Chapter 5

Exponential and Logarithmic Functions

Exercise Set 5.1

1. We interchange the first and second coordinates of each ordered pair to find the inverse of the relation. It is

$$\{(8, 7), (8, -2), (-4, 3), (-8, 8)\}.$$

2. $\{(1, 0), (6, 5), (-4, -2)\}$

3. We interchange the first and second coordinates of each ordered pair to find the inverse of the relation. It is

$$\{(-1, -1), (4, -3)\}.$$

4. $\{(3, -1), (5, 2), (5, -3), (0, 2)\}$

5. Interchange x and y.

$$\begin{array}{l} y = 4x - 5 \\ \downarrow \quad \downarrow \\ x = 4y - 5 \end{array}$$

6. $2y^2 + 5x^2 = 4$

7. Interchange x and y.

$$\begin{array}{l} x^3 y = -5 \\ \downarrow \downarrow \\ y^3 x = -5 \end{array}$$

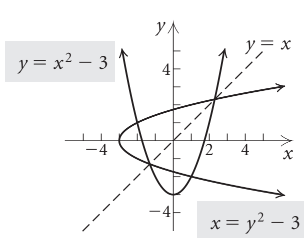
8. $x = 3y^2 - 5y + 9$

9. Interchange x and y.

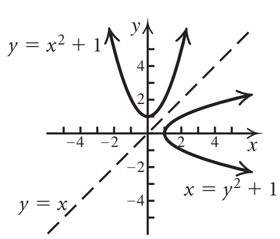
$$\begin{array}{l} x = y^2 - 2y \\ \downarrow \quad \downarrow \quad \downarrow \\ y = x^2 - 2x \end{array}$$

10. $y = \frac{1}{2}x + 4$

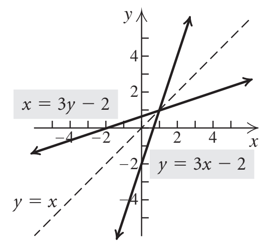
11. Graph $x = y^2 - 3$. Some points on the graph are $(-3, 0)$, $(-2, -1)$, $(-2, 1)$, $(1, -2)$, and $(1, 2)$. Plot these points and draw the curve. Then reflect the graph across the line $y = x$.



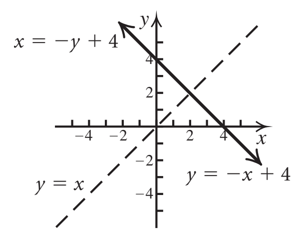
12.



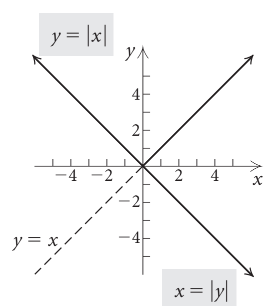
13. Graph $y = 3x - 2$. The intercepts are $(0, -2)$ and $\left(\frac{2}{3}, 0\right)$. Plot these points and draw the line. Then reflect the graph across the line $y = x$.



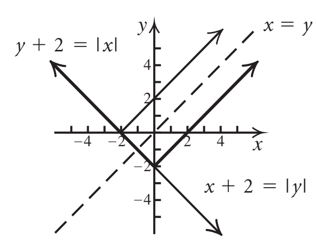
14.



15. Graph $y = |x|$. Some points on the graph are $(0, 0)$, $(-2, 2)$, $(2, 2)$, $(-5, 5)$, and $(5, 5)$. Plot these points and draw the graph. Then reflect the graph across the line $y = x$.



16.



17. We show that if $f(a) = f(b)$, then $a = b$.

$$\begin{aligned} \frac{1}{3}a - 6 &= \frac{1}{3}b - 6 \\ \frac{1}{3}a &= \frac{1}{3}b \qquad \text{Adding 6} \\ a &= b \qquad \text{Multiplying by 3} \end{aligned}$$

Thus f is one-to-one.

