MATH 100
DR. MCLOUGHLIN'S HANDY DANDY
SYSTEMATIC GRAPHING GUIDE PART II

Here we discuss basic graphing techniques of functions from Part I.

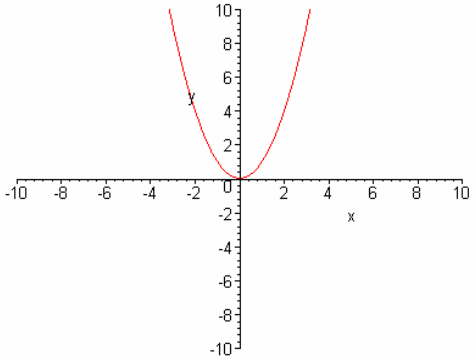
$y = A\,f(B(x - C)) + D$

A stretches or contracts and flips along the y – axis
B stretches or contracts and flips along the x – axis
C shifts left or right along the x – axis
D shifts down or up along the y – axis

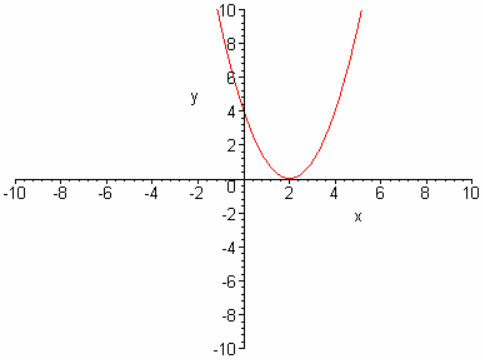
Example 1:
Systematically graph $y = 3(x - 2)^2 - 4$

Note the domain is $\mathbb{R}$ and the codomain is $\mathbb{R}$.

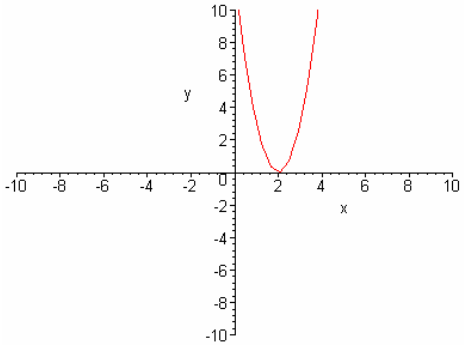
First graph $y = x^2$ notice the point (0, 0)



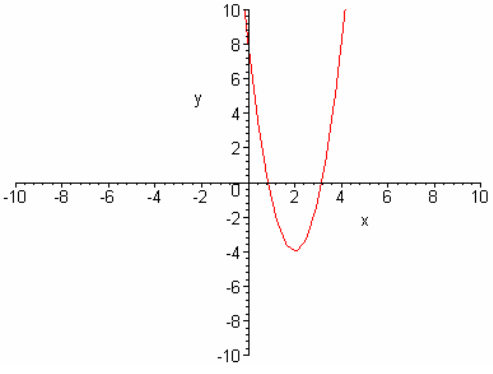
then $y = (x - 2)^2$ which shifts the graph right 2 (0, 0) moves to (2, 0)



then $y = 3(x - 2)^2$ which stretches the graph with respect to y (2, 0) doesn't move



and finally $y = 3(x - 2)^2 - 4$ which shifts the graph down 4 (2, 0) moves to (2, -4)



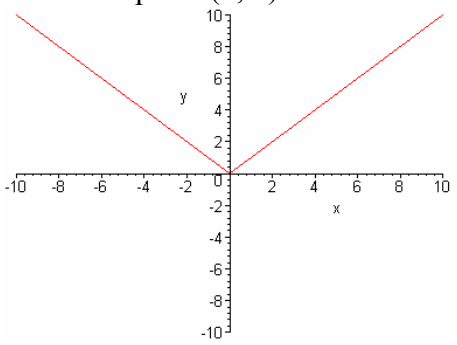
Now the domain is $\mathbb{R}$, the codomain is $\mathbb{R}$, and the range is $[-4, \infty)$

