2.3 CONCEPT QUESTIONS, page 103

1. a. There may be no solution, a unique solution, or infinitely many solutions.
   b. There may be no solution or infinitely many solutions.
2. No

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1. a. The system has one solution. b. The solution is (3, –1, 2).

2. a. The system has one solution. b. The solution is (3, –2, 1).

3. a. The system has one solution. b. The solution is (2, 4).

4. a. The system has one solution. b. The solution is (3, 1).

5. a. The system has infinitely many solutions.
   b. Letting $x_3 = t$, we see that the solutions are given by $(4 - t, -2, t)$, where t is a parameter.

6. a. The system has infinitely many solutions.
   b. Letting $x_3 = t$, we see that the solutions are given by $(3, -1 - t, t, 2)$, where t is a parameter.

7. a. The system has no solution. The last row contains all zeros to the left of the vertical line and a nonzero number (1) to its right.

8. a. The system has no solution. The last row contains all zeros to the left of the vertical line and a nonzero number (1) to its right.

9. a. The system has infinitely many solutions.
   b. Letting $x_4 = t$, we see that the solutions are given by $(2, -1, 2 - t, t)$, where t is a parameter.

10. a. The system has infinitely many solutions.
    b. Letting $x_1 = s$ and $x_4 = t$, we see that the solutions are given by $(s, 3 - t, 4 + 2t, t)$, where s and t are parameters.

11. a. The system has infinitely many solutions.

b. Letting $x_3 = s$ and $x_4 = t$, the solutions are given by $(2 - 3s, 1 + s, s, t)$, where s and t are parameters.

12. a. The system has infinitely many solutions.
b. Letting $x_3 = s$ and $x_4 = t$, we see that the solutions are given by $(4 - 3s + t, 2 + 2s - 3t, s, t)$, where s and t are parameters.

13. Using the Gauss-Jordan method, we have

$$\left[\begin{array}{rr|r} 2 & -1 & 3 \\ 1 & 2 & 4 \\ 2 & 3 & 7 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{rr|r} 1 & 2 & 4 \\ 2 & -1 & 3 \\ 2 & 3 & 7 \end{array}\right] \xrightarrow[R_3 - 2R_1]{R_2 - 2R_1} \left[\begin{array}{rr|r} 1 & 2 & 4 \\ 0 & -5 & -5 \\ 0 & -1 & -1 \end{array}\right] \xrightarrow{-\frac{1}{5}R_2}$$

$$\left[\begin{array}{rr|r} 1 & 2 & 4 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{array}\right] \xrightarrow[R_3 + R_2]{R_1 - 2R_2} \left[\begin{array}{rr|r} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{array}\right].$$ The solution is (2,1).

14. Using the Gauss-Jordan method, we have

$$\left[\begin{array}{rr|r} 1 & 2 & 3 \\ 2 & -3 & -8 \\ 1 & -4 & -9 \end{array}\right] \xrightarrow[R_3 - R_1]{R_2 - 2R_1} \left[\begin{array}{rr|r} 1 & 2 & 3 \\ 0 & -7 & -14 \\ 0 & -6 & -12 \end{array}\right] \xrightarrow{-\frac{1}{7}R_2} \left[\begin{array}{rr|r} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -6 & -12 \end{array}\right]$$

$$\xrightarrow[R_3 + 6R_2]{R_1 - 2R_2} \left[\begin{array}{rr|r} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right].$$ We conclude that the solution is (–1, 2).

15. Using the Gauss-Jordan method, we have

$$\left[\begin{array}{rr|r} 3 & -2 & -3 \\ 2 & 1 & 3 \\ 1 & -2 & -5 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{rr|r} 1 & -2 & -5 \\ 2 & 1 & 3 \\ 3 & -2 & -3 \end{array}\right] \xrightarrow[R_3 - 3R_1]{R_2 - 2R_1} \left[\begin{array}{rr|r} 1 & -2 & -5 \\ 0 & 5 & 13 \\ 0 & 4 & 12 \end{array}\right] \xrightarrow{\frac{1}{5}R_2}$$

$$\left[\begin{array}{cc|c} 1 & -2 & -5 \\ 0 & 1 & \frac{13}{5} \\ 0 & 4 & 12 \end{array}\right] \xrightarrow[R_3-4R_2]{R_1+2R_2} \left[\begin{array}{cc|c} 1 & 0 & \frac{1}{5} \\ 0 & 1 & \frac{13}{5} \\ 0 & 0 & \frac{8}{5} \end{array}\right].$$

Since the last row implies the 0 = 8/5, we conclude that the system of equations is inconsistent and has no solution.

