2.2 CONCEPT QUESTIONS, page 90

1. a. The two systems are equivalent to each other if they have precisely the same solutions.
   b. (i) Interchanging row i with row j (ii) replacing row j by c times row j
   (iii) replacing row i with the sum of row i and a times row j.
2. a. The coefficient matrix is the $m \times n$ matrix made up of the coefficients of the system of m linear equations in the n variables. The augmented matrix for the system is obtained from the matrix of coefficients by adjoining the column of constants to it. A column in the coefficient matrix is a unit column if one of the entries in the column is a 1 and all other entries are zero.
   b. To pivot about an element means to transform the column containing that element into a unit colum with a 1 in the position previously occupied by that element.
3. a. It lies below any other row having nonzero entries.
   b. It is a 1.
   c. The leading 1 in the lower row lies to the right of the leading 1 in the upper row.
   d. They are all zero.

EXERCISES 2.2, page 91

1. $\left[\begin{array}{rr|r} 2 & -3 & 7 \\ 3 & 1 & 4 \end{array}\right]$ 2. $\left[\begin{array}{rrr|r} 3 & 7 & -8 & 5 \\ 1 & 0 & 3 & -2 \\ 4 & -3 & 0 & 7 \end{array}\right]$ 3. $\left[\begin{array}{rrr|r} 0 & -1 & 2 & 6 \\ 2 & 2 & -8 & 7 \\ 0 & 3 & 4 & 0 \end{array}\right]$

4. $\left[\begin{array}{rrr|r} 3 & 2 & 0 & 0 \\ 1 & -1 & 2 & 4 \\ 0 & 2 & -3 & 5 \end{array}\right]$

5. $\begin{aligned} 3x + 2y &= -4 \\ x - y &= 5 \end{aligned}$ 6. $\begin{aligned} 3y + 2z &= 4 \\ x - y - 2z &= -3 \\ 4x + 3z &= 2. \end{aligned}$ 7. $\begin{aligned} x + 3y + 2z &= 4 \\ 2x &= 5 \\ 3x - 3y + 2z &= 6 \end{aligned}$

8. $\begin{aligned} 2x + 3y + z &= 6 \\ 4x + 3y + 2z &= 5 \end{aligned}$

9. Yes. Conditions 1-4 are satisfied (see page 85 of the text).

10. Yes. Conditions 1-4 are satisfied.

11. No. Condition 3 is violated. The first nonzero entry in the second row does not lie to the right of the first nonzero entry 1 in row 1.

12. Yes. Conditions 1-4 are satisfied.

13. Yes. Conditions 1-4 are satisfied.

14. No. Condition 2 is violated. The first nonzero entry in the third row is not a 1.

15. No. Condition 2 and consequently condition 4 are not satisfied.The first nonzero entry in the last row is not a 1 and the column containing that entry does not have zeros elsewhere.

16. Yes. Conditions 1-4 are satisfied.

17. No. Condition 1 is violated. The first row consists entirely of zeros and it lies above row 2.

18. No. Conditions 2 and 3 are violated. Row 4 shouuld lie above row 3, and the entry in row 3, column 4, should be a 1, not a 4.

19. $$\left[\begin{array}{cc|c} \boxed{2} & 4 & 8 \\ 3 & 1 & 2 \end{array}\right] \xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{cc|c} 1 & 2 & 4 \\ \boxed{3} & 1 & 2 \end{array}\right] \xrightarrow{R_2-3R_1} \left[\begin{array}{cc|c} 1 & 2 & 4 \\ 0 & -5 & -10 \end{array}\right]$$

20. $$\left[\begin{array}{cc|c} 3 & 2 & 6 \\ \boxed{4} & 2 & 5 \end{array}\right] \xrightarrow{\frac{1}{4}R_2} \left[\begin{array}{cc|c} 3 & 2 & 6 \\ \boxed{1} & \frac{1}{2} & \frac{5}{4} \end{array}\right] \xrightarrow{R_1-3R_2} \left[\begin{array}{cc|c} 0 & \frac{1}{2} & \frac{9}{4} \\ 1 & \frac{1}{2} & \frac{5}{4} \end{array}\right]$$

21. $$\left[\begin{array}{cc|c} \boxed{-1} & 2 & 3 \\ 6 & 4 & 2 \end{array}\right] \xrightarrow{-R_1} \left[\begin{array}{cc|c} 1 & -2 & -3 \\ 6 & 4 & 2 \end{array}\right] \xrightarrow{R_2-6R_1} \left[\begin{array}{cc|c} 1 & -2 & -3 \\ 0 & 16 & 20 \end{array}\right]$$

22. $$\left[\begin{array}{cc|c} \boxed{1} & 3 & 4 \\ 2 & 4 & 6 \end{array}\right] \xrightarrow{R_2-2R_1} \left[\begin{array}{cc|c} 1 & 3 & 4 \\ 0 & -2 & -2 \end{array}\right]$$

23. $$\left[\begin{array}{ccc|c} \boxed{2} & 4 & 6 & 12 \\ 2 & 3 & 1 & 5 \\ 3 & -1 & 2 & 4 \end{array}\right] \xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 2 & 3 & 1 & 5 \\ 3 & -1 & 2 & 4 \end{array}\right] \xrightarrow[R_3-3R_1]{R_2-2R_1} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & -1 & -5 & -7 \\ 0 & -7 & -7 & -14 \end{array}\right]$$

24. $$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 4 \\ \boxed{2} & 4 & 8 & 6 \\ -1 & 2 & 3 & 4 \end{array}\right] \xrightarrow{\frac{1}{2}R_2} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 4 \\ 1 & 2 & 4 & 3 \\ -1 & 2 & 3 & 4 \end{array}\right] \xrightarrow[R_3+R_2]{R_1-R_2} \left[\begin{array}{ccc|c} 0 & 1 & -2 & 1 \\ 1 & 2 & 4 & 3 \\ 0 & 4 & 7 & 7 \end{array}\right]$$

25. $$\left[\begin{array}{ccc|c} 0 & 1 & 3 & 4 \\ 2 & 4 & \boxed{1} & 3 \\ 5 & 6 & 2 & -4 \end{array}\right] \xrightarrow[R_3-2R_2]{R_1-3R_2} \left[\begin{array}{ccc|c} -6 & -11 & 0 & -5 \\ 2 & 4 & 1 & 3 \\ 1 & -2 & 0 & -10 \end{array}\right]$$

26. $$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & \boxed{-3} & 3 & 2 \\ 0 & 4 & -1 & 3 \end{array}\right] \xrightarrow{-\frac{1}{3}R_2} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & 1 & -1 & -\frac{2}{3} \\ 0 & 4 & -1 & 3 \end{array}\right] \xrightarrow[R_3-4R_2]{R_1-2R_2} \left[\begin{array}{ccc|c} 1 & 0 & 5 & \frac{19}{3} \\ 0 & 1 & -1 & -\frac{2}{3} \\ 0 & 0 & 3 & \frac{17}{3} \end{array}\right]$$

27. $$\left[\begin{array}{cc|c} 3 & 9 & 6 \\ 2 & 1 & 4 \end{array}\right] \xrightarrow{\frac{1}{3}R_1} \left[\begin{array}{cc|c} 1 & 3 & 2 \\ 2 & 1 & 4 \end{array}\right] \xrightarrow{R_2-2R_1} \left[\begin{array}{cc|c} 1 & 3 & 2 \\ 0 & -5 & 0 \end{array}\right] \xrightarrow{-\frac{1}{5}R_2}$$

$$\left[\begin{array}{cc|c} 1 & 3 & 2 \\ 0 & 1 & 0 \end{array}\right] \xrightarrow{R_1-3R_2} \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 0 \end{array}\right]$$

28. $$\left[\begin{array}{cc|c} 1 & 2 & 1 \\ 2 & 3 & -1 \end{array}\right] \xrightarrow{R_2-2R_1} \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & -1 & -3 \end{array}\right] \xrightarrow{-R_2} \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & 1 & 3 \end{array}\right] \xrightarrow{R_1-2R_2} \left[\begin{array}{cc|c} 1 & 0 & -5 \\ 0 & 1 & 3 \end{array}\right]$$