Example 2:

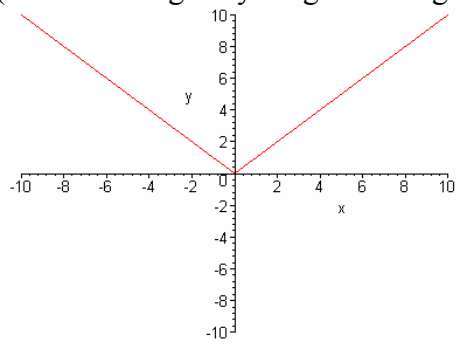
Systematically graph $y = \frac{1}{2}|-x+1| + \pi$

Note the domain is $\mathbb{R}$ and the codomain is $\mathbb{R}$

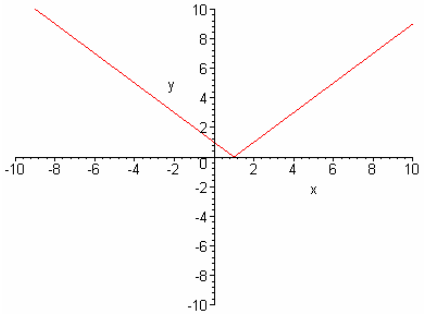
First graph y = | x |
notice the point (0, 0)



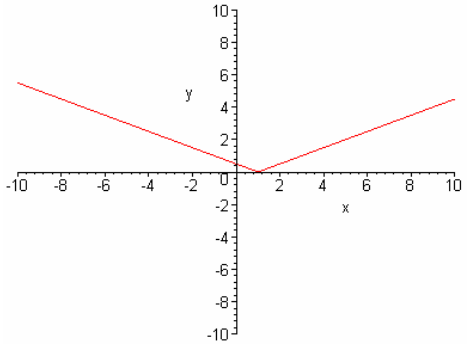
then y = | -x | which reflects it across the y-axis
(doesn't change anything including the point (0, 0))



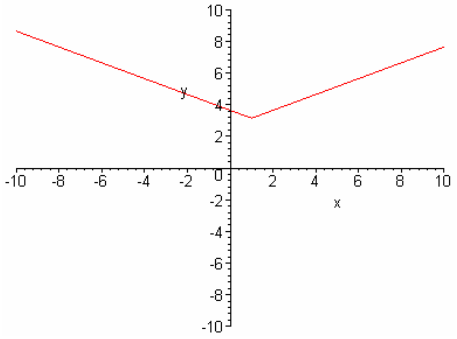
then $y = |-x+1|$ which shifts the graph right one
(0, 0) moves to (1, 0)



then $y = \frac{1}{2}|-x+1|$ which contracts the graph with respect to the y-axis ("squeezes it down some')
the point (1, 0) doesn't change



and finally, $y = \frac{1}{2}|-x+1| + \pi$ which shifts the graph up π.
notice the point (1, 0) moves up to (1, π).



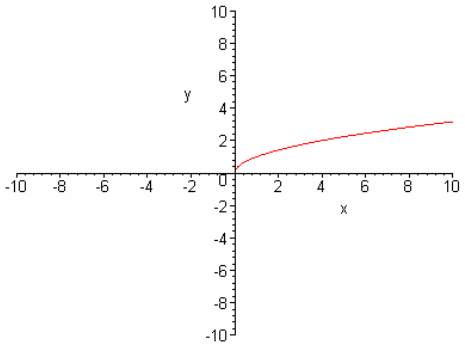
Note the domain is $\mathbb{R}$, the codomain is $\mathbb{R}$, and the range is $[\pi, \infty)$

Example 3:

