HANDOUT IX
PROBABILITY

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Recall from Set Theory that elements, a, are members of sets, A, and a set, A, is a subset of some well defined universe, *U*.
The universe is defined first then we can talk about sets.
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Set Theory	Probability Theory
Universe, *U*	Sample space, *S*
example $U = \{1, 2, 3, 4, 5\}$	example $S = \{1, 2, 3, 4, 5\}$
Sets, A, B, and C.	Events E, F, and G
$A = \{1, 2\}$	$E = \{1, 2\}$
$B = \{2, 3, 4\}$	$F = \{2, 3, 4\}$
$C = \{1, 4\}$	$G = \{1, 4\}$
Elements	Outcomes
$1 \in A$	$1 \in E$
$3 \notin A$	$1 \notin F$
$2, 3 \in B$	$2 \in E \wedge 2 \in F$
$5 \in C^C$	$3 \in E^C$

So, for probability theory we rename the universe as a sample space [the space from whence a sample may be chosen], an arbitrary set is called an event, and an arbitrary element of a set is an outcome.

The Axioms of Probability

Let S denote the sample space, E, E_i, F, etc. events and the notation Pr(•) the probability of whatever.

Axiom 1 — S is the space $\Rightarrow \Pr(S) = 1$

Axiom 2 — E is an event $\Rightarrow 0 \le \Pr(E) \le 1$

Axiom 3 — Let I be an index set. The collection $\{E_i\}_{i \in I}$ being mutually exclusive

$$\Rightarrow \Pr(\bigcup_{i \in I} E_i) = \sum_{i \in I} \Pr(E_i)$$

Corollary 1 — E is an event $\Rightarrow \Pr(E') = 1 - \Pr(E)$

Corollary 2 — E and F are events $\ni E \subseteq F \Rightarrow \Pr(E) \le \Pr(F)$

Corollary 3 — Let I be an index set. The collection $\{E_i\}_{i \in I}$ being mutually exclusive and exhaustive $\Rightarrow \Pr(\bigcup_{i \in I} E_i) = \sum_{i \in I} \Pr(E_i) = 1$

Theorem 1 — E and F are events. $\Pr(E \cup F) = \Pr(E) + \Pr(F) - \Pr(E \cap F)$