16. Using the Gauss-Jordan method, we have

$$\left[\begin{array}{cc|c} 2 & 3 & 2 \\ 1 & 3 & -2 \\ 1 & -1 & 3 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{cc|c} 1 & 3 & -2 \\ 2 & 3 & 2 \\ 1 & -1 & 3 \end{array}\right] \xrightarrow[R_3-R_1]{R_2-2R_1} \left[\begin{array}{cc|c} 1 & 3 & -2 \\ 0 & -3 & 6 \\ 0 & -4 & 5 \end{array}\right] \xrightarrow{-\frac{1}{3}R_2}$$

$$\left[\begin{array}{cc|c} 1 & 3 & -2 \\ 0 & 1 & -2 \\ 0 & -4 & 5 \end{array}\right] \xrightarrow[R_3+4R_2]{R_1-3R_2} \left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & -2 \\ 0 & 0 & -3 \end{array}\right].$$

This last row implies that 0 = –3, which is impossible. We conclude that the system of equations is inconsistent and has no solution.

17. $$\left[\begin{array}{cc|c} 3 & -2 & 5 \\ -1 & 3 & -4 \\ 2 & -4 & 6 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{cc|c} -1 & 3 & -4 \\ 3 & -2 & 5 \\ 2 & -4 & 6 \end{array}\right] \xrightarrow{-R_1} \left[\begin{array}{cc|c} 1 & -3 & 4 \\ 3 & -2 & 5 \\ 2 & -4 & 6 \end{array}\right] \xrightarrow[R_3-2R_1]{R_2-3R_1}$$

$$\left[\begin{array}{cc|c} 1 & -3 & 4 \\ 0 & 7 & -7 \\ 0 & 2 & -2 \end{array}\right] \xrightarrow{\frac{1}{7}R_2} \left[\begin{array}{cc|c} 1 & -3 & 4 \\ 0 & 1 & -1 \\ 0 & 2 & -2 \end{array}\right] \xrightarrow[R_3-2R_2]{R_1+3R_2} \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{array}\right].$$

We conclude that the solution is (1,–1).

18. $$\left[\begin{array}{cc|c} 4 & 6 & 8 \\ 3 & -2 & -7 \\ 1 & 3 & 5 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{cc|c} 1 & 3 & 5 \\ 3 & -2 & -7 \\ 4 & 6 & 8 \end{array}\right] \xrightarrow[R_3-4R_1]{R_2-3R_1} \left[\begin{array}{cc|c} 1 & 3 & 5 \\ 0 & -11 & -22 \\ 0 & -6 & -12 \end{array}\right] \xrightarrow{-\frac{1}{11}R_2}$$

$$\left[\begin{array}{cc|c} 1 & 3 & 5 \\ 0 & 1 & 2 \\ 0 & -6 & -12 \end{array}\right] \xrightarrow[R_3+6R_2]{R_1-3R_2} \left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right].$$ We conclude that the solution is (–1,2).

19. $$\left[\begin{array}{cc|c} 1 & -2 & 2 \\ 7 & -14 & 14 \\ 3 & -6 & 6 \end{array}\right] \xrightarrow[R_3-3R_1]{R_2-7R_1} \left[\begin{array}{cc|c} 1 & -2 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right].$$

We conclude that the infinitely many solutions are given by $(2t + 2, t)$, where t is a parameter.

20. $$\left[\begin{array}{ccc|c} 1 & 2 & 1 & -2 \\ -2 & -3 & -1 & 1 \\ 2 & 4 & 2 & -4 \end{array}\right] \xrightarrow[R_3-2R_1]{R_2+2R_1} \left[\begin{array}{ccc|c} 1 & 2 & 1 & -2 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{R_1-2R_2} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 4 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

Let $x_3 = t$ and we find that $x_1 = 4 + t$ and $x_2 = -3 - t$. The infinitely many solutions are given by $(4 + t, -3 - t, t)$.

21. $$\left[\begin{array}{cc|c} 3 & 2 & 4 \\ -\frac{3}{2} & -1 & -2 \\ 6 & 4 & 8 \end{array}\right] \xrightarrow{\frac{1}{3}R_1} \left[\begin{array}{cc|c} 1 & \frac{2}{3} & \frac{4}{3} \\ -\frac{3}{2} & -1 & -2 \\ 6 & 4 & 8 \end{array}\right] \xrightarrow[R_3-6R_1]{R_2+\frac{3}{2}R_1} \left[\begin{array}{cc|c} 1 & \frac{2}{3} & \frac{4}{3} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right].$$

We conclude that the infinitely many solutions are given by $(\frac{4}{3} - \frac{2}{3}t, t)$, where t is a parameter.