29. $$\left[\begin{array}{ccc|c} 1 & 3 & 1 & 3 \\ 3 & 8 & 3 & 7 \\ 2 & -3 & 1 & -10 \end{array}\right] \xrightarrow[R_3-2R_1]{R_2-3R_1} \left[\begin{array}{ccc|c} 1 & 3 & 1 & 3 \\ 0 & -1 & 0 & -2 \\ 0 & -9 & -1 & -16 \end{array}\right] \xrightarrow{-R_2} \left[\begin{array}{ccc|c} 1 & 3 & 1 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & -9 & -1 & -16 \end{array}\right]$$

$$\xrightarrow[R_3+9R_2]{R_1-3R_2}\left[\begin{array}{ccc|c}1&0&1&-3\\0&1&0&2\\0&0&-1&2\end{array}\right]\xrightarrow[-R_3]{R_1+R_3}\left[\begin{array}{ccc|c}1&0&0&-1\\0&1&0&2\\0&0&1&-2\end{array}\right]$$

30. $$\left[\begin{array}{ccc|c}0&1&3&-4\\1&2&1&7\\1&-2&0&1\end{array}\right]\xleftrightarrow{R_1\leftrightarrow R_2}\left[\begin{array}{ccc|c}1&2&1&7\\0&1&3&-4\\1&-2&0&1\end{array}\right]\xrightarrow{R_3-R_1}\left[\begin{array}{ccc|c}1&2&1&7\\0&1&3&-4\\0&-4&-1&-6\end{array}\right]$$

$$\xrightarrow[R_3+4R_2]{R_1+\frac{1}{2}R_3}\left[\begin{array}{ccc|c}1&0&\frac{1}{2}&4\\0&1&3&-4\\0&0&11&-22\end{array}\right]\xrightarrow{\frac{1}{11}R_3}\left[\begin{array}{ccc|c}1&0&\frac{1}{2}&4\\0&1&3&-4\\0&0&1&-2\end{array}\right]\xrightarrow[R_2-3R_3]{R_1-\frac{1}{2}R_3}\left[\begin{array}{ccc|c}1&0&0&5\\0&1&0&2\\0&0&1&-2\end{array}\right]$$

31. The augmented matrix is equivalent to the system of linear equations

$$\begin{aligned}3x+9y&=6\\2x+y&=4\end{aligned}$$

The ordered pair (2,0) is the solution to the system.

32. The augmented matrix is equivalent to the system of linear equations

$$\begin{aligned}x+2y&=1\\2x+3y&=-1\end{aligned}$$

and $x = -5$ and $y = 3$ is the solution to the system.

33. The augmented matrix is equivalent to the system of linear equations

$$\begin{aligned}x+3y+z&=3\\3x+8y+3z&=7\\2x-3y+z&=-10\end{aligned}$$

Reading off the solution from the last augmented matrix,

$$\left[\begin{array}{ccc|c}1&0&0&-1\\0&1&0&2\\0&0&1&-2\end{array}\right],$$

which is in row-reduced form, we have $x = -1$, $y = 2$, and $z = -2$.

34. The augmented matrix is equivalent to the system of linear equations

$$\begin{aligned} y + 3z &= -4 \\ x + 2y + z &= 7 \\ x - 2y &= 1 \end{aligned}$$

$x = 5$, $y = 2$, and $z = -2$ is the solution to the system.

35. Using the Gauss-Jordan method, we have

$$\left[\begin{array}{cc|c} 1 & -2 & 8 \\ 3 & 4 & 4 \end{array}\right] \xrightarrow{R_2 - 3R_1} \left[\begin{array}{cc|c} 1 & -2 & 8 \\ 0 & 10 & -20 \end{array}\right] \xrightarrow{\frac{1}{10}R_2} \left[\begin{array}{cc|c} 1 & -2 & 8 \\ 0 & 1 & -2 \end{array}\right] \xrightarrow{R_1 + 2R_2} \left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & -2 \end{array}\right]$$

The solution is (4,–2).

36. Using the Gauss-Jordan method, we have

$$\left[\begin{array}{cc|c} 3 & 1 & 1 \\ -7 & -2 & -1 \end{array}\right] \xrightarrow{\frac{1}{3}R_1} \left[\begin{array}{cc|c} 1 & \frac{1}{3} & \frac{1}{3} \\ -7 & -2 & -1 \end{array}\right] \xrightarrow{R_2 + 7R_1} \left[\begin{array}{cc|c} 1 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} & \frac{4}{3} \end{array}\right] \xrightarrow{3R_2}$$

$$\left[\begin{array}{cc|c} 1 & \frac{1}{3} & \frac{1}{3} \\ 0 & 1 & 4 \end{array}\right] \xrightarrow{R_1 - \frac{1}{3}R_2} \left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 4 \end{array}\right]$$

The solution is (–1,4).

37. Using the Gauss-Jordan method, we have

$$\left[\begin{array}{cc|c} 2 & -3 & -8 \\ 4 & 1 & -2 \end{array}\right] \xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{cc|c} 1 & -\frac{3}{2} & -4 \\ 4 & 1 & -2 \end{array}\right] \xrightarrow{R_2 - 4R_1} \left[\begin{array}{cc|c} 1 & -\frac{3}{2} & -4 \\ 0 & 7 & 14 \end{array}\right] \xrightarrow{\frac{1}{7}R_2}$$

$$\left[\begin{array}{cc|c} 1 & -\frac{3}{2} & -4 \\ 0 & 1 & 2 \end{array}\right] \xrightarrow{R_1 + \frac{3}{2}R_2} \left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 2 \end{array}\right].$$

The solution is (–1,2).

38. Using the Gauss-Jordan method, we have

$$\left[\begin{array}{cc|c} 5 & 3 & 9 \\ -2 & 1 & -8 \end{array}\right] \xrightarrow{\frac{1}{5}R_1} \left[\begin{array}{cc|c} 1 & \frac{3}{5} & \frac{9}{5} \\ -2 & 1 & -8 \end{array}\right] \xrightarrow{R_2+2R_1} \left[\begin{array}{cc|c} 1 & \frac{3}{5} & \frac{9}{5} \\ 0 & \frac{11}{5} & -\frac{22}{5} \end{array}\right] \xrightarrow{\frac{5}{11}R_2}$$

$$\left[\begin{array}{cc|c} 1 & \frac{3}{5} & \frac{9}{5} \\ 0 & 1 & -2 \end{array}\right] \xrightarrow{R_1-\frac{3}{5}R_2} \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & -2 \end{array}\right].$$ The solution is (3,–2).

39. Using the Gauss-Jordan method, we have

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & -1 & 1 & 1 \\ 1 & 1 & -2 & 2 \end{array}\right] \xrightarrow[R_3-R_1]{R_2-2R_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -3 & -1 & 1 \\ 0 & 0 & -3 & 2 \end{array}\right] \xrightarrow{-\frac{1}{3}R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & \frac{1}{3} & -\frac{1}{3} \\ 0 & 0 & -3 & 2 \end{array}\right] \xrightarrow{R_1-R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{1}{3} & -\frac{1}{3} \\ 0 & 0 & -3 & 2 \end{array}\right] \xrightarrow{-\frac{1}{3}R_3} \left[\begin{array}{ccc|c} 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{1}{3} & -\frac{1}{3} \\ 0 & 0 & 1 & -\frac{2}{3} \end{array}\right] \xrightarrow[R_2-\frac{1}{3}R_3]{R_1-\frac{2}{3}R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{7}{9} \\ 0 & 1 & 0 & -\frac{1}{9} \\ 0 & 0 & 1 & -\frac{2}{3} \end{array}\right].$$

The solution is $\left(\frac{7}{9}, -\frac{1}{9}, -\frac{2}{3}\right)$.