18. Assume $f(a) = f(b)$.

$$\begin{aligned} 4 - 2a &= 4 - 2b \\ -2a &= -2b \\ a &= b \end{aligned}$$

Then f is one-to-one.

19. We show that if $f(a) = f(b)$, then $a = b$.

$$\begin{aligned} a^3 + \frac{1}{2} &= b^3 + \frac{1}{2} \\ a^3 &= b^3 \quad \text{Subtracting } \frac{1}{2} \\ a &= b \quad \text{Taking cube roots} \end{aligned}$$

Thus f is one-to-one.

20. Assume $f(a) = f(b)$.

$$\begin{aligned} \sqrt[3]{a} &= \sqrt[3]{b} \\ a &= b \quad \text{Using the principle of powers} \end{aligned}$$

Then f is one-to-one.

21. $g(-1) = 1 - (-1)^2 = 1 - 1 = 0$ and $g(1) = 1 - 1^2 = 1 - 1 = 0$, so $g(-1) = g(1)$ but $-1 \neq 1$. Thus the function is not one-to-one.

22. $g(-1) = 4$ and $g(1) = 4$, so $g(-1) = g(1)$ but $-1 \neq 1$. Thus the function is not one-to-one.

23. $f(-2) = (-2)^4 - (-2)^2 = 16 - 4 = 12$ and $f(2) = 2^4 - 2^2 = 16 - 4 = 12$, so $f(-2) = f(2)$ but $-2 \neq 2$. Thus the function is not one-to-one.

24. $g(-1) = 1$ and $g(1) = 1$ so $g(-1) = g(1)$ but $-1 \neq 1$. Thus the function is not one-to-one.

25. The function is one-to-one, because no horizontal line crosses the graph more than once.

26. The function is one-to-one, because no horizontal line crosses the graph more than once.

27. The function is not one-to-one, because there are many horizontal lines that cross the graph more than once.

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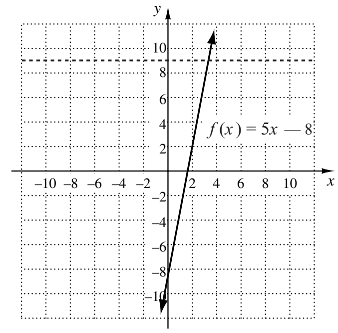
31. The function is one-to-one, because no horizontal line crosses the graph more than once.

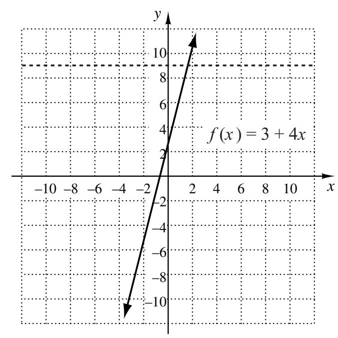
32. The function is one-to-one, because no horizontal line crosses the graph more than once.

33. The graph of $f(x) = 5x - 8$ is shown below.



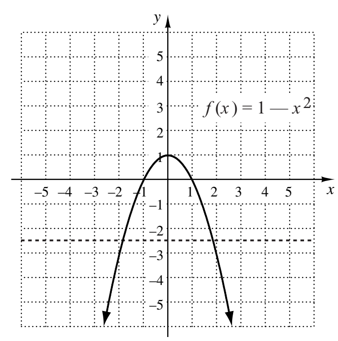
Since there is no horizontal line that crosses the graph more than once, the function is one-to-one.

34. The graph of $f(x) = 3 + 4x$ is shown below.



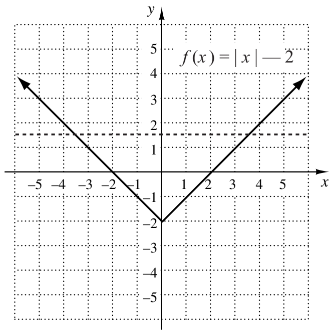
Since there is no horizontal line that crosses the graph more than once, the function is one-to-one.