22. $$\left[\begin{array}{ccc|c} 0 & 3 & 2 & 4 \\ 2 & -1 & -3 & 3 \\ 2 & 2 & -1 & 7 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 2 & -1 & -3 & 3 \\ 0 & 3 & 2 & 4 \\ 2 & 2 & -1 & 7 \end{array}\right] \xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & -\frac{3}{2} & \frac{3}{2} \\ 0 & 3 & 2 & 4 \\ 2 & 2 & -1 & 7 \end{array}\right]$$

$$\xrightarrow{R_3-2R_1} \left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & -\frac{3}{2} & \frac{3}{2} \\ 0 & 3 & 2 & 4 \\ 0 & 3 & 2 & 4 \end{array}\right] \xrightarrow{\frac{1}{3}R_2} \left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & -\frac{3}{2} & \frac{3}{2} \\ 0 & 1 & \frac{2}{3} & \frac{4}{3} \\ 0 & 3 & 2 & 4 \end{array}\right] \xrightarrow[R_3-3R_2]{R_1+\frac{1}{2}R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -\frac{7}{6} & \frac{13}{6} \\ 0 & 1 & \frac{2}{3} & \frac{4}{3} \\ 0 & 0 & 0 & 0 \end{array}\right].$$

Letting $z = t$, we see that the infinitely many solutions are given by $(\frac{13}{6} + \frac{7t}{6}, \frac{4}{3} - \frac{2t}{3}, t)$.

23. $$\left[\begin{array}{ccc|c} 2 & -1 & 1 & -4 \\ 3 & -\frac{3}{2} & \frac{3}{2} & -6 \\ -6 & 3 & -3 & 12 \end{array}\right] \xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & \frac{1}{2} & -2 \\ 3 & -\frac{3}{2} & \frac{3}{2} & -6 \\ -6 & 3 & -3 & 12 \end{array}\right] \xrightarrow[R_3+6R_1]{R_2-3R_1} \left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & \frac{1}{2} & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

We conclude that the infinitely many solutions are given by $(-2 + \frac{1}{2}s - \frac{1}{2}t, s, t)$ where s and t are parameters.

24. $$\left[\begin{array}{ccc|c} 1 & 1 & -2 & -3 \\ 2 & -1 & 3 & 7 \\ 1 & -2 & 5 & 0 \end{array}\right] \xrightarrow[R_3-R_1]{R_2-2R_1} \left[\begin{array}{ccc|c} 1 & 1 & -2 & -3 \\ 0 & -3 & 7 & 13 \\ 0 & -3 & 7 & 3 \end{array}\right] \xrightarrow{-\frac{1}{3}R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -2 & -3 \\ 0 & 1 & -\frac{7}{3} & -\frac{13}{3} \\ 0 & -3 & 7 & 3 \end{array}\right] \xrightarrow[R_3+3R_2]{R_1-R_2} \left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{3} & \frac{4}{3} \\ 0 & 1 & -\frac{7}{3} & -\frac{13}{3} \\ 0 & 0 & 0 & -10 \end{array}\right].$$

This last row implies that 0 = –10, which is impossible. We conclude that the system of equations is inconsistent and has no solution.

25. $$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 4 \\ 2 & 3 & -1 & 2 \\ 1 & 2 & -3 & -6 \end{array}\right] \xrightarrow[R_3-R_1]{R_2-2R_1} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 4 \\ 0 & 7 & -7 & -6 \\ 0 & 4 & -6 & -10 \end{array}\right] \xrightarrow{\frac{1}{7}R_2} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 4 \\ 0 & 1 & -1 & -\frac{6}{7} \\ 0 & 4 & -6 & -10 \end{array}\right]$$

$$\xrightarrow[R_3-4R_2]{R_1+2R_2} \left[\begin{array}{ccc|c} 1 & 0 & 1 & \frac{16}{7} \\ 0 & 1 & -1 & -\frac{6}{7} \\ 0 & 0 & -2 & -\frac{46}{7} \end{array}\right] \xrightarrow{-\frac{1}{2}R_3} \left[\begin{array}{ccc|c} 1 & 0 & 1 & \frac{16}{7} \\ 0 & 1 & -1 & -\frac{6}{7} \\ 0 & 0 & 1 & \frac{23}{7} \end{array}\right] \xrightarrow[R_2+R_3]{R_1-R_3}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & \frac{17}{7} \\ 0 & 0 & 1 & \frac{23}{7} \end{array}\right].$$

We conclude that the solution is $(-1, \frac{17}{7}, \frac{23}{7})$.