40. $$\left[\begin{array}{ccc|c} 2 & 1 & -2 & 4 \\ 1 & 3 & -1 & -3 \\ 3 & 4 & -1 & 7 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 1 & 3 & -1 & -3 \\ 2 & 1 & -2 & 4 \\ 3 & 4 & -1 & 7 \end{array}\right] \xrightarrow[R_3-3R_1]{R_2-2R_1} \left[\begin{array}{ccc|c} 1 & 3 & -1 & -3 \\ 0 & -5 & 0 & 10 \\ 0 & -5 & 2 & 16 \end{array}\right] \xrightarrow{-\frac{1}{5}R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & -3 \\ 0 & 1 & 0 & -2 \\ 0 & -5 & 2 & 16 \end{array}\right] \xrightarrow[R_3+5R_2]{R_1-3R_2} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 2 & 6 \end{array}\right] \xrightarrow{\frac{1}{2}R_3} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right] \xrightarrow{R_1+R_3}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right].$$ The solution is (6,–2,3).

41. $$\left[\begin{array}{ccc|c} 2 & 2 & 1 & 9 \\ 1 & 0 & 1 & 4 \\ 0 & 4 & -3 & 17 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 2 & 2 & 1 & 9 \\ 0 & 4 & -3 & 17 \end{array}\right] \xrightarrow{R_2-2R_1} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 2 & -1 & 1 \\ 0 & 4 & -3 & 17 \end{array}\right] \xrightarrow{\frac{1}{2}R_2}$$

$$\left[\begin{array}{ccc|c}1 & 0 & 1 & 4\\ 0 & 1 & -\frac{1}{2} & \frac{1}{2}\\ 0 & 4 & -3 & 17\end{array}\right] \xrightarrow{R_3-4R_2} \left[\begin{array}{ccc|c}1 & 0 & 1 & 4\\ 0 & 1 & -\frac{1}{2} & \frac{1}{2}\\ 0 & 0 & -1 & 15\end{array}\right] \xrightarrow{-R_3} \left[\begin{array}{ccc|c}1 & 0 & 1 & 4\\ 0 & 1 & -\frac{1}{2} & \frac{1}{2}\\ 0 & 0 & 1 & -15\end{array}\right] \xrightarrow[R_2+\frac{1}{2}R_3]{R_1-R_3}$$

$$\left[\begin{array}{ccc|c}1 & 0 & 0 & 19\\ 0 & 1 & 0 & -7\\ 0 & 0 & 1 & -15\end{array}\right].$$ The solution is (19,–7,–15).

42. $$\left[\begin{array}{ccc|c}2 & 3 & -2 & 10\\ 3 & -2 & 2 & 0\\ 4 & -1 & 3 & -1\end{array}\right] \xrightarrow{R_1\leftrightarrow R_2} \left[\begin{array}{ccc|c}3 & -2 & 2 & 0\\ 2 & 3 & -2 & 10\\ 4 & -1 & 3 & -1\end{array}\right] \xrightarrow{R_1-R_2} \left[\begin{array}{ccc|c}1 & -5 & 4 & -10\\ 2 & 3 & -2 & 10\\ 4 & -1 & 3 & -1\end{array}\right]$$

$$\xrightarrow[R_3-4R_1]{R_2-2R_1} \left[\begin{array}{ccc|c}1 & -5 & 4 & -10\\ 0 & 13 & -10 & 30\\ 0 & 19 & -13 & 39\end{array}\right] \xrightarrow{\frac{1}{13}R_2} \left[\begin{array}{ccc|c}1 & -5 & 4 & -10\\ 0 & 1 & -\frac{10}{13} & \frac{30}{13}\\ 0 & 19 & -13 & 39\end{array}\right] \xrightarrow[R_3-19R_2]{R_1+5R_2}$$

$$\left[\begin{array}{ccc|c}1 & 0 & \frac{2}{13} & \frac{20}{13}\\ 0 & 1 & -\frac{10}{13} & \frac{30}{13}\\ 0 & 0 & \frac{21}{13} & -\frac{63}{13}\end{array}\right] \xrightarrow{\frac{13}{21}R_3} \left[\begin{array}{ccc|c}1 & 0 & \frac{2}{13} & \frac{20}{13}\\ 0 & 1 & -\frac{10}{13} & \frac{30}{13}\\ 0 & 0 & 1 & -3\end{array}\right] \xrightarrow[R_2+\frac{10}{13}R_3]{R_1-\frac{2}{13}R_3} \left[\begin{array}{ccc|c}1 & 0 & 0 & 2\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & -3\end{array}\right]$$

The solution is (2,0,–3).

43. $$\left[\begin{array}{ccc|c}0 & -1 & 1 & 2\\ 4 & -3 & 2 & 16\\ 3 & 2 & 1 & 11\end{array}\right] \xrightarrow{R_1\leftrightarrow R_2} \left[\begin{array}{ccc|c}4 & -3 & 2 & 16\\ 0 & -1 & 1 & 2\\ 3 & 2 & 1 & 11\end{array}\right] \xrightarrow{R_1-R_3} \left[\begin{array}{ccc|c}1 & -5 & 1 & 5\\ 0 & -1 & 1 & 2\\ 3 & 2 & 1 & 11\end{array}\right]$$

$$\xrightarrow[R_3-3R_1]{-R_2} \left[\begin{array}{ccc|c}1 & -5 & 1 & 5\\ 0 & 1 & -1 & -2\\ 0 & 17 & -2 & -4\end{array}\right] \xrightarrow[R_3-17R_2]{R_1+5R_2} \left[\begin{array}{ccc|c}1 & 0 & -4 & -5\\ 0 & 1 & -1 & -2\\ 0 & 0 & 15 & 30\end{array}\right] \xrightarrow{\frac{1}{15}R_3}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -4 & -5 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 1 & 2 \end{array}\right] \xrightarrow[R_2+R_3]{R_1+4R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right].$$

The solution is (3,0,2).

44. $$\left[\begin{array}{ccc|c} 2 & 4 & -6 & 38 \\ 1 & 2 & 3 & 7 \\ 3 & -4 & 4 & -19 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 7 \\ 2 & 4 & -6 & 38 \\ 3 & -4 & 4 & -19 \end{array}\right] \xrightarrow[R_3-3R_1]{R_2-2R_1} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 7 \\ 0 & 0 & -12 & 24 \\ 0 & -10 & -5 & -40 \end{array}\right]$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 7 \\ 0 & -10 & -5 & -40 \\ 0 & 0 & -12 & 24 \end{array}\right] \xrightarrow{-\frac{1}{10}R_2} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 7 \\ 0 & 1 & \frac{1}{2} & 4 \\ 0 & 0 & -12 & 24 \end{array}\right] \xrightarrow{R_1-2R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & -1 \\ 0 & 1 & \frac{1}{2} & 4 \\ 0 & 0 & -12 & 24 \end{array}\right] \xrightarrow{-\frac{1}{12}R_3} \left[\begin{array}{ccc|c} 1 & 0 & 2 & -1 \\ 0 & 1 & \frac{1}{2} & 4 \\ 0 & 0 & 1 & -2 \end{array}\right] \xrightarrow[R_2-\frac{1}{2}R_3]{R_1-2R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -2 \end{array}\right].$$

The solution is (3,5,–2).

45. Using the Gauss-Jordan method, we have

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 6 \\ 2 & 1 & -3 & -3 \\ 1 & -3 & 3 & 10 \end{array}\right] \xrightarrow[R_3-R_1]{R_2-2R_1} \left[\begin{array}{ccc|c} 1 & -2 & 1 & 6 \\ 0 & 5 & -5 & -15 \\ 0 & -1 & 2 & 4 \end{array}\right] \xrightarrow{\frac{1}{5}R_2} \left[\begin{array}{ccc|c} 1 & -2 & 1 & 6 \\ 0 & 1 & -1 & -3 \\ 0 & -1 & 2 & 4 \end{array}\right]$$

$$\xrightarrow[R_3+R_2]{R_1+2R_2} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 1 & 1 \end{array}\right] \xrightarrow[R_2+R_3]{R_1+R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{array}\right].$$

Therefore, the solution is (1,–2,1).