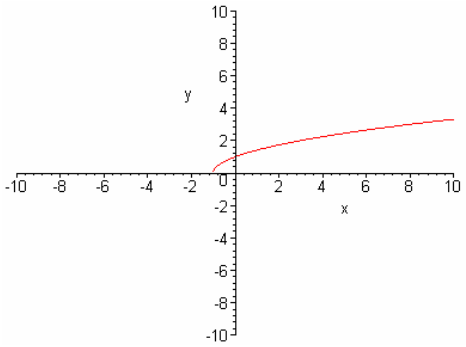
Systematically graph $y = \frac{-5}{3}\sqrt{x+1}$

Note the domain is [-1, ∞) and the codomain is $\mathbb{R}$

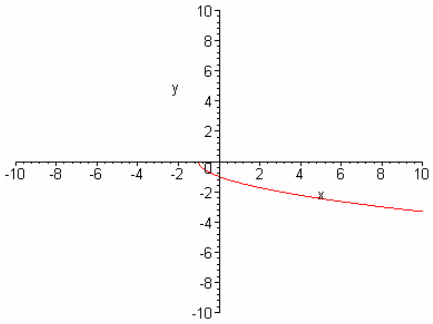
First graph $y = \sqrt{x}$ (the domain is [0, ∞)) notice the point (0, 0)



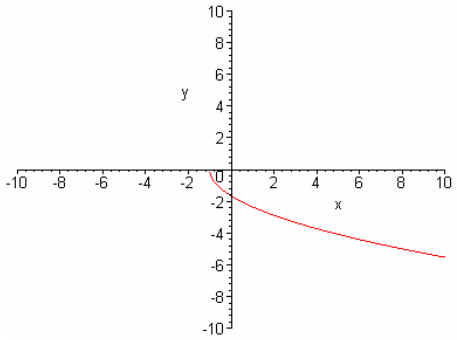
then $y = \sqrt{x+1}$ (the domain is now [-1, ∞)) which shifts it left 1 and (0, 0) moves to (-1, 0)



then $y = -\sqrt{x+1}$ which reflects it across the x-axis (-1, 0) doesn't move



then $y = \frac{-5}{3}\sqrt{x+1}$ which stretches the graph with respect to the y-axis (-1, 0) doesn't move



Note the domain is [-1, ∞), the codomain is $\mathbb{R}$, and the range is $(-\infty, 0]$.

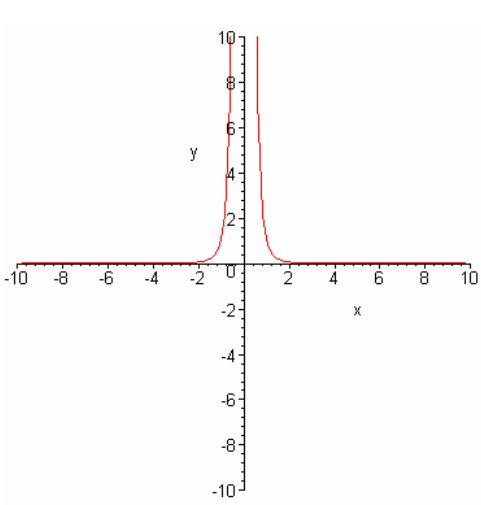
Example 4:

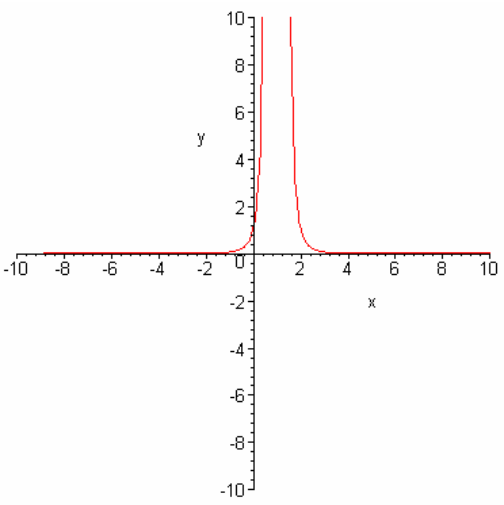
Systematically graph $y = 2(x-1)^{-4} + 3$

