FUNDAMENTALS OF MATHEMATICS HANDOUT § 1.3
DR. MCLOUGHLIN

Truth Table for 'Not'

K	$\neg$K
T	F
F	T

Truth Table for 'And'

B	M	B $\wedge$ M
T	T	T
T	F	F
F	T	F
F	F	F

Truth Table for 'Or'

R	S	R $\vee$ S
T	T	T
T	F	T
F	T	T
F	F	F

Highest precedence   parentheses
not
or/and and/or (from left to right *only*)   Lowest precedence

Compound statement:

Truth Table for 'It is not the case P or M'

P	M	P $\vee$ M	$\neg$(P $\vee$ M)
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

Truth Table for $P \vee M \wedge L$

P	M	L	$P \vee M$	$(P \vee M) \wedge L$
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	F
F	F	F	F	F

Truth Table $P \vee (M \wedge L)$

P	M	L	$M \wedge L$	$P \vee (M \wedge L)$
T	T	T	T	T
T	T	F	F	T
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

Note the truth values obtained for the statement in this table are different than in the previous table.

Two statements are said to be **equivalent** (or synonymous, the same, or **logically equivalent**) only in the instance where the final column of the compleat truth tables are the same where the prime statements were assigned truth values in the exact same order. Suppose the statement X is equivalent to Y, we symbolise this as $X \equiv Y$. Two statements are said to **non- equivalent** in the instance where they are not equivalent (duh). Suppose the statement X is not equivalent to Y, we symbolise this as $X \not\equiv Y$. Finally two statements, X and Y, are said to be **logical opposites** in the instance where $X \equiv \neg Y$ (and $Y \equiv \neg X$).

As with any convention, when we wish to symbolise not a particular property in symbol form, we slash through the symbol to represent such a scenario.

Note, that the negation of the conjunction $\neg (p \wedge q)$ is equivalent to the disjunction $\neg p \vee \neg q$ (it is left as an exercise to verify). Also note, the parentheses are necessary, for the statement $\neg (p \wedge q)$ is not the same as $\neg p \wedge q$ (see truth tables 1.7 and 1.8)! This is problematic for some people for they might erroneously think the two are the same. For colloquial statements in English, this can be a problem, but for proper statements in logic it is not. Let us assign to the symbol p the simple statement "it is pouring" and assign to the symbol q the simple statement Natasha is quick. Now, the statement, it is not pouring and Natasha is quick is $\neg p \wedge q$; whereas, the statement, it is not the case that it is pouring and Natasha is quick is, $\neg (p \wedge q)$. Since $\neg (p \wedge q)$ is equivalent to $\neg p \vee \neg q$, for many it would be clearer if one simply said, "it is not pouring or Natasha is not quick."

Truth Table 7

P	Q	P ∨ Q	¬(P ∨ Q)
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

Truth Table 8

P	Q	¬P	¬P ∨ Q
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

§ 1.1 EXERCISES.

1. Construct a <u>compleat</u> truth table for the following statements, identify which statements are logically equivalent, which statements are logical opposites, and which are neither:

A. ¬p ∧ q
B. p ∧ ~q
C. p ∧ ~q
D. p ∧ q
E. ¬q ∨ p
F. ¬p ∨ q
G. p ∧ ~p
H. ~ (p ∨ q)
I. ~ (p ∧ q)
J. p ∨ ¬p

2. Construct a <u>compleat</u> truth table for the following statements:

A. p ∧ q ∨ p ∧ r
B. p ∧ (q ∨ p) ∧ r
C. p ∧ q ∨ (p ∧ r)
D. (p ∧ q) ∨ (p ∧ r)
E. ¬p ∧ q ∨ p ∧ r
F. p ∧ ~(q ∨ p) ∧ r
G. $p \wedge q \vee \overline{p \wedge r}$
H. (p ∧ ¬q) ∨ (p ∧ r)

CONDITIONAL AND BICONDITIONAL STATEMENTS

Truth Table for ‘if p, then q.’

p	q	p ⇒ q
T	T	T
T	F	F
F	T	T
F	F	T

Truth Table for ‘if p, then q; and, if q, then p.’’

p	q	p ⇒ q	q ⇒ p	(p ⇒ q) ∧ (q ⇒ p)
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Which can be simplified to:

Truth Table for ‘p if and only if q.’

p	Q	p ⇔ q
T	T	T
T	F	F
F	T	F
F	F	T

by replacing (p ⇒ q) ∧ (q ⇒ p) with the more parsimonious symbol p ⇔ q.

