MATH 351 ADVANCED CALCULUS (REAL ANALYSIS) I
DR. MCLOUGHLIN MORE OF A DISCUSSION OF SEQUENCES §16 – §18
MORE DEFINITIONS, AXIOMS, LEMMAS, COROLLARIES, OR THEOREMS
HANDOUT 5 ¼

Let $U = \mathbb{R}$

Ms. Hutton's Claim: Let sequence $f(\mathrm{n}),\ f:\mathbb{N} \longrightarrow \mathbb{R}$ be well defined. Let us use the notation $\{f_{\mathrm{n}}\}_{\mathrm{n}=1}^{\infty}$. Let $\{f_{\mathrm{n_i}}\}_{\mathrm{i}\in\mathbb{N}}$ be a convergent subsequence of the sequence f,

$$f = \{(1, f(1)),(2, f(2)),(3, f(3)),(4, f(4)),(5, f(5)),\ldots(\mathrm{m}, f(\mathrm{m})),((\mathrm{m}+1), f(\mathrm{m}+1))\ldots\}$$

$$f_{\mathrm{n_i}} = \{(\mathrm{n}_1, f(\mathrm{n}_1)),(\mathrm{n}_2, f(\mathrm{n}_2)),(\mathrm{n}_3, f(\mathrm{n}_3)),\ldots,(\mathrm{n}_k, f(\mathrm{n}_k)),\ldots\}$$

Mr. Sheeler's Claim (1): Let sequence Let $\{\mathrm{b_n}\}_{\mathrm{n}=1}^{\infty}$ be a well defined sequence such that for $\{\mathrm{b_n}\}_{\mathrm{n}=1}^{\infty}$ $\lim\limits_{\mathrm{n}\to\infty}(\mathrm{b_n})$ exists (and is 0). Let $\{\mathrm{c_n}\}_{\mathrm{n}=1}^{\infty}$ be the well defined sequence such that $\mathrm{c_n} = (-1)^{\mathrm{n}}\cdot \mathrm{b_n}$.
It is the case that $\lim\limits_{\mathrm{n}\to\infty}(\mathrm{c_n}) = 0$.

Mr. Sheeler's Claim (2): Let sequence Let $\{\mathrm{b_n}\}_{\mathrm{n}=1}^{\infty}$ be a well defined sequence such that for $\{\mathrm{b_n}\}_{\mathrm{n}=1}^{\infty}$ $\lim\limits_{\mathrm{n}\to\infty}(\mathrm{b_n}) = \mathrm{m}$ where $\mathrm{m} \in \mathbb{R}$. Let $\{\mathrm{c_n}\}_{\mathrm{n}=1}^{\infty}$ be the well defined sequence such that $\mathrm{c_n} = (-1)^{\mathrm{n}}\cdot \mathrm{b_n}$.
It is the case that $\lim\limits_{\mathrm{n}\to\infty}(\mathrm{c_n}) = \mathrm{m}$.

Mr. Sheeler's Claim (3): Let sequence Let $\{\mathrm{b_n}\}_{\mathrm{n}=1}^{\infty}$ be a well defined sequence such that $\{\mathrm{b_n}\}_{\mathrm{n}=1}^{\infty}$ is bounded. Let $\{\mathrm{c_n}\}_{\mathrm{n}=1}^{\infty}$ be the well defined sequence such that $\mathrm{c_n} = (-1)^{\mathrm{n}}\cdot \mathrm{b_n}$.
It is the case that $\{\mathrm{c_n}\}_{\mathrm{n}=1}^{\infty}$ is bounded.

Mr. Connor claims he has an example to solve Exercise 18.01:

Exercise 18.01: Construct a sequence $f(\mathrm{n}),\ f:\mathbb{N} \longrightarrow \mathbb{R}$, that has the 'shrinking difference' property but is NOT a Cauchy sequence.