

Ms. Grace Hajjar
Exercise 11.3 (f)

Let $U = \mathbb{R}$.

Claim: $0 < 1$

Proof: Assume the premises.

We note $0 \neq 1$ from the axiom of the multiplicative identity.

1. Suppose $0 \geq 1$

2. $0 > 1$ or $0 = 1$

Case A: $0 = 1$

3A. $0 = 1$

4A. $0 \neq 1$

5A. $0 = 1$ and $0 \neq 1$

3A. Cases

4A. Axiom of existence of multiplicative inverse

5A. Adjunction. So we have a contradiction of the trichotomy law.

Case B: $1 < 0$

3B. $1 < 0$

4B. $1 + (-1) < 0 + (-1)$

5B. $0 < 0 + (-1)$

6B. $0 < -1$

7B. So, -1 is positive

8B. $0(-1) > (-1)(1)$

9B. $0 > (-1)(1)$

10B. $0 > -1$

11B. $-1 > 0$ and $0 > -1$

3B. Cases

4B. Axiom of preservation of order under addition

5B. Axiom of Additive inverse

6B. Axiom of Additive identity

7B. By the definition of positive Reals

8B. Axiom of preservation of order under a pos. mult.

9B. Lemma 2

10B. Axiom of identity of multiplication

11B. Adjunction. So we have a contradiction of the trichotomy law.

Thus, $0 < 1$

QED