46. Using the Gauss-Jordan method, we have

$$\left[\begin{array}{ccc|c} 2 & 3 & -6 & -11 \\ 1 & -2 & 3 & 9 \\ 3 & 1 & 0 & 7 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ 2 & 3 & -6 & -11 \\ 3 & 1 & 0 & 7 \end{array}\right] \xrightarrow[R_3-3R_1]{R_2-2R_1} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ 0 & 7 & -12 & -29 \\ 0 & 7 & -9 & -20 \end{array}\right]$$

$$\xrightarrow{\frac{1}{7}R_2} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ 0 & 1 & -\frac{12}{7} & -\frac{29}{7} \\ 0 & 7 & -9 & -20 \end{array}\right] \xrightarrow[R_3-7R_2]{R_1+2R_2} \left[\begin{array}{ccc|c} 1 & 0 & -\frac{3}{7} & \frac{5}{7} \\ 0 & 1 & -\frac{12}{7} & -\frac{29}{7} \\ 0 & 0 & 3 & 9 \end{array}\right] \xrightarrow{\frac{1}{3}R_3}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -\frac{3}{7} & \frac{5}{7} \\ 0 & 1 & -\frac{12}{7} & -\frac{29}{7} \\ 0 & 0 & 1 & 3 \end{array}\right] \xrightarrow[R_2+\frac{12}{7}R_3]{R_1+\frac{3}{7}R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{array}\right]$$. Therefore, the solution is (2,1,3).

47. Using the Gauss-Jordan method, we have

$$\left[\begin{array}{ccc|c} 2 & 0 & 3 & -1 \\ 3 & -2 & 1 & 9 \\ 1 & 1 & 4 & 4 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & 1 & 4 & 4 \\ 3 & -2 & 1 & 9 \\ 2 & 0 & 3 & -1 \end{array}\right] \xrightarrow[R_3-2R_1]{R_2-3R_1} \left[\begin{array}{ccc|c} 1 & 1 & 4 & 4 \\ 0 & -5 & -11 & -3 \\ 0 & -2 & -5 & -9 \end{array}\right]$$

$$\xrightarrow{-\frac{1}{5}R_2} \left[\begin{array}{ccc|c} 1 & 1 & 4 & 4 \\ 0 & 1 & \frac{11}{5} & \frac{3}{5} \\ 0 & -2 & -5 & -9 \end{array}\right] \xrightarrow[R_3+2R_2]{R_1-R_2} \left[\begin{array}{ccc|c} 1 & 0 & \frac{9}{5} & \frac{17}{5} \\ 0 & 1 & \frac{11}{5} & \frac{3}{5} \\ 0 & 0 & -\frac{3}{5} & -\frac{39}{5} \end{array}\right] \xrightarrow{-\frac{5}{3}R_3}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{9}{5} & \frac{17}{5} \\ 0 & 1 & \frac{11}{5} & \frac{3}{5} \\ 0 & 0 & 1 & 13 \end{array}\right] \xrightarrow[R_2-\frac{11}{5}R_3]{R_1-\frac{9}{5}R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -20 \\ 0 & 1 & 0 & -28 \\ 0 & 0 & 1 & 13 \end{array}\right].$$

Therefore, the solution is (–20, –28, 13).

48. Using the Gauss-Jordan methods of solution, we have

$$\left[\begin{array}{ccc|c} 2 & -1 & 3 & -4 \\ 1 & -2 & 1 & -1 \\ 1 & -5 & 2 & -3 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 2 & -1 & 3 & -4 \\ 1 & -5 & 2 & -3 \end{array}\right] \xrightarrow[R_3-R_1]{R_2-2R_1} \left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 0 & 3 & 1 & -2 \\ 0 & -3 & 1 & -2 \end{array}\right]$$

$$\xrightarrow{\frac{1}{3}R_2}\left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 0 & 1 & \frac{1}{3} & -\frac{2}{3} \\ 0 & -3 & 1 & -2 \end{array}\right]\xrightarrow[R_3+3R_2]{R_1+2R_2}\left[\begin{array}{ccc|c} 1 & 0 & \frac{5}{3} & -\frac{7}{3} \\ 0 & 1 & \frac{1}{3} & -\frac{2}{3} \\ 0 & 0 & 2 & -4 \end{array}\right]\xrightarrow{\frac{1}{2}R_3}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{5}{3} & -\frac{7}{3} \\ 0 & 1 & \frac{1}{3} & -\frac{2}{3} \\ 0 & 0 & 1 & -2 \end{array}\right]\xrightarrow[R_2-\frac{1}{3}R_3]{R_1-\frac{5}{3}R_3}\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \end{array}\right].$$ Therefore, the solution is (1,0,–2).

49. Using the Gauss-Jordan method, we have

$$\left[\begin{array}{ccc|c} 1 & -1 & 3 & 14 \\ 1 & 1 & 1 & 6 \\ -2 & -1 & 1 & -4 \end{array}\right]\xrightarrow[R_3+2R_1]{R_2-R_1}\left[\begin{array}{ccc|c} 1 & -1 & 3 & 14 \\ 0 & 2 & -2 & -8 \\ 0 & -3 & 7 & 24 \end{array}\right]\xrightarrow{\frac{1}{2}R_2}\left[\begin{array}{ccc|c} 1 & -1 & 3 & 14 \\ 0 & 1 & -1 & -4 \\ 0 & -3 & 7 & 24 \end{array}\right]$$

$$\xrightarrow[R_3+3R_2]{R_1+R_2}\left[\begin{array}{ccc|c} 1 & 0 & 2 & 10 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & 4 & 12 \end{array}\right]\xrightarrow{\frac{1}{4}R_3}\left[\begin{array}{ccc|c} 1 & 0 & 2 & 10 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & 1 & 3 \end{array}\right]\xrightarrow[R_2+R_3]{R_1-2R_3}\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

Therefore, the solution is (4, –1, 3).

50. Using the Gauss-Jordan method, we have

$$\left[\begin{array}{ccc|c} 2 & -1 & -1 & 0 \\ 3 & 2 & 1 & 7 \\ 1 & 2 & 2 & 5 \end{array}\right]\xrightarrow{R_1\leftrightarrow R_3}\left[\begin{array}{ccc|c} 1 & 2 & 2 & 5 \\ 3 & 2 & 1 & 7 \\ 2 & -1 & -1 & 0 \end{array}\right]\xrightarrow[R_3-2R_1]{R_2-3R_1}\left[\begin{array}{ccc|c} 1 & 2 & 2 & 5 \\ 0 & -4 & -5 & -8 \\ 0 & -5 & -5 & -10 \end{array}\right]$$

$$\xrightarrow{-\frac{1}{4}R_2}\left[\begin{array}{ccc|c} 1 & 2 & 2 & 5 \\ 0 & 1 & \frac{5}{4} & 2 \\ 0 & -5 & -5 & -10 \end{array}\right]\xrightarrow[R_3+5R_2]{R_1-2R_2}\left[\begin{array}{ccc|c} 1 & 0 & -\frac{1}{2} & 1 \\ 0 & 1 & \frac{5}{4} & 2 \\ 0 & 0 & \frac{5}{4} & 0 \end{array}\right]\xrightarrow{\frac{4}{5}R_3}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -\frac{1}{2} & 1 \\ 0 & 1 & \frac{5}{4} & 2 \\ 0 & 0 & 1 & 0 \end{array}\right]\xrightarrow[R_2-\frac{5}{4}R_3]{R_1+\frac{1}{2}R_3}\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{array}\right].$$ Therefore, the solution is (1, 2, 0).

