INTRODUCTION TO MATHEMATICS
HANDOUT § 3
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All prime statements P, Q, R, etc. are statements.

If P is a statement, then $\neg P$ is a statement.
If P and Q are statements, then $P \vee Q$ is a statement.
If P and Q are statements, then $P \wedge Q$ is a statement.
If P and Q are statements, then $P \Rightarrow Q$ is a statement.
If P and Q are statements, then $P \Leftrightarrow Q$ is a statement.

Idempotent Law (1) $P \vee P \equiv P$
Idempotent Law (2) $P \wedge P \equiv P$

Law of double negation $\neg(\neg P) \equiv P$ [same as $\neg(\neg P) \Leftrightarrow P$]

Or form of implication $P \Rightarrow Q \equiv \neg P \vee Q$
(when changing from implication to or form
reference or form; but when changing from or form to implication
reference implication form)

Contrapositive form of implication $P \Rightarrow Q \equiv \neg Q \Rightarrow \neg P$

De Morgan Law (1) $\neg P \vee \neg Q \equiv \neg(P \wedge Q)$
De Morgan Law (2) $\neg P \wedge \neg Q \equiv \neg(P \vee Q)$

Direct Proof Law $P \wedge R \Rightarrow Q \equiv (P \Rightarrow (R \Rightarrow Q))$

Indirect Proof Law $P \wedge \neg Q \Rightarrow$ always false $\equiv P \Rightarrow Q$

Law of the Excluded Middle (1) $P \wedge \neg P$ always false
Law of the Excluded Middle (2) $P \vee \neg P$ always true[1]

Commutative Law of "or" (1) $P \vee Q \equiv Q \vee P$
Commutative Law of "and" (2) $P \wedge Q \equiv Q \wedge P$

Associative Law of "or" (1) $P \vee (Q \vee R) \equiv (P \vee Q) \vee R \equiv P \vee Q \vee R$
Associative Law of "and" (2) $P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge R \equiv P \wedge Q \wedge R$

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[1] This is obvious, but let's take a closer look. Note $P \vee \neg P$ is logically equivalent to $\neg P \vee P$ which by the or form into implication form is $P \Rightarrow P$!!!! Now, if anyone (usually in the Social Sciences) says the Law of the Excluded Middle is an antiquated, outdated, or invalid law ask them, " 'If - - - -, then - - - -' [fill in the blank] is a fallacy?"

Distributive Law of "and over or" (1) $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$
Distributive Law of "or over and" (2) $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$

Law of Addition $P \Rightarrow P \vee Q$

Law of Simplification $P \wedge Q \Rightarrow P$

Modus Ponens $[(P \Rightarrow Q) \wedge P] \Rightarrow Q$

Modus Tollens $[(P \Rightarrow Q) \wedge \neg Q] \Rightarrow \neg P$

Disjunctive Syllogism $[(P \vee Q) \wedge \neg Q] \Rightarrow P$

Hypothetical Syllogism $[(P \Rightarrow Q) \wedge (Q \Rightarrow R)] \Rightarrow [P \Rightarrow R]$
(Transitivity)

Assume the hypothesis of the conclusion $(P \Rightarrow (R \Rightarrow Q)) \Rightarrow (P \wedge R) \Rightarrow Q$

FALLACIES:

Asserting the conclusion $[(P \Rightarrow Q) \wedge Q] \Rightarrow P$
(assuming the conclusion) (fallacy of the converse)
It is actually the case that $[(P \Rightarrow Q) \wedge Q] \not\Rightarrow P$ necessarily!

Asserting the premise $(P \Rightarrow Q) \Rightarrow P$
(assuming the premise must always be true)
It is actually the case that $(P \Rightarrow Q) \not\Rightarrow P$ necessarily!

Fallacy of the inverse $[(P \Rightarrow Q) \wedge \neg P] \Rightarrow \neg Q$
It is actually the case that $[(P \Rightarrow Q) \wedge \neg P] \not\Rightarrow \neg Q$ necessarily!

Fallacy (1) $P \vee Q \Rightarrow P$
(they reversed the law of addition)

Fallacy (2) $P \Rightarrow P \wedge Q$
(they reversed the law of simplification)

There are MANY more fallacies we could list; but, these are the most common. Avoid them!