Note the domain is $(-\infty, 1) \cup (1, \infty)$ and the codomain is $\mathbb{R}$

First graph $y = x^{-4} = \dfrac{1}{x^4}$ then $y = (x-1)^{-4}$

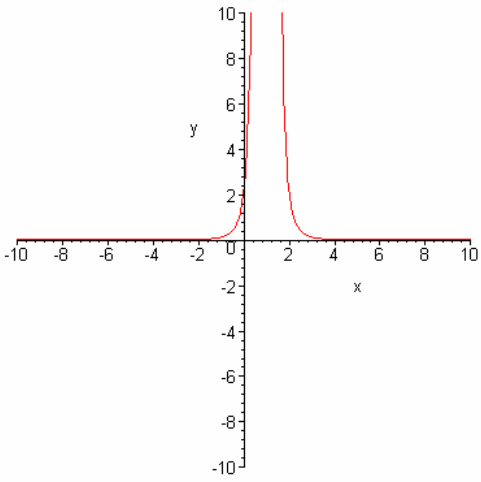
(the domain is $(-\infty, 0) \cup (0, \infty)$) (the domain is now $(-\infty, 1) \cup (1, \infty)$)
notice there isn't the point (0, 0) which shifts it right 1
on this graph – - there is a vertical asymptote x = 0 the vert. asy. x = 0 moves to x = 1.

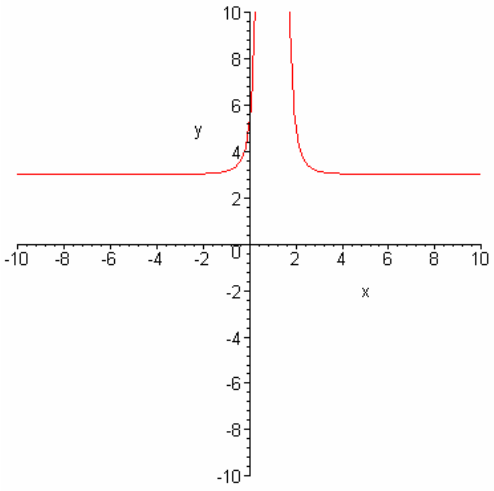




Now graph $y = 2(x-1)^{-4}$ and, finally, graph $y = 2(x-1)^{-4} + 3$

which stretches it with respect which shifts it up 3
to the y-axis (hardly noticeable) so the horizontal asymptote y = 0
the vert. asy. is x = 1 shits up to y = 3





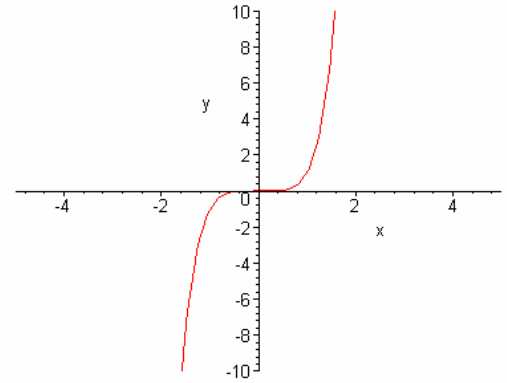
Note the domain is $(-\infty, 1) \cup (1, \infty)$, the codomain is $\mathbb{R}$, and the range is $(3, \infty)$.