51. We wish to solve the system of equations

$$\begin{aligned} x + y &= 500 \quad (x = \text{the number of acres of corn planted}) \\ 42x + 30y &= 18{,}600 \quad (y = \text{the number of acres of wheat planted}) \end{aligned}$$

Using the Gauss-Jordan method, we find

$$\left[\begin{array}{cc|c} 1 & 1 & 500 \\ 42 & 30 & 18600 \end{array}\right] \xrightarrow{R_2-42R_1} \left[\begin{array}{cc|c} 1 & 1 & 500 \\ 0 & -12 & -2400 \end{array}\right] \xrightarrow{-\frac{1}{12}R_2} \left[\begin{array}{cc|c} 1 & 1 & 500 \\ 0 & 1 & 200 \end{array}\right]$$

$$\xrightarrow{R_1-R_2} \left[\begin{array}{cc|c} 1 & 0 & 300 \\ 0 & 1 & 200 \end{array}\right].$$

The solution to this system of equations is $x = 300$ and $y = 200$. We conclude that Jacob should plant 300 acres of corn and 200 acres of wheat.

52. We wish to solve the system of equations

$$\begin{aligned} x + y &= 2000 \quad (x = \text{the amount invested at 6 percent}) \\ 0.06x + 0.08y &= 144 \quad (y = \text{the amount invested at 8 percent}) \end{aligned}$$

Using the Gauss-Jordan method, we find

$$\left[\begin{array}{cc|c} 1 & 1 & 2000 \\ 0.06 & 0.08 & 144 \end{array}\right] \xrightarrow{R_2-0.06R_1} \left[\begin{array}{cc|c} 1 & 1 & 2000 \\ 0 & 0.02 & 24 \end{array}\right] \xrightarrow{50R_2} \left[\begin{array}{cc|c} 1 & 1 & 2000 \\ 0 & 1 & 1200 \end{array}\right]$$

$$\xrightarrow{R_1-R_2} \left[\begin{array}{cc|c} 1 & 0 & 800 \\ 0 & 1 & 1200 \end{array}\right].$$

The solution to this system of equations is $x = 800$ and $y = 1200$. We conclude that Michael should invest \$800 at 6 percent per year and \$1200 at 8 percent per year.

53. Let x denote the number of pounds of the \$2.50/lb coffee and y denote the number of pounds of the \$3.00/lb coffee. Then we are required to solve the system

$$\begin{aligned} x + y &= 100 \\ 2.50x + 3.00y &= 280 \end{aligned}$$

Using the Gauss-Jordan method of elimination, we have

$$\left[\begin{array}{cc|c} 1 & 1 & 100 \\ 2.5 & 3 & 280 \end{array}\right] \xrightarrow{R_2-2.5R_1} \left[\begin{array}{cc|c} 1 & 1 & 100 \\ 0 & 0.5 & 30 \end{array}\right] \xrightarrow{2R_2} \left[\begin{array}{cc|c} 1 & 1 & 100 \\ 0 & 1 & 60 \end{array}\right]$$

$$\xrightarrow{R_1-R_2}\left[\begin{array}{cc|c}1 & 0 & 40\\0 & 1 & 60\end{array}\right].$$

Therefore, 40 pounds of the \$2.50/lb coffee and 60 pounds of the \$3.00/lb coffee should be used in the 100 lb mixture.

54. Let the amount of money invested in the bonds yielding 8% be x dollars and the amount of money invested in the bonds yielding 10% be y dollars. Then the solution
to the problem can be found by solving the system of equations

$$\begin{aligned} x + \quad y &= 30{,}000 \\ 0.08x + 0.10y &= 2{,}640 \end{aligned}$$

Using the Gauss-Jordan method, we have

$$\left[\begin{array}{cc|c}1 & 1 & 30{,}000\\0.08 & 0.10 & 2{,}640\end{array}\right]\xrightarrow{R_2-0.08R_1}\left[\begin{array}{cc|c}1 & 1 & 30{,}000\\0 & 0.02 & 240\end{array}\right]\xrightarrow{50R_2}\left[\begin{array}{cc|c}1 & 1 & 30{,}000\\0 & 1 & 12{,}000\end{array}\right]$$

$$\xrightarrow{R_1-R_2}\left[\begin{array}{cc|c}1 & 0 & 18{,}000\\0 & 1 & 12{,}000\end{array}\right].$$

Then the amount she has invested in bonds yielding 8% is \$18,000 and the amount she has invested in bonds yielding 10% is \$12,000.

55. Let x and y denote the number of children and adults who rode the bus during the morning shift, respectively. Then the solution to the problem can be found by solving the system of equations

$$\begin{aligned} x + \quad y &= 1000 \\ 0.25x + 0.75y &= 650 \end{aligned}$$

Using the Gauss-Jordan elimination method, we have

$$\left[\begin{array}{cc|c}1 & 1 & 1000\\0.25 & 0.75 & 650\end{array}\right]\xrightarrow{R_2-0.25R_1}\left[\begin{array}{cc|c}1 & 1 & 1000\\0 & 0.5 & 400\end{array}\right]\xrightarrow{2R_2}\left[\begin{array}{cc|c}1 & 1 & 1000\\0 & 1 & 800\end{array}\right]$$

$$\xrightarrow{R_1-R_2}\left[\begin{array}{cc|c}1 & 0 & 200\\0 & 1 & 800\end{array}\right].$$

We conclude that 800 adults and 200 children rode the bus during the morning shift.

56. Let x, y, and z denote the number of one-bedroom units, two-bedroom townhouses, and three-bedroom townhouses, respectively. Then we are required to solve the system

$$\begin{aligned} x+y+z &= 192 \\ x-y-z &= 0 \\ x-\quad 3z &= 0 \end{aligned}$$

Using the Gauss-Jordan method, we find

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 192 \\ 1 & -1 & -1 & 0 \\ 1 & 0 & -3 & 0 \end{array}\right] \xrightarrow[R_3-R_1]{R_2-R_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 192 \\ 0 & -2 & -2 & -192 \\ 0 & -1 & -4 & -192 \end{array}\right] \xrightarrow{-\frac{1}{2}R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 192 \\ 0 & 1 & 1 & 96 \\ 0 & -1 & -4 & -192 \end{array}\right]$$

$$\xrightarrow[R_3+R_2]{R_1-R_2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 96 \\ 0 & 1 & 1 & 96 \\ 0 & 0 & -3 & -96 \end{array}\right] \xrightarrow{-\frac{1}{3}R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 96 \\ 0 & 1 & 1 & 96 \\ 0 & 0 & 1 & 32 \end{array}\right] \xrightarrow{R_2-R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 96 \\ 0 & 1 & 0 & 64 \\ 0 & 0 & 1 & 32 \end{array}\right].$$

Therefore, 96 one-bedroom, 64 two-bedroom, and 32 three-bedroom units should be built.

57. Let x, y, and z, denote the amount of money he should invest in a savings account, in mutual funds, and in bonds, respectively. Then, we are required to solve the system

$$\begin{aligned} 0.06x + 0.08y + 0.12z &= 21{,}600 \\ 2x \qquad\qquad - \quad z &= 0 \\ 0.08y - 0.12z &= 0 \end{aligned}$$