26. $$\left[\begin{array}{ccc|c} 1 & -2 & 1 & -3 \\ 2 & 1 & -2 & 2 \\ 1 & 3 & -3 & 5 \end{array}\right] \xrightarrow[R_3 - R_1]{R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & -2 & 1 & -3 \\ 0 & 5 & -4 & 8 \\ 0 & 5 & -4 & 8 \end{array}\right] \xrightarrow{\frac{1}{5}R_2} \left[\begin{array}{ccc|c} 1 & -2 & 1 & -3 \\ 0 & 1 & -\frac{4}{5} & \frac{8}{5} \\ 0 & 5 & -4 & 8 \end{array}\right] \xrightarrow[R_3 - 5R_2]{R_1 + 2R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -\frac{3}{5} & \frac{1}{5} \\ 0 & 1 & -\frac{4}{5} & \frac{8}{5} \\ 0 & 0 & 0 & 0 \end{array}\right].$$

We conclude that the infinitely many solutions to this system are $(\frac{1}{5} + \frac{3}{5}t,\ \frac{8}{5} + \frac{4}{5}t,\ t)$.

27. $$\left[\begin{array}{ccc|c} 4 & 1 & -1 & 4 \\ 8 & 2 & -2 & 8 \end{array}\right] \xrightarrow{\frac{1}{4}R_1} \left[\begin{array}{ccc|c} 1 & \frac{1}{4} & \frac{1}{4} & 1 \\ 8 & 2 & -2 & 8 \end{array}\right] \xrightarrow{R_2 - 8R_1} \left[\begin{array}{ccc|c} 1 & \frac{1}{4} & -\frac{1}{4} & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

We conclude that the infinitely many solutions are given by $(1 - \frac{1}{4}s + \frac{1}{4}t,\ s,\ t)$, where s and t are parameters.

28. $$\left[\begin{array}{ccc|c} 1 & 2 & 4 & 2 \\ 1 & 1 & 2 & 1 \end{array}\right] \xrightarrow{R_2 - R_1} \left[\begin{array}{ccc|c} 1 & 2 & 4 & 2 \\ 0 & -1 & -2 & -1 \end{array}\right] \xrightarrow{-R_2} \left[\begin{array}{ccc|c} 1 & 2 & 4 & 2 \\ 0 & 1 & 2 & 1 \end{array}\right]$$

$$\xrightarrow{R_1 - 2R_2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 \end{array}\right].$$

We conclude that the infinitely many solutions are given by $(0,\ 1 - 2t,\ t)$, where t is a parameter.

29. $$\left[\begin{array}{ccc|c} 2 & 1 & -3 & 1 \\ 1 & -1 & 2 & 1 \\ 5 & -2 & 3 & 6 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 2 & 1 & -3 & 1 \\ 5 & -2 & 3 & 6 \end{array}\right] \xrightarrow[R_3 - 5R_1]{R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 0 & 3 & -7 & -1 \\ 0 & 3 & -7 & 1 \end{array}\right] \xrightarrow{\frac{1}{3}R_2}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 0 & 1 & -\frac{7}{3} & -\frac{1}{3} \\ 0 & 3 & -7 & 1 \end{array}\right] \xrightarrow[R_3 - 3R_2]{R_1 + R_2} \left[\begin{array}{ccc|c} 1 & 0 & -\frac{1}{3} & \frac{2}{3} \\ 0 & 1 & -\frac{7}{3} & -\frac{1}{3} \\ 0 & 0 & 0 & 2 \end{array}\right].$$

This last row implies that $0 = 2$, which is impossible. We conclude that the system of equations is inconsistent and has no solution.

30. $$\left[\begin{array}{ccc|c} 3 & -9 & 6 & -12 \\ 1 & -3 & 2 & -4 \\ 2 & -6 & 4 & 8 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 1 & -3 & 2 & -4 \\ 3 & -9 & 6 & -12 \\ 2 & -6 & 4 & 8 \end{array}\right] \xrightarrow{\substack{R_2-3R_1 \\ R_3-2R_1}} \left[\begin{array}{ccc|c} 1 & -3 & 2 & -4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 16 \end{array}\right].$$

This last row implies that 0 = 16, which is impossible. We conclude that the system of equations is inconsistent and has no solution.