Example 5:

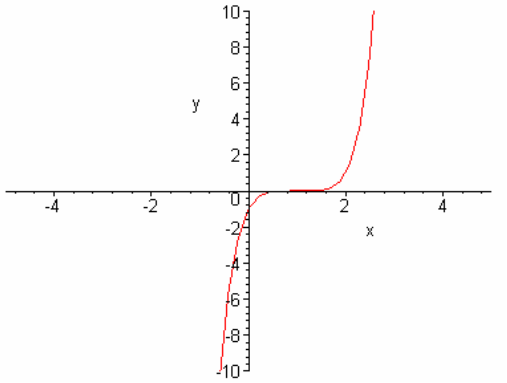
Systematically graph $y = 2(x-1)^5 + 3$

Note the domain is $\mathbb{R}$ and the codomain is $\mathbb{R}$

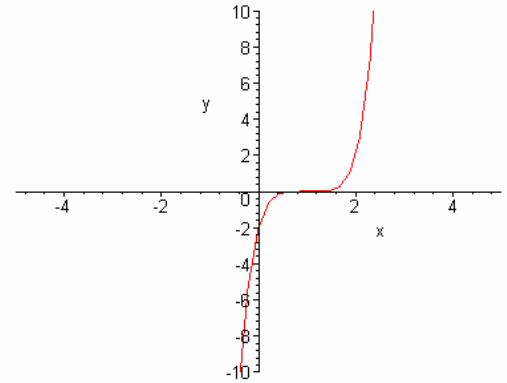
First graph $y = x^5$
notice the point (0, 0)



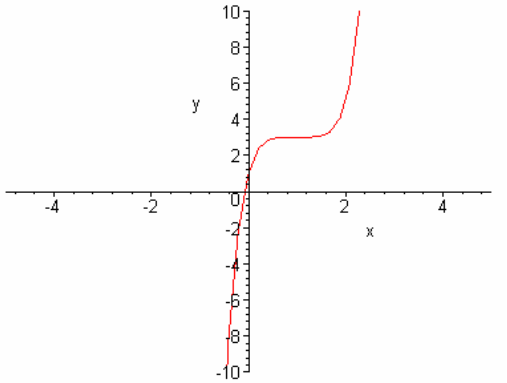
then $y = (x-1)^5$
which shifts it right 1
(0, 0) moves to (1, 0)



Now graph $y = 2(x-1)^5$
which stretches it with respect to the y-axis
(1, 0) doesn't move



and, finally, graph $y = 2(x-1)^5 + 3$
which shifts it up 3
(1, 0) moves to (1, 3)



Note the domain is $\mathbb{R}$, the codomain is $\mathbb{R}$, and the range is $\mathbb{R}$.

Exercises:

1. Systematically graph $y = 3(x-2)^2 + 5$, 'track' the point (0, 0),
note the domain and the codomain before graphing, and the range after finishing.

2. Systematically graph $y = \frac{1}{3}(x+2)^4 + 6$, 'track' the point (0, 0),
note the domain and the codomain before graphing, and the range after finishing.

3. Systematically graph $y = \frac{4}{3}\sqrt[3]{x-2} + 5$, 'track' the point (0, 0),
note the domain and the codomain before graphing, and the range after finishing.

4. Systematically graph $y = \frac{3}{4}\sqrt{x-3} - 1$, 'track' the point (0, 0),
note the domain and the codomain before graphing, and the range after finishing.

5. Systematically graph $y = (x+1)^{-1} + 2$ 'track' vertical and horizontal asymptotes,
note the domain and the codomain before graphing, and the range after finishing.

6. Systematically graph $y = 2(x+1)^{1/4} + 3$, 'track' the point (0, 0),
note the domain and the codomain before graphing, and the range after finishing.

7. Systematically graph $y = 4(x+1)^2 + 3$, 'track' the point (0, 0),
note the domain and the codomain before graphing, and the range after finishing.

8. Systematically graph $y = \frac{1}{4}(x-1)^2 + 3$, 'track' the point (0, 0),
note the domain and the codomain before graphing, and the range after finishing.

9. Systematically graph $y = -\frac{1}{4}(x+1)^2 - 3$, 'track' the point (0, 0),
note the domain and the codomain before graphing, and the range after finishing.

End, last revised 9 October 2004.