Using the Gauss-Jordan method, we find

$$\left[\begin{array}{ccc|c} 0.06 & 0.08 & 0.12 & 21{,}600 \\ 2 & 0 & -1 & 0 \\ 0 & 0.08 & -0.12 & 0 \end{array}\right] \xrightarrow[\frac{1}{0.08}R_3]{\frac{1}{0.06}R_1} \left[\begin{array}{ccc|c} 1 & \frac{4}{3} & 2 & 360{,}000 \\ 2 & 0 & -1 & 0 \\ 0 & 1 & -\frac{3}{2} & 0 \end{array}\right] \xrightarrow{R_2-2R_1}$$

$$\left[\begin{array}{ccc|c} 1 & \frac{4}{3} & 2 & 360{,}000 \\ 0 & -\frac{8}{3} & -5 & -720{,}000 \\ 0 & 1 & -\frac{3}{2} & 0 \end{array}\right] \xrightarrow{-\frac{3}{8}R_2} \left[\begin{array}{ccc|c} 1 & \frac{4}{3} & 2 & 360{,}000 \\ 0 & 1 & \frac{15}{8} & 270{,}000 \\ 0 & 1 & -\frac{3}{2} & 0 \end{array}\right]$$

$$\xrightarrow[R_3-R_2]{R_1-\frac{4}{3}R_2}\left[\begin{array}{ccc|c}1 & 0 & -\frac{1}{2} & 0\\ 0 & 1 & \frac{15}{8} & 270{,}000\\ 0 & 0 & -\frac{27}{8} & -270{,}000\end{array}\right]\xrightarrow{-\frac{8}{27}R_3}\left[\begin{array}{ccc|c}1 & 0 & -\frac{1}{2} & 0\\ 0 & 1 & \frac{15}{8} & 270{,}000\\ 0 & 0 & 1 & 80{,}000\end{array}\right]$$

$$\xrightarrow[R_2-\frac{15}{8}R_3]{R_1+\frac{1}{2}R_3}\left[\begin{array}{ccc|c}1 & 0 & 0 & 40{,}000\\ 0 & 1 & 0 & 120{,}000\\ 0 & 0 & 1 & 80{,}000\end{array}\right]$$

Therefore, Sid should invest \$40,000 in a savings account, \$120,000 in mutual funds, and \$80,000 in bonds.

58. Refer to Exercise 22, page 75. We obtain the following augmented matrices.

$$\left[\begin{array}{ccc|c}1 & 1 & 1 & 200000\\ 15 & 10 & 6 & 2000000\\ 1 & 1 & -1 & 0\end{array}\right]\xrightarrow[R_3-R_1]{R_2-15R_1}\left[\begin{array}{ccc|c}1 & 1 & 1 & 200000\\ 0 & -5 & -9 & -1000000\\ 0 & 0 & -2 & -200000\end{array}\right]\xrightarrow{-\frac{1}{5}R_2}$$

$$\left[\begin{array}{ccc|c}1 & 1 & 1 & 200000\\ 0 & 1 & \frac{9}{5} & 200000\\ 0 & 0 & -2 & -200000\end{array}\right]\xrightarrow[-\frac{1}{2}R_3]{R_1-R_2}\left[\begin{array}{ccc|c}1 & 0 & -\frac{4}{5} & 0\\ 0 & 1 & \frac{9}{5} & 200000\\ 0 & 0 & 1 & 100000\end{array}\right]\xrightarrow[R_2-\frac{9}{5}R_3]{R_1+\frac{4}{5}R_3}\left[\begin{array}{ccc|c}1 & 0 & 0 & 80000\\ 0 & 1 & 0 & 20000\\ 0 & 0 & 1 & 100000\end{array}\right]$$

We see that $x = 80{,}000$, $y = 20{,}000$, and $z = 100{,}000$. Therefore, they should invest \$80,000 in high-risk, \$20,000 in medium-risk, and \$100,000 in low-risk stocks.

59. Refer to Exercise 23, page 75. We obtain the following augmented matrices.

$$\left[\begin{array}{ccc|c}18 & 20 & 24 & 26400\\ 4 & 4 & 3 & 4900\\ 5 & 4 & 6 & 6200\end{array}\right]\xrightarrow{R_1\leftrightarrow R_3}\left[\begin{array}{ccc|c}5 & 4 & 6 & 6200\\ 4 & 4 & 3 & 4900\\ 18 & 20 & 24 & 26400\end{array}\right]\xrightarrow{R_1-R_2}$$

$$\left[\begin{array}{ccc|c}1 & 0 & 3 & 1300\\ 4 & 4 & 3 & 4900\\ 18 & 20 & 24 & 26400\end{array}\right]\xrightarrow[R_3-18R_1]{R_2-4R_1}\left[\begin{array}{ccc|c}1 & 0 & 3 & 1300\\ 0 & 4 & -9 & -300\\ 0 & 20 & -30 & 3000\end{array}\right]\xrightarrow{\frac{1}{4}R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 1300 \\ 0 & 1 & -\frac{9}{4} & -75 \\ 0 & 20 & -30 & 3000 \end{array}\right] \xrightarrow{R_3-20R_2} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 1300 \\ 0 & 1 & -\frac{9}{4} & -75 \\ 0 & 0 & 15 & 4500 \end{array}\right] \xrightarrow{\frac{1}{15}R_3}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 1300 \\ 0 & 1 & -\frac{9}{4} & -75 \\ 0 & 0 & 1 & 300 \end{array}\right] \xrightarrow[R_2+\frac{9}{4}R_3]{R_1-3R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 400 \\ 0 & 1 & 0 & 600 \\ 0 & 0 & 1 & 300 \end{array}\right]$$

We see that $x = 400$, $y = 600$, and $z = 300$. Therefore, Lawnco should produce 400, 600, and 300 100-lb bags of grade-A, grade-B, and grade-C fertilizer.

60. Let x, y, and z denote the number of tickets sold to children, students and adults, respectively. Then the solution to the problem can be found by solving the system

$$\begin{aligned} x + y + z &= 900 \\ x + y - 2z &= 0 \\ 2x + 3y + 4z &= 2800 \end{aligned}$$

Using the Gauss-Jordan method, we have

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 900 \\ 1 & 1 & -2 & 0 \\ 2 & 3 & 4 & 2800 \end{array}\right] \xrightarrow[R_3-2R_1]{R_2-R_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 900 \\ 0 & 0 & -3 & -900 \\ 0 & 1 & 2 & 1000 \end{array}\right] \xrightarrow{R_2\leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 900 \\ 0 & 1 & 2 & 1000 \\ 0 & 0 & -3 & -900 \end{array}\right]$$

$$\xrightarrow{R_1-R_2} \left[\begin{array}{ccc|c} 1 & 0 & -1 & -100 \\ 0 & 1 & 2 & 1000 \\ 0 & 0 & -3 & -900 \end{array}\right] \xrightarrow{-\frac{1}{3}R_3} \left[\begin{array}{ccc|c} 1 & 0 & -1 & -100 \\ 0 & 1 & 2 & 1000 \\ 0 & 0 & 1 & 300 \end{array}\right] \xrightarrow[R_2-2R_3]{R_1+R_3}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 200 \\ 0 & 1 & 0 & 400 \\ 0 & 0 & 1 & 300 \end{array}\right].$$

We conclude that 200 children attended the show.

61. Let x, y, and z denote the number of compact, intermediate, and full-size cars, respectively, to be purchased. Then the problem can be solved by solving the system

$$\begin{aligned} 12000x + 18{,}000y + 24{,}000z &= 1{,}500{,}000 \\ x - \quad 2y \quad\quad &= 0 \\ x + \quad y + \quad z &= 100 \end{aligned}$$

Using the Gauss-Jordan method, we have

$$\left[\begin{array}{ccc|c} 12{,}000 & 18{,}000 & 24{,}000 & 1{,}500{,}000 \\ 1 & -2 & 0 & 0 \\ 1 & 1 & 1 & 100 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 100 \\ 1 & -2 & 0 & 0 \\ 12{,}000 & 18{,}000 & 24{,}000 & 1{,}500{,}000 \end{array}\right]$$

$$\xrightarrow[R_3 - 12{,}000R_1]{R_2 - R_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 100 \\ 0 & -3 & -1 & -100 \\ 0 & 6000 & 12{,}000 & 300{,}000 \end{array}\right] \xrightarrow{-\frac{1}{3}R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 100 \\ 0 & 1 & \frac{1}{3} & \frac{100}{3} \\ 0 & 6000 & 12{,}000 & 300{,}000 \end{array}\right]$$

$$\xrightarrow[R_3 - 6000R_2]{R_1 - R_2} \left[\begin{array}{ccc|c} 1 & 0 & \frac{2}{3} & \frac{200}{3} \\ 0 & 1 & \frac{1}{3} & \frac{100}{3} \\ 0 & 0 & 10000 & 100{,}000 \end{array}\right] \xrightarrow{\frac{1}{10{,}000}R_3} \left[\begin{array}{ccc|c} 1 & 0 & \frac{2}{3} & \frac{200}{3} \\ 0 & 1 & \frac{1}{3} & \frac{100}{3} \\ 0 & 0 & 1 & 10 \end{array}\right] \xrightarrow[R_2 - \frac{1}{3}R_3]{R_1 - \frac{2}{3}R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 60 \\ 0 & 1 & 0 & 30 \\ 0 & 0 & 1 & 10 \end{array}\right].$$

We conclude that 60 compact cars, 30 intermediate-size cars, and 10 full-size cars will be purchased.

