

## Calculus III

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### HOMEWORK MACLAURIN OR TAYLOR REMAINDER

Here are some more fun problems to sink your teeth into.

Find the following:

1. Consider the function  $f : \mathbb{R} \rightarrow \mathbb{R} \ni f(x) = \sin x$   
 Create a Taylor series for  $f$  centered at  $x = \frac{\pi}{4}$  of degree 4.

2. Consider the function  $g : \mathbb{R} \rightarrow \mathbb{R} \ni g(x) = e^{x^2}$   
 Create a Taylor series for  $f$  centered at  $x = \ln 4$  of degree 4.

3. Consider the function  $g : \mathbb{R} \rightarrow \mathbb{R} \ni g(x) = e^{x^2}$   
 Create a Maclaurin series for  $f$  of degree 4.

4. Consider the Maclaurin series representation for  $f(x) = e^x$ .  
 In order to approximate  $f\left(\frac{1}{4}\right) = e^{.25}$ , what is the lowest degree Maclaurin polynomial one can use in order to approximate  $e^{.25}$  with error less than or equal to 0.001?

5. Find a power series representation for the function  $f : (0, \infty) \rightarrow \mathbb{R} \ni f(x) = \cos(\sqrt{x})$  (you may express your solution in non-sigma notation therefore, explicitly show the first 4 significant terms).

6. Let us consider  $\sum_{n=1}^{\infty} \left(\frac{-x}{3}\right)^n$  find its interval of convergence.

7. Let  $h : D \rightarrow \mathbb{R} \ni h(x) = \sum_{n=1}^{\infty} \left(\frac{-x}{3}\right)^n$  where  $D \subseteq \mathbb{R}$  and  $D$  is the set found in problem 6.  
 Find a power series representation for

$$\int (h(x)) dx$$