31. $$\left[\begin{array}{ccc|c} 1 & 2 & -1 & -4 \\ 2 & 1 & 1 & 7 \\ 1 & 3 & 2 & 7 \\ 1 & -3 & 1 & 9 \end{array}\right] \xrightarrow{\substack{R_2-2R_1 \\ R_3-R_1 \\ R_4-R_1}} \left[\begin{array}{ccc|c} 1 & 2 & -1 & -4 \\ 0 & -3 & 3 & 15 \\ 0 & 1 & 3 & 11 \\ 0 & -5 & 2 & 13 \end{array}\right] \xrightarrow{-\frac{1}{3}R_2} \left[\begin{array}{ccc|c} 1 & 2 & -1 & -4 \\ 0 & 1 & -1 & -5 \\ 0 & 1 & 3 & 11 \\ 0 & -5 & 2 & 13 \end{array}\right]$$

$$\xrightarrow{\substack{R_1-2R_2 \\ R_3-R_2 \\ R_4+5R_2}} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 6 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & 4 & 16 \\ 0 & 0 & -3 & -12 \end{array}\right] \xrightarrow{\frac{1}{4}R_3} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 6 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & -3 & -12 \end{array}\right] \xrightarrow{\substack{R_1-R_3 \\ R_2+R_3 \\ R_4+3R_3}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

We conclude that the solution of the system is (2,–1,4).

32. $$\left[\begin{array}{ccc|c} 3 & -2 & 1 & 4 \\ 1 & 3 & -4 & -3 \\ 2 & -3 & 5 & 7 \\ 1 & -8 & 9 & 10 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 1 & 3 & -4 & -3 \\ 3 & -2 & 1 & 4 \\ 2 & -3 & 5 & 7 \\ 1 & -8 & 9 & 10 \end{array}\right] \xrightarrow{\substack{R_2-3R_1 \\ R_3-2R_1 \\ R_4-R_1}} \left[\begin{array}{ccc|c} 1 & 3 & -4 & -3 \\ 0 & -11 & 13 & 13 \\ 0 & -9 & 13 & 13 \\ 0 & -11 & 13 & 13 \end{array}\right]$$

$$\xrightarrow{-\frac{1}{11}R_2} \left[\begin{array}{ccc|c} 1 & 3 & -4 & -3 \\ 0 & 1 & -\frac{13}{11} & -\frac{13}{11} \\ 0 & -9 & 13 & 13 \\ 0 & -11 & 13 & 13 \end{array}\right] \xrightarrow{\substack{R_1-3R_2 \\ R_3+9R_2 \\ R_4+11R_2}} \left[\begin{array}{ccc|c} 1 & 0 & -\frac{5}{11} & \frac{6}{11} \\ 0 & 1 & -\frac{13}{11} & -\frac{13}{11} \\ 0 & 0 & \frac{26}{11} & \frac{26}{11} \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{\frac{11}{26}R_3}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -\frac{5}{11} & \frac{6}{11} \\ 0 & 1 & -\frac{13}{11} & -\frac{13}{11} \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{\substack{R_1+\frac{5}{11}R_3 \\ R_2+\frac{13}{11}R_3}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

We conclude that the solution of the system is (1,0,1).

33. Let x, y, and z represent the number of compact, mid-sized, and full-size cars, respectively, to be purchased. Then the problem can be solved by solving the system

$$\begin{aligned} x + y + z &= 60 \\ 12000x + 19200y + 26400z &= 1008000 \end{aligned}.$$

Using the Gauss-Jordan method, we have

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 60 \\ 12000 & 19200 & 26400 & 1008000 \end{array}\right] \xrightarrow{R_2 - 12,000R_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 60 \\ 0 & 7200 & 14400 & 288000 \end{array}\right]$$

$$\xrightarrow{\frac{1}{7200}R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 60 \\ 0 & 1 & 2 & 40 \end{array}\right] \xrightarrow{R_1 - R_2} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 20 \\ 0 & 1 & 2 & 40 \end{array}\right]$$

and we conclude that the solution is $(20 + z, 40 - 2z, z)$. Letting $z = 5$, we see that one possible solution is (25,30,5); that is Hartman should buy 25 compact, 30 mid-sized cars, and 5 full-sized cars. Letting $z = 10$, we see that another possible solution is (30,20,10); that is, 30 compact cars, 20 mid-sized cars, and 10 full-sized cars.

34. Let x, y, and z denote the number of ounces of Food *I*, Food *II*, and Food *III*, respectively, that the dietician includes in the meal. Then the problem can be solved by solving the system

$$\begin{aligned} 400x + 1200y + 800z &= 8800 \\ 110x + 570y + 340z &= 3380 \\ 90x + 30y + 60z &= 1020. \end{aligned}$$