62. Let x, y, and z denote the amount of money invested in high-risk stocks, medium-risk stocks, and low-risk stocks, respectively. Then the problem can be solved by solving the system

$$\begin{aligned} x + \quad y + \quad z &= 200{,}000 \\ 2x + 2y - \quad z &= 0 \\ 6x + \quad y - 3z &= 0 \end{aligned}$$

Using the Gauss-Jordan method, we have

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 200{,}000 \\ 2 & 2 & -1 & 0 \\ 6 & 1 & -3 & 0 \end{array}\right] \xrightarrow[R_3 - 6R_1]{R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 200{,}000 \\ 0 & 0 & -3 & -400{,}000 \\ 0 & -5 & -9 & -1{,}200{,}000 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_3}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 200{,}000 \\ 0 & -5 & -9 & -1{,}200{,}000 \\ 0 & 0 & -3 & -400{,}000 \end{array}\right] \xrightarrow{-\frac{1}{5}R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 200{,}000 \\ 0 & 1 & \frac{9}{5} & 240{,}000 \\ 0 & 0 & -3 & -400{,}000 \end{array}\right] \xrightarrow{R_1 - R_2}$$

$$\begin{bmatrix} 1 & 0 & -\frac{4}{5} & -40{,}000 \\ 0 & 1 & \frac{9}{5} & 240{,}000 \\ 0 & 0 & -3 & -400{,}000 \end{bmatrix} \xrightarrow{-\frac{1}{3}R_3} \begin{bmatrix} 1 & 0 & -\frac{4}{5} & -40{,}000 \\ 0 & 1 & \frac{9}{5} & 240{,}000 \\ 0 & 0 & 1 & \frac{400{,}000}{3} \end{bmatrix} \xrightarrow[R_2-\frac{9}{5}R_3]{R_1+\frac{4}{5}R_3}$$

$$\begin{bmatrix} 1 & 0 & 0 & \frac{200{,}000}{3} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{400{,}000}{3} \end{bmatrix}.$$

We conclude that the investment club should invest \$66,666.67 in high risk stocks, \$0 in medium risk stocks, and \$133,333.33 in low risk stocks.

63. Let x, y, and z, represent the number of ounces of Food *I*, Food *II*, and Food *III* used in the meal, respectively. Then the problem reduces to solving the following system of linear equations:

$$\begin{aligned} 10x + 6y + 8z &= 100 \\ 10x + 12y + 6z &= 100 \\ 5x + 4y + 12z &= 100. \end{aligned}$$

Using the Gauss-Jordan method, we obtain

$$\begin{bmatrix} 10 & 6 & 8 & 100 \\ 10 & 12 & 6 & 100 \\ 5 & 4 & 12 & 100 \end{bmatrix} \xrightarrow{\frac{1}{10}R_1} \begin{bmatrix} 1 & \frac{3}{5} & \frac{4}{5} & 10 \\ 10 & 12 & 6 & 100 \\ 5 & 4 & 12 & 100 \end{bmatrix} \xrightarrow[R_3-5R_1]{R_2-10R_1}$$

$$\begin{bmatrix} 1 & \frac{3}{5} & \frac{4}{5} & 10 \\ 0 & 6 & -2 & 0 \\ 0 & 1 & 8 & 50 \end{bmatrix} \xrightarrow{\frac{1}{6}R_2} \begin{bmatrix} 1 & \frac{3}{5} & \frac{4}{5} & 10 \\ 0 & 1 & -\frac{1}{3} & 0 \\ 0 & 1 & 8 & 50 \end{bmatrix} \xrightarrow[R_3-R_2]{R_1-\frac{3}{5}R_2}$$

$$\begin{bmatrix} 1 & 0 & 1 & 10 \\ 0 & 1 & -\frac{1}{3} & 0 \\ 0 & 0 & \frac{25}{3} & 50 \end{bmatrix} \xrightarrow{\frac{3}{25}R_3} \begin{bmatrix} 1 & 0 & 1 & 10 \\ 0 & 1 & -\frac{1}{3} & 0 \\ 0 & 0 & 1 & 6 \end{bmatrix} \xrightarrow[R_2+\frac{1}{3}R_3]{R_1-R_3} \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 6 \end{bmatrix}.$$

We conclude that 4 oz of Food *I*, 2 oz of Food *II*, and 6 oz of Food *III* should be used to prepare the meal.

64. Let x, y, and z denote the amount of money invested in stocks, bonds, and a money market account, respectively. Then the problem can be solved by solving the system

$$\begin{aligned} x + \quad y + \quad z &= 100{,}000 \\ 12x + 8y + 4z &= 1{,}000{,}000 \\ 20x + 10y - 100z &= 0. \end{aligned}$$

Using the Gauss-Jordan method, we have

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 100{,}000 \\ 12 & 8 & 4 & 1{,}000{,}000 \\ 20 & 10 & -100 & 0 \end{array}\right] \xrightarrow[R_3-20R_1]{R_2-12R_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 100{,}000 \\ 0 & -4 & -8 & -200{,}000 \\ 0 & -10 & -120 & -2{,}000{,}000 \end{array}\right] \xrightarrow{-\frac{1}{4}R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 100{,}000 \\ 0 & 1 & 2 & 50{,}000 \\ 0 & -10 & -120 & -2{,}000{,}000 \end{array}\right] \xrightarrow[R_3+10R_2]{R_1-R_2} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 50{,}000 \\ 0 & 1 & 2 & 50{,}000 \\ 0 & 0 & -100 & -1{,}500{,}000 \end{array}\right] \xrightarrow{-\frac{1}{100}R_3}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 50{,}000 \\ 0 & 1 & 2 & 50{,}000 \\ 0 & 0 & 1 & 15{,}000 \end{array}\right] \xrightarrow[R_2-2R_3]{R_1+R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 65{,}000 \\ 0 & 1 & 0 & 20{,}000 \\ 0 & 0 & 1 & 15{,}000 \end{array}\right]$$

We conclude that the Garcias should invest \$65,000 in stocks,\$20,000 in bonds, and \$15,000 in a money-market account.

65. Let x = the number of front orchestra seats sold
y = the number of rear orchestra seats sold
and z = the number of front balcony seats sold for this performance.
Then, we are required to solve the system

$$\begin{aligned} x + \quad y + \quad z &= 1{,}000 \\ 80x + 60y + 50z &= 62{,}800 \\ x + \quad y - \;2z &= 400. \end{aligned}$$

Using the Gauss-Jordan method, we find

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1{,}000 \\ 80 & 60 & 50 & 62{,}800 \\ 1 & 1 & -2 & 400 \end{array}\right] \xrightarrow[R_3-R_1]{R_2-80R_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1{,}000 \\ 0 & -20 & -30 & -17{,}200 \\ 0 & 0 & -3 & -600 \end{array}\right] \xrightarrow[-\frac{1}{3}R_3]{-\frac{1}{20}R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1{,}000 \\ 0 & 1 & \frac{3}{2} & 860 \\ 0 & 0 & 1 & 200 \end{array}\right] \xrightarrow{R_1-R_2} \left[\begin{array}{ccc|c} 1 & 0 & -\frac{1}{2} & 140 \\ 0 & 1 & \frac{3}{2} & 860 \\ 0 & 0 & 1 & 200 \end{array}\right] \xrightarrow[R_2-\frac{3}{2}R_3]{R_1+\frac{1}{2}R_3}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 240 \\ 0 & 1 & 0 & 560 \\ 0 & 0 & 1 & 200 \end{array}\right].$$

We conclude that tickets for 240 front orchestra seats, 560 rear orchestra seats, and 200 front balcony seats were sold.

