Worksheet I
Foundations and Calculus
Pre-requisite Concepts
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Definition: $\mathbb{R} = \{x|$ where x is a point on the line$\}$.

Let $U = \mathbb{R}$ for the line and let $U = \mathbb{R} \times \mathbb{R}$ for the plane.
Definition: $\mathbb{N}^* = \{0, 1, 2, 3, \ldots, (k-1), k, \ldots\}$
Definition: $\mathbb{N} = \{1, 2, 3, \ldots, (k-1), k, \ldots\}$
Definition: $\mathbb{N}_k^* = \{0, 1, 2, 3, \ldots, (k-1), k\}$
Definition: $\mathbb{N}_k = \{1, 2, 3, \ldots, (k-1), k\}$
Definition: Let $a \in \mathbb{R}$, $b \in \mathbb{R}$ such that $a < b$.
Then a **segment** is (a, b) and an **interval** is $[a, b]$.
A **half-segment or half-interval** is $(a, b]$ or $[a, b)$.

Prove means to rigorously prove a claim is so from the premises of the claim to the conclusion if the cliam is a universal claim or produce an example to prove an existential claim is true. Recall this also means that the claim is conditioned on a set of axioms. The axioms we assume for Math 351 are the axioms of set theory, the Peano axioms, and the axioms of the reals.

Show means to outine a proof of a claim is so from the premises of the claim to the conclusion. recall this also means that the claim is conditioned on a set of axioms but such might not be explicitly shown. Show is a weaker term than prove and some steps are usually skipped.

Demonstrate means to work out a probblem as was done in Math 171-273. Demonstrate is a weaker term than show or prove and many steps are usually skipped.

Disprove means to rigorously produce a counterexample to a UNIVERSAL CLAIM or prove the negation of an EXISTENTIAL CLAIM is true.

Assigment Due Monday, 31 August 2009

1. Download the the axioms of the reals handout and memorise the axioms (by name not number).

2. Read Sections 1 - 8 of the book, they are review and we assume students are familiar with the topics. Section 9 is optional so we won't bother with it. The course begins with §10. Read it in preparation for Monday 31 August 2009.

3. (Math 171). By definition of the derivative show that $g : \mathbb{R} \longrightarrow \mathbb{R} \ni g(x) = sin(x)$ has the property that $g\prime : \mathbb{R} \longrightarrow \mathbb{R} \ni g\prime(x) = cos(x)$

4. (Math 172). By definition of the area between curves (Riemann Sums) show that the region, R, defined as the area between the curves$f : \mathbb{R} \longrightarrow \mathbb{R} \quad \ni \quad f(x) = x^2, y = 0, x = -1, x = 2$ has the property that said area exists (and is therefore obviously finite). Find the area of R.

5. (Math 273). By the integral test show that $\sum_{i=1}^{\infty} i^{-2}$ exists (and is therefore obviously finite).

6. (Math 273). By the integral test show that $\sum_{i=1}^{\infty} i^{-\frac{1}{3}}$ does not exist (and is therefore obviously a divergent series).

7. (Math 273). Assume that $\sum_{i=1}^{\infty} i^{-2}$ exists (is convergent) show therefore it is the case based on this assumption that $\sum_{i=1}^{\infty} \frac{1}{i^2+6i+11}$ exists (and is therefore obviously a convergent series).

8. (Math 224) Prove or disprove the following claim.
Claim: Let $U =$ $\mathbb{R}$. $\forall n \in \mathbb{N}$ it is the case that $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$.

9. (Math 224) Prove or disprove the following claim.
Claim: Let U be a well defined universe. Let A, B, and C be sets. If $A \subseteq B$, then $A \cup C \subseteq B \cup C$.

10. (Math 224) Prove or disprove the following claim.
Claim: Let U be a well defined universe. Let A, B, and C be sets. If $A \subseteq B$, then $A \cap C \subseteq B \cap C$.

11. (Math 224) Prove or disprove the following claim.
Claim: Let U be a well defined universe, sets A, B, and C non-empty subsets of that universe, and $A \cap C \neq \emptyset$. It is the case that $(A \cap C \subseteq C - B)$ and $(A \cap B \subseteq C)$ if and only if $(A \cap B = \emptyset)$

12. (Math 224) Prove or disprove the following claim.
$h : \mathbb{N} \longrightarrow \mathbb{R}$ ∋ $f(x) = x^2$.
A. Claim: h is a well defined sequence.
B. Claim: h is injective from $\mathbb{N}$ to $\mathbb{R}$.
C. Claim: h is surjective from $\mathbb{N}$ to $\mathbb{R}$.

13. (Math 224) Prove or disprove the following claim.
$|\mathbb{Z}| = \aleph_0$.

Assume axioms of set theory, the Peano axioms, and the axioms of the reals for the rest of the questions.

14. Let $U = \mathbb{R}$ Prove or disprove
Lemma 1: $0 < 1$.

15. Let $U = \mathbb{R}$ Prove or disprove
Lemma 2: x ∈ $\mathbb{R}$ It is the case that $x \cdot 0 = 0$.

16. Let $U = \mathbb{R}$ Prove or disprove
Lemma 3: $(-1) \cdot (-1) = 1$.

17. Let $U = \mathbb{R}$ Prove or disprove
Lemma 4: $x \in \mathbb{R} \Rightarrow (-1) \cdot x = -x$

18. Let $U = \mathbb{R}$ Prove or disprove
Lemma 5: x ∈ $\mathbb{R}$, y ∈ $\mathbb{R} \Rightarrow x - y = x + (-y)$.