Using the Gauss-Jordan method, we have

$$\left[\begin{array}{ccc|c} 400 & 1200 & 800 & 8800 \\ 110 & 570 & 340 & 3380 \\ 90 & 30 & 60 & 1020 \end{array}\right] \xrightarrow{\frac{1}{400}R_1} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 22 \\ 110 & 570 & 340 & 3380 \\ 90 & 30 & 60 & 1020 \end{array}\right] \xrightarrow[R_3 - 90R_1]{R_2 - 110R_1}$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 22 \\ 0 & 240 & 120 & 960 \\ 0 & -240 & -120 & -960 \end{array}\right] \xrightarrow{\frac{1}{240}R_2} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 22 \\ 0 & 1 & \frac{1}{2} & 4 \\ 0 & -240 & -120 & -960 \end{array}\right] \xrightarrow[R_3 + 240R_2]{R_1 - 3R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & 10 \\ 0 & 1 & \frac{1}{2} & 4 \\ 0 & 0 & 0 & 0 \end{array}\right],$$

and we conclude that the solution is $(10-\frac{1}{2}z, 4-\frac{1}{2}z, z)$. Letting $z = 2$, we see that one possible solution is a meal prepared with 9 ounces of Food I, 3 ounces of Food II, and 2 ounces of Food III. Another possible solution is obtained by letting $z = 4$, In this case, 8 ounces of Food I, 2 ounces of Food II, and 4 ounces of Food III would be used to prepare the meal.

35. Let x, y, and z denote the number of ounces of Food I, Food II, and Food III, respectively, that the dietician includes in the meal. Then the problem can be solved by solving the system

$$\begin{aligned} 400x + 1200y + 800z &= 8800 \\ 110x + 570y + 340z &= 2160 \\ 90x + 30y + 60z &= 1020. \end{aligned}$$

Using the Gauss-Jordan method, we have

$$\left[\begin{array}{ccc|c} 400 & 1200 & 800 & 8800 \\ 110 & 570 & 340 & 2160 \\ 90 & 30 & 60 & 1020 \end{array}\right] \xrightarrow{\frac{1}{400}R_1} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 22 \\ 110 & 570 & 340 & 2160 \\ 90 & 30 & 60 & 1020 \end{array}\right] \xrightarrow[R_3-90R_1]{R_2-110R_1}$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 22 \\ 0 & 240 & 120 & -260 \\ 0 & -240 & -120 & -960 \end{array}\right] \xrightarrow{\frac{1}{240}R_2} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 22 \\ 0 & 1 & \frac{1}{2} & -\frac{13}{12} \\ 0 & -240 & -120 & -960 \end{array}\right] \xrightarrow[R_3+240R_2]{R_1-3R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & \frac{101}{4} \\ 0 & 1 & \frac{1}{2} & -\frac{13}{12} \\ 0 & 0 & 0 & -1220 \end{array}\right].$$

This last row implies that $0 = -1220$, which is impossible. We conclude that the system of equations is inconsistent and has no solution--that is, the dietician cannot prepare a meal from these foods and meet the given requirements.

36. Let x, y, and z denote the amount of money invested in stocks, bonds, and a money-market account, respectively. Then the problem can be solved by solving the system

$$\begin{aligned} x + y + z &= 100{,}000 \\ 12x + 8y + 4z &= 1{,}000{,}000 \\ x - y - 3z &= 0. \end{aligned}$$

Using the Gauss-Jordan method, we have

$$\left[\begin{array}{rrr|r} 1 & 1 & 1 & 100000 \\ 12 & 8 & 4 & 1000000 \\ 1 & -1 & -3 & 0 \end{array}\right] \xrightarrow[R_3-R_1]{R_2-12R_1} \left[\begin{array}{rrr|r} 1 & 1 & 1 & 100000 \\ 0 & -4 & -8 & -200000 \\ 0 & -2 & -4 & -100000 \end{array}\right] \xrightarrow{-\frac{1}{4}R_2}$$

$$\left[\begin{array}{rrr|r} 1 & 1 & 1 & 100000 \\ 0 & 1 & 2 & 50000 \\ 0 & -2 & -4 & -100000 \end{array}\right] \xrightarrow[R_3+2R_2]{R_1-R_2} \left[\begin{array}{rrr|r} 1 & 0 & -1 & 50000 \\ 0 & 1 & 2 & 50000 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

We conclude that the solution is $(50000 + z, 50000 - 2z, z)$. Therefore, one possible solution for the Garcias is to invest \$10,000 in a money-market account, \$60,000 in stocks and \$30,000 in bonds. Another possible solution is for the Garcias to invest \$20,000 in a money-market account, \$70,000 in stocks and \$10,000 in bonds.