66. Let x = the number of dozens of sleeveless blouses produced per day
y = the number of dozens of short-sleeve blouses produced per day
and z = the number of dozens of long-sleeve blouses produced per day.
Then, we want to solve the system

$$\begin{aligned} 9x+12y+15z &= 4800 \\ 22x+24y+28z &= 9600 \\ 6x+\ 8y+\ 8z &= 2880 \end{aligned}$$

Using the Gauss-Jordan method of elimination, we find

$$\left[\begin{array}{ccc|c} 9 & 12 & 15 & 4800 \\ 22 & 24 & 28 & 9600 \\ 6 & 8 & 8 & 2880 \end{array}\right] \xrightarrow{\frac{1}{9}R_1} \left[\begin{array}{ccc|c} 1 & \frac{4}{3} & \frac{5}{3} & \frac{1600}{3} \\ 22 & 24 & 28 & 9600 \\ 6 & 8 & 8 & 2880 \end{array}\right] \xrightarrow[R_3-6R_1]{R_2-22R_1}$$

$$\left[\begin{array}{ccc|c} 1 & \frac{4}{3} & \frac{5}{3} & \frac{1600}{3} \\ 0 & -\frac{16}{3} & -\frac{26}{3} & -\frac{6400}{3} \\ 0 & 0 & -2 & -320 \end{array}\right] \xrightarrow{-\frac{3}{16}R_2} \left[\begin{array}{ccc|c} 1 & \frac{4}{3} & \frac{5}{3} & \frac{1600}{3} \\ 0 & 1 & \frac{13}{8} & 400 \\ 0 & 0 & -2 & -320 \end{array}\right] \xrightarrow[-\frac{1}{2}R_3]{R_1-\frac{4}{3}R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & \frac{13}{8} & 400 \\ 0 & 0 & 1 & 160 \end{array}\right] \xrightarrow[R_2-\frac{13}{8}R_3]{R_1+\frac{1}{2}R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 80 \\ 0 & 1 & 0 & 140 \\ 0 & 0 & 1 & 160 \end{array}\right].$$

Therefore, the manufacturer should produce 80 dozen sleeveless, 140 dozen short-sleeve and 160 dozen long-sleeve blouses per day.

67. Let x, y, and z denote the number of days he spent in London, Paris, and Rome, respectively. We have

$$\begin{aligned} 180x+230y+160z&=2660\\ 110x+120y+\ \ 90z&=1520\\ x-\quad y-\quad z&=\quad 0 \qquad \text{(since } x=y+z\text{)} \end{aligned}$$

Using the Gauss-Jordan method to solve the system, we have

$$\left[\begin{array}{rrr|r} 180 & 230 & 160 & 2660\\ 110 & 120 & 90 & 1520\\ 1 & -1 & -1 & 0 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{rrr|r} 1 & -1 & -1 & 0\\ 110 & 120 & 90 & 1520\\ 180 & 230 & 160 & 2660 \end{array}\right] \xrightarrow[R_3-180R_1]{R_2-110R_1}$$

$$\left[\begin{array}{rrr|r} 1 & -1 & -1 & 0\\ 0 & 230 & 200 & 1520\\ 0 & 410 & 340 & 2660 \end{array}\right] \xrightarrow{\frac{1}{230}R_2} \left[\begin{array}{rrr|r} 1 & -1 & -1 & 0\\ 0 & 1 & \frac{20}{23} & \frac{152}{23}\\ 0 & 410 & 340 & 2660 \end{array}\right] \xrightarrow[R_3-410R_2]{R_1+R_2}$$

$$\left[\begin{array}{rrr|r} 1 & 0 & -\frac{3}{23} & \frac{152}{23}\\ 0 & 1 & \frac{20}{23} & \frac{152}{23}\\ 0 & 0 & -\frac{380}{23} & -\frac{1140}{23} \end{array}\right] \xrightarrow{-\frac{23}{380}R_3} \left[\begin{array}{rrr|r} 1 & 0 & -\frac{3}{23} & \frac{152}{23}\\ 0 & 1 & \frac{20}{23} & \frac{152}{23}\\ 0 & 0 & 1 & 3 \end{array}\right] \xrightarrow[R_2-\frac{20}{23}R_1]{R_1+\frac{3}{23}R_3}$$

$$\left[\begin{array}{rrr|r} 1 & 0 & 0 & 7\\ 0 & 1 & 0 & 4\\ 0 & 0 & 1 & 3 \end{array}\right]$$

The solution is $x = 7$, $y = 4$, and $z = 3$. Therefore, he spent 7 days in London, 4 days in Paris, and 3 days in Rome.

68. Let x, y, z, and w denote the number of days they spent in Boston, NYC, Philadelphia, and Washington D.C., respectively. The given information leads to the following system of equations:

$$\begin{aligned} x+\quad y+\quad z+\quad w&=\ \ 14\\ 120x+200y+80z+100w&=2020\\ y\qquad\qquad&=x+w\\ y\qquad\qquad&=3z \end{aligned}$$

or, upon rewriting,

$$\begin{aligned} x + \quad y + \quad z + \quad w &= 14 \\ x - \quad y + \qquad\quad w &= 0 \\ y - 3z \qquad\quad &= 0 \\ 120x + 200y + 80z + 100w &= 2020 \end{aligned}$$

We obtain the following augmented matrices:

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 14 \\ 1 & -1 & 0 & 1 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 120 & 200 & 80 & 100 & 2020 \end{array}\right] \xrightarrow[R_3-120R_1]{R_2-R_1} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 14 \\ 0 & -2 & -1 & 0 & -14 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 80 & -40 & -20 & 340 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_3}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 14 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & -2 & -1 & 0 & -14 \\ 0 & 80 & -40 & -20 & 340 \end{array}\right] \xrightarrow[\substack{R_3+2R_2 \\ R_4-80R_2}]{R_1-R_2} \left[\begin{array}{cccc|c} 1 & 0 & 4 & 1 & 14 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & -7 & 0 & -14 \\ 0 & 0 & 200 & -20 & 340 \end{array}\right] \xrightarrow{-\frac{1}{7}R_3}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4 & 1 & 14 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 200 & -20 & 340 \end{array}\right] \xrightarrow[\substack{R_2+3R_3 \\ R_4-200R_3}]{R_1-4R_3} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 6 \\ 0 & 1 & 0 & 0 & 6 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & -20 & -60 \end{array}\right] \xrightarrow{-\frac{1}{20}R_4}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 6 \\ 0 & 1 & 0 & 0 & 6 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right] \xrightarrow{R_1-R_4} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 & 6 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right]$$

We see that $x = 3$, $y = 6$, $z = 2$, and $w = 3$. Therefore, we conclude that they spent 3 days each in Boston and Washington D.C., 6 days in NYC, and 2 days in Philadelphia.

69. False. The constant cannot be zero. The system

$$\begin{aligned} 2x + y &= 1 \\ 3x - y &= 2 \end{aligned}$$

is not equivalent to

$$\begin{aligned} 2x + y &= 1 \\ 0(3x - y) &= 0(2) \end{aligned} \quad \text{or} \quad \begin{aligned} 2x + y &= 1 \\ 0 &= 0 \end{aligned}.$$

70. True. The row with the given form says

$$0x + 0y + 0z = a$$

or $$0 = a$$

But if $a \neq 0$, we have a contradiction.

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1. (3,1,–1,2)
2. (1,0,–2,–1)
3. (5,4,–3,–4)
4. (–256, –33, –12, 167)
5. (1,–1,2,0,3)
6. (1.2, –0.8, 3.6, 4.7, 2.1)