35. The graph of $f(x) = 1 - x^2$ is shown below.



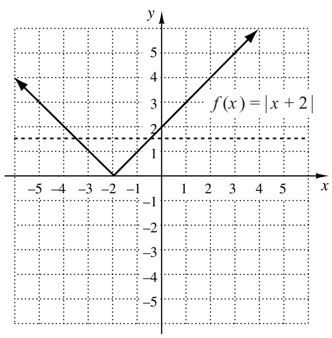
Since there are many horizontal lines that cross the graph more than once, the function is not one-to-one.

36. The graph of $f(x) = |x| - 2$ is shown below.



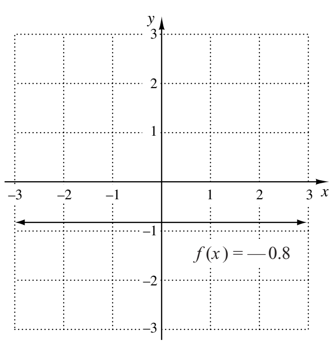
Since there are many horizontal lines that cross the graph more than once, the function is not one-to-one.

37. The graph of $f(x) = |x + 2|$ is shown below.



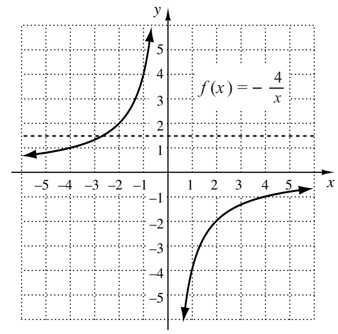
Since there are many horizontal lines that cross the graph more than once, the function is not one-to-one.

38. The graph of $f(x) = -0.8$ is shown below.



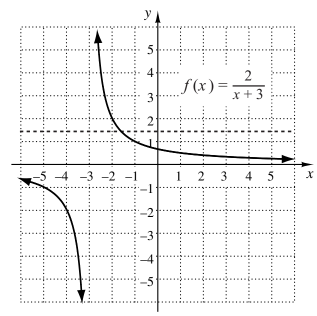
Since the horizontal line $y = -0.8$ crosses the graph more than once, the function is not one-to-one.

39. The graph of $f(x) = -\dfrac{4}{x}$ is shown below.



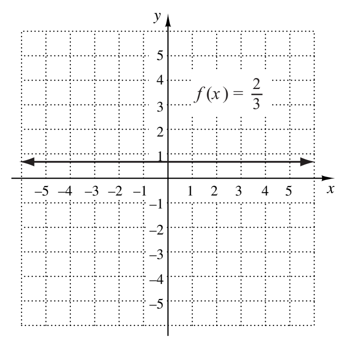
Since there is no horizontal line that crosses the graph more than once, the function is one-to-one.

40. The graph of $f(x) = \dfrac{2}{x+3}$ is shown below.



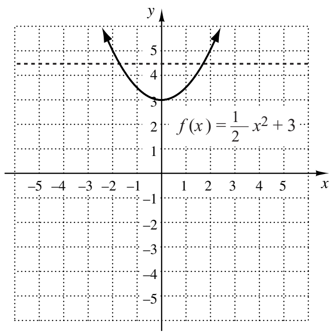
Since there is no horizontal line that crosses the graph more than once, the function is one-to-one.

41. The graph of $f(x) = \dfrac{2}{3}$ is shown below.



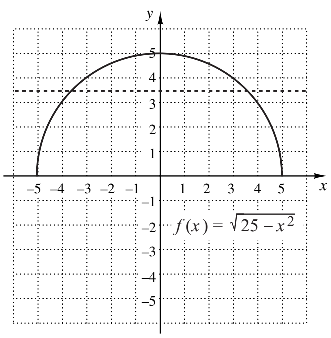
Since the horizontal line $y = \dfrac{2}{3}$ crosses the graph more than once, the function is not one-to-one.