37. a.

$$\begin{aligned} x_1 - x_2 &= 200 \\ x_1 - x_5 &= 100 \\ -x_2 + x_3 + x_6 &= 600 \\ -x_3 + x_4 &= 200 \\ x_4 - x_5 + x_6 &= 700. \end{aligned}$$

b.

$$\left[\begin{array}{rrrrrr|r} 1 & -1 & 0 & 0 & 0 & 0 & 200 \\ 1 & 0 & 0 & 0 & -1 & 0 & 100 \\ 0 & -1 & 1 & 0 & 0 & 1 & 600 \\ 0 & 0 & -1 & 1 & 0 & 0 & 200 \\ 0 & 0 & 0 & 1 & -1 & 1 & 700 \end{array}\right] \xrightarrow{R_2-R_1} \left[\begin{array}{rrrrrr|r} 1 & -1 & 0 & 0 & 0 & 0 & 200 \\ 0 & 1 & 0 & 0 & -1 & 0 & -100 \\ 0 & -1 & 1 & 0 & 0 & 1 & 600 \\ 0 & 0 & -1 & 1 & 0 & 0 & 200 \\ 0 & 0 & 0 & 1 & -1 & 1 & 700 \end{array}\right] \xrightarrow[R_1+R_2]{R_3+R_2}$$

$$\left[\begin{array}{rrrrrr|r} 1 & 0 & 0 & 0 & -1 & 0 & 100 \\ 0 & 1 & 0 & 0 & -1 & 0 & -100 \\ 0 & 0 & 1 & 0 & -1 & 1 & 500 \\ 0 & 0 & -1 & 1 & 0 & 0 & 200 \\ 0 & 0 & 0 & 1 & -1 & 1 & 700 \end{array}\right] \xrightarrow{R_4+R_3} \left[\begin{array}{rrrrrr|r} 1 & 0 & 0 & 0 & -1 & 0 & 100 \\ 0 & 1 & 0 & 0 & -1 & 0 & -100 \\ 0 & 0 & 1 & 0 & -1 & 1 & 500 \\ 0 & 0 & 0 & 1 & -1 & 1 & 700 \\ 0 & 0 & 0 & 1 & -1 & 1 & 700 \end{array}\right] \xrightarrow{R_5-R_4}$$

$$\left[\begin{array}{cccccc|c} 1 & 0 & 0 & 0 & -1 & 0 & 100 \\ 0 & 1 & 0 & 0 & 1 & 0 & -100 \\ 0 & 0 & 1 & 0 & -1 & 1 & 500 \\ 0 & 0 & 0 & 1 & -1 & 1 & 700 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right].$$

We conclude that the solution is

$(s+100,\ s-100,\ s-t+500,\ s-t+700,\ s,\ t)$.

Taking $s = 150$ and $t = 50$, we see that one possible traffic pattern is

(250, 50, 600, 800, 150, 50).

Similarly, taking $s = 200$, and $t = 100$, we see that another possible traffic pattern is

(300, 100, 600, 800, 200, 100).

c. Taking $t = 0$ and $s = 200$, we see that another possible traffic pattern is

(300, 100, 700, 900, 200, 0).

38. a.

$$\begin{aligned} x_1 \quad\quad\quad\quad\quad\quad + x_6 \quad\quad &= 1700 \\ x_1 - x_2 \quad\quad\quad\quad\quad\quad + x_7 &= 700 \\ x_2 - x_3 \quad\quad\quad\quad\quad &= 300 \\ -x_3 + x_4 \quad\quad\quad\quad &= 400 \\ -x_4 + x_5 \quad + x_7 &= 700 \\ x_5 + x_6 \quad\quad &= 1800. \end{aligned}$$

b.

$$\left[\begin{array}{ccccccc|c} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1700 \\ 1 & -1 & 0 & 0 & 0 & 0 & 1 & 700 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 300 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 400 \\ 0 & 0 & 0 & -1 & 1 & 0 & 1 & 700 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1800 \end{array}\right] \xrightarrow{R_2 - R_1} \left[\begin{array}{ccccccc|c} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1700 \\ 0 & -1 & 0 & 0 & 0 & -1 & 1 & -1000 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 300 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 400 \\ 0 & 0 & 0 & -1 & 1 & 0 & 1 & 700 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1800 \end{array}\right]$$

$$\xrightarrow{-R_2} \left[\begin{array}{ccccccc|c} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1700 \\ 0 & 1 & 0 & 0 & 0 & 1 & -1 & 1000 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 300 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 400 \\ 0 & 0 & 0 & -1 & 1 & 0 & 1 & 700 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1800 \end{array}\right] \xrightarrow{R_3 - R_2}$$

$$\left[\begin{array}{ccccccc|c} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1700 \\ 0 & 1 & 0 & 0 & 0 & 1 & -1 & 1000 \\ 0 & 0 & -1 & 0 & 0 & -1 & 1 & -700 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 400 \\ 0 & 0 & 0 & -1 & 1 & 0 & 1 & 700 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1800 \end{array}\right] \xrightarrow{-R_3} \left[\begin{array}{ccccccc|c} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1700 \\ 0 & 1 & 0 & 0 & 0 & 1 & -1 & 1000 \\ 0 & 0 & 1 & 0 & 0 & 1 & -1 & 700 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 400 \\ 0 & 0 & 0 & -1 & 1 & 0 & 1 & 700 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1800 \end{array}\right]$$