The biconditional P ⇔ Q or P ↔ Q translates to:

(1) P if and only if Q.
(2) P iff Q (this is just a shorthand for version 1).
(3) P is necessary and sufficient for Q.
(4) P and Q are logically equivalent.
(5) If P then Q and if Q then P.

Condition (4) establishes that two statements, P and Q, are logically equivalent in the instance where P ⇔ Q is true for note the only time when P ⇔ Q is true is when both P and Q are true or when both P and Q are false.

Highest precedence parentheses ( )
not ~, ¬, or ‾
and/or (from left to right) $\wedge$ $\vee$
conditional $\Rightarrow$ $\rightarrow$
biconditional $\Leftrightarrow$ $\leftrightarrow$
Lowest precedence

Various types of statements are of interest to mathematicians. A compound statement is a **tautology** when the compound statement is true for every true-false combination. A statement is a **fallacy** when the statement is true for at least one true-false combination and is false for at least one true-false combination. A **contradiction** is a compound statement that is false for every true-false combination for the prime statements.

We are interested in discerning what statement forms are tautologies, fallacies, or contradictions so that when we begin investigating argument forms, we can use tautologies or contradictions and avoid fallacies. A **tautological** argument is that which we attempt to construct when we prove an assertion.

Finally there is one other type of disjunction, the **exclusive disjunction**, which is symbolised as $P \veebar Q$. We reference this because there are times in mathematics when we do not want to have the possibility of both conditions being satisfied but wish to have one or the other exclusively satisfied.

P or Q, but not both.
P exclusive or Q.
P exor Q.

Truth Table 1.2.4

p	q	$p \veebar q$
T	T	F
T	F	T
F	T	T
F	F	F

Note that $P \veebar Q$ is the logical opposite of $P \Leftrightarrow Q$.

§ 1.2 EXERCISES.

1. Construct a <u>compleat</u> truth table for the following statements, identify which statements are logically equivalent, and which statements are logical opposites:

A. $\neg p \wedge q$
B. $p \Rightarrow q$
C. $\neg p \rightarrow \neg q$
D. $p \Leftarrow q$
E. $p \wedge \sim q$
F. $p \Leftrightarrow \neg q$
G. $p \wedge q$
H. $\neg q \vee p$
I. $\neg p \vee q$
J. $\neg p \rightarrow q$
K. $\neg q \rightarrow \neg p$
L. $q \rightarrow p$
M. $\neg q \Rightarrow q$
N. $p \wedge \sim p$
O. $p \Rightarrow p$
P. $\sim (p \vee q)$
Q. $\sim (p \wedge q)$
R. $p \rightarrow p$

2. Construct a <u>compleat</u> truth table for the following statements:

A. $(p \wedge \neg q) \vee (p \wedge r)$
B. $p \Rightarrow q \vee p \wedge r$
C. $p \wedge (q \rightarrow p) \wedge r$
D. $p \rightarrow q \vee (p \wedge r)$
E. $(p \wedge q) \Rightarrow (p \wedge r)$
F. $(p \wedge q) \Rightarrow p$
G. $(p \wedge q) \Rightarrow q$
H. $(p \vee q) \Rightarrow p$
I. $(p \vee q) \Rightarrow q$
J. $p \Rightarrow p \vee q$
K. $q \Rightarrow p \vee q$
L. $q \Rightarrow p \vee q \vee r$

3. Let p be "it is foggy" and q be "it is cold."
Translate each of the following into symbolic logic.
A. It is foggy or it is cold.
B. It is not foggy and it is not cold.
C. If it is foggy, then it is warm or hot.
D. The day being foggy is a necessary and sufficient condition for it to be cold.
E. It is false that it is both cold and foggy.