# WORKSHEET QUESTIONS 3
# MATH 021 FUNDAMENTALS OF MATHEMATICS
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## The Field Axioms of $\mathbb{R}$

Axiom 1 (closure of addition): $\forall$ x, y $\in \mathbb{R}$, x + y $\in \mathbb{R}$ and (x = w $\wedge$ y = v) $\Rightarrow$ (x + y = w + v)

Axiom 2 (commutative of addition): $\forall$ x, y $\in$ \\$\mathbb{R}$, x + y = y + x.

Axiom 3 (associative of addition): $\forall$ x, y, z $\in \mathbb{R}$, (x + y) + z = x + (y + z)

Axiom 4 (existence of identity of addition): $\exists$ a unique number 0 $\ni$ x + 0 = x $\forall$ x $\in \mathbb{R}$

Axiom 5 (existence of additive inverse): $\forall$ x $\in \mathbb{R}$ $\exists$ a unique number -x $\ni$ x + (-x) = 0

Axiom 6 (closure of multiplication): $\forall$ x, y $\in \mathbb{R}$, x $\cdot$ y $\in \mathbb{R}$ and (x = w $\wedge$ y = v) $\Rightarrow$ (x $\cdot$ y = w $\cdot$ v)

Axiom 7 (commutative of multiplication): $\forall$ x, y $\in \mathbb{R}$, x $\cdot$ y = y $\cdot$ x.

Axiom 8 (associative of multiplication): $\forall$ x, y, z $\in \mathbb{R}$, (x $\cdot$ y) $\cdot$ z = x $\cdot$ (y $\cdot$ z)

Axiom 9 (existence of identity of multiplication): $\exists$ a unique number 1 $\ni$ x $\cdot$ 1 = x $\forall$ x $\in \mathbb{R}$

(1 $\neq$ 0).

Axiom 10 (existence of multiplicative inverse): $\forall$ x $\in \mathbb{R}$ $\ni$ x $\neq$ 0 $\exists$ a unique number $x^{-1}$

$\ni$ x $\cdot$ ($x^{-1}$) = 1

Axiom 11 (distributive of multiplication over addition): $\forall$ x, y, z $\in \mathbb{R}$, x $\cdot$ (y + z) = (x $\cdot$ y) + (x$\cdot$ z)

# The Order Axioms of $\mathbb{R}$

Axiom 12 (trichotomy): $\forall$ x, y $\in$ $\mathbb{R}$, exactly one of the following relationships exists between x and y : x < y, x = y, $\vee$ x > y. [(x < y) exor (x = y) exor (x > y)]

Axiom 13 (transitive): $\forall$ x, y, z $\in$ $\mathbb{R}$, [ (x < y) $\wedge$ (y < z)] $\Rightarrow$ (x < z)

Axiom 14 (preservation of order under addition): $\forall$ x, y, z $\in$ $\mathbb{R}$ , (x < y) $\Rightarrow$ (x + z < y + z)

Axiom 15 (preservation of order for positive multiplier): $\forall$ x, y $\in$ $\mathbb{R}$, [(x < y) $\wedge$ (0 < z)] $\Rightarrow$ (x · z < y · z)

# The Completeness Axiom of $\mathbb{R}$

Axiom 16 (completeness): $\forall$ A $\subseteq$ $\mathbb{R}$ $\ni$ A is bounded above $\exists$ a number m which is the supremum of the set

1. Which of the axioms holds for $\mathbb{N}$ ?

2. Which of the axioms holds for $\mathbb{N}^*$ ?

3. Which of the axioms holds for $\mathbb{Z}$ ?

4. Create an example where one of the axioms ***does not*** holds for $\mathbb{N}$ .

5. Create an example where one of the axioms ***does not*** holds for $\mathbb{N}^*$ .

6. Create an example where one of the axioms ***does not*** holds for $\mathbb{Z}$ .