$$\xrightarrow{R_4+R_3} \left[\begin{array}{ccccccc|c} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1700 \\ 0 & 1 & 0 & 0 & 0 & 1 & -1 & 1000 \\ 0 & 0 & 1 & 0 & 0 & 1 & -1 & 700 \\ 0 & 0 & 0 & 1 & 0 & 1 & -1 & 1100 \\ 0 & 0 & 0 & -1 & 1 & 0 & 1 & 700 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1800 \end{array}\right] \xrightarrow{R_5+R_4}$$

$$\left[\begin{array}{ccccccc|c} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1700 \\ 0 & 1 & 0 & 0 & 0 & 1 & -1 & 1000 \\ 0 & 0 & 1 & 0 & 0 & 1 & -1 & 700 \\ 0 & 0 & 0 & 1 & 0 & 1 & -1 & 1100 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1800 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1800 \end{array}\right] \xrightarrow{R_6-R_5} \left[\begin{array}{ccccccc|c} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1700 \\ 0 & 1 & 0 & 0 & 0 & 1 & -1 & 1000 \\ 0 & 0 & 1 & 0 & 0 & 1 & -1 & 700 \\ 0 & 0 & 0 & 1 & 0 & 1 & -1 & 1100 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1800 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

We conclude that the solution of the system is

$$(1700-s, 1000-s+t, 700-s+t, 1100-s+t, 1800-s, s, t)$$

Two possible traffic patterns are (900, 1000, 700, 1100, 1000, 800, 800) and (1000, 1100, 800, 1200, 1100, 700, 800).

c. x_6 must have at least 300 cars/hour.

39. We solve the given system by using the Gauss-Jordan method. We have

$$\left[\begin{array}{cc|c} 2 & 3 & 2 \\ 1 & 4 & 6 \\ 5 & k & 2 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{cc|c} 1 & 4 & 6 \\ 2 & 3 & 2 \\ 5 & k & 2 \end{array}\right] \xrightarrow[R_3-5R_1]{R_2-2R_1} \left[\begin{array}{cc|c} 1 & 4 & 6 \\ 0 & -5 & -10 \\ 0 & k-20 & -28 \end{array}\right] \xrightarrow{-\frac{1}{5}R_2}$$

$$\left[\begin{array}{cc|c} 1 & 4 & 6 \\ 0 & 1 & 2 \\ 0 & k-20 & -28 \end{array}\right] \xrightarrow[R_3+aR_2]{R_1-4R_2} \left[\begin{array}{cc|c} 1 & 0 & -2 \\ 0 & 1 & 2 \\ 0 & k+a-20 & -28+2a \end{array}\right]$$

From the last matrix, we see that the system has a solution if and only if $x = -2$, $y = 2$, and

$$-28 + 2a = 0, \text{ or } a = 14$$

and $$k + a - 20 = k - 6 = 0, \text{ or } k = 6.$$

(All the entries in the last row of the matrix must be equal to zero.)

40. We solve the given system by using the Gauss-Jordan method. We have

$$\left[\begin{array}{rrr|r} 3 & -2 & 4 & 12 \\ -9 & 6 & -12 & k \end{array}\right] \xrightarrow{\frac{1}{3}R_1} \left[\begin{array}{rrr|r} 1 & -\frac{2}{3} & \frac{4}{3} & 4 \\ -9 & 6 & -12 & k \end{array}\right] \xrightarrow{R_2+9R_1}$$

$$\left[\begin{array}{rrr|r} 1 & -\frac{2}{3} & \frac{4}{3} & 4 \\ 0 & 0 & 0 & k+36 \end{array}\right].$$

Since this system has a solution only if the last row has all zero entries, we see that $k = -36$. We conclude that the solution is $(4 + \frac{2}{3}y - \frac{4}{3}z, y, z)$ and $k = -36$.

41. False. Such a system cannot have a unique solution.

42. True. See Theorem 1, page 99.

USING TECHNOLOGY EXERCISES 2.3, page 107

1. $(1+t, 2+t, t)$; t, a parameter 2. No solution

3. $(-\frac{17}{7} + \frac{6}{7}t, 3 - t, -\frac{18}{7} + \frac{1}{7}t, t)$ 4. $(2t, 1 - t, 1 + t, t)$ 5. No solution

6. $(2.5810 - 0.0406t, 3.2462 - 3.8226t, 1.6619 - 2.1548t, 0.2942 + 0.3531t, t)$