. Claim: $x \le y \Rightarrow x^{-1} \ge y^{-1}$
11. Claim: $x + x = 0 \Rightarrow x = 0$.
12. Claim: $0 < 1$
13. Claim: $-1 < 0$
14. Claim: $x^2 \le y^2 \Rightarrow x \le y$
15. Claim: $x \le y \Rightarrow x^2 \le y^2$

16. <u>Claim</u>: $(-x)(-y) = xy$

17. <u>Claim</u>: $1 \leq y \Rightarrow 1 \geq y^{-1}$

18. <u>Claim</u>: $ad + bc \leq ac + bd$, if $a \leq b$ and $c \leq d$

19. <u>Claim</u>: Let $b \neq 0$ and $d \neq 0$, $0 < \frac{a}{b} < \frac{c}{d} \Rightarrow ad < bc$

20. <u>Claim</u>: Let $x \neq 0$, so $(x^{-1})^{-1} = x$.

21. <u>Claim</u>: $xy = 0$ iff $x = 0$ or $y = 0$.

22. <u>Claim</u>: Let $x \in \mathbb{R}$ It is the case that $-x$ is less than zero.

23. <u>Claim</u>: Let $p \in \mathbb{Q} \wedge f \in \mathbb{I}$

A. $(p + f) \in \mathbb{I}$

B. $(p \cdot f) \in \mathbb{I}$

C. $(p + f) \in \mathbb{Q}$

D. $(p \cdot f) \in \mathbb{Q}$

24. <u>Claim</u>: Let $x \in \mathbb{R}$ It is the case that $\frac{1}{x} \in \mathbb{R}$.

25. <u>Claim</u>: Let $x \in \mathbb{R}$ such that $x < 0$ It is the case that $\frac{1}{x} < 0$.

26. <u>Claim</u>: $x \leq y \Rightarrow x^3 \leq y^3$

27. <u>Claim</u>: $\exists\, p \in \mathbb{Q} \ni p > 0$ where $\nexists\, r \in \mathbb{Q} \ni 0 < r < p$.

28. <u>Claim</u>: $\exists\, m \in \mathbb{N} \ni m > 0$ where $\nexists\, n \in \mathbb{N} \ni 0 < n < m$.

28. <u>Claim</u>: $\exists\, m \in \mathbb{N} \ni m > 0$ where $\nexists\, n \in \mathbb{N} \ni 0 < n < m$.

29. <u>Claim</u>: $\pi = 3.14159265358979323846264338327950$

30. <u>Claim</u>: Let $x < 0 \wedge y < 0$. It is the case that $x \cdot y > 0$.

31. <u>Claim</u>: Let $x < 0 \wedge y < 0$. It is the case that $\frac{1}{xy} > 0$.