42. The graph of $f(x) = \frac{1}{2}x^2 + 3$ is shown below.



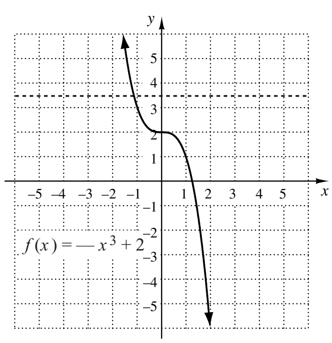
Since there are many horizontal lines that cross the graph more than once, the function is not one-to-one.

43. The graph of $f(x) = \sqrt{25 - x^2}$ is shown below.



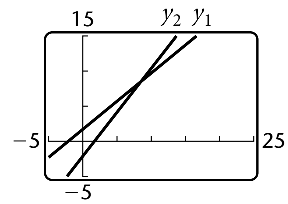
Since there are many horizontal lines that cross the graph more than once, the function is not one-to-one.

44. The graph of $f(x) = -x^3 + 2$ is shown below.



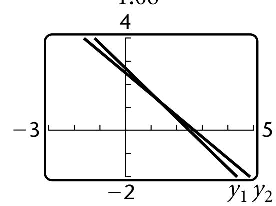
Since there is no horizontal line that crosses the graph more than once, the function is one-to-one.

45. $y_1 = 0.8x + 1.7$,
$y_2 = \frac{x - 1.7}{0.8}$



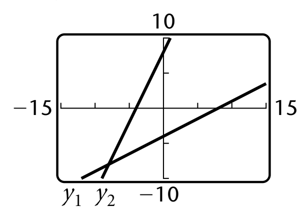
Both the domain and the range of f are the set of all real numbers. Then both the domain and the range of f^{-1} are also the set of all real numbers.

46. $y_1 = 2.7 - 1.08x$,
$y_2 = \frac{2.7 - x}{1.08}$



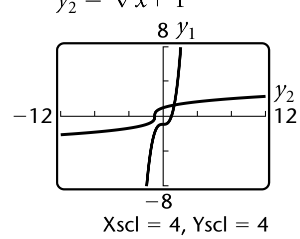
Both the domain and the range of f are the set of all real numbers. Then both the domain and the range of f^{-1} are also the set of all real numbers.

47. $y_1 = \frac{1}{2}x - 4$,
$y_2 = 2x + 8$



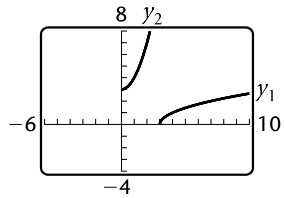
Both the domain and the range of f are the set of all real numbers. Then both the domain and the range of f^{-1} are also the set of all real numbers.

48. $y_1 = x^3 - 1$,
$y_2 = \sqrt[3]{x + 1}$



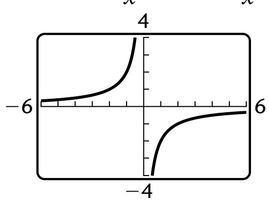
Both the domain and the range of f are the set of all real numbers. Then both the domain and the range of f^{-1} are also the set of all real numbers.

49. $y_1 = \sqrt{x-3}$,
$y_2 = x^2 + 3, x \geqslant 0$



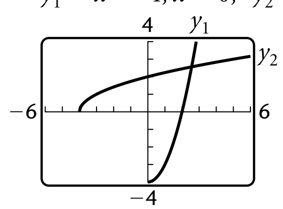
The domain of f is $[3, \infty)$ and the range of f is $[0, \infty)$. Then the domain of f^{-1} is $[0, \infty)$ and the range of f^{-1} is $[3, \infty)$.

50. $y_1 = -\dfrac{2}{x}$, $y_2 = -\dfrac{2}{x}$



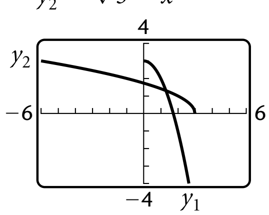
Both the domain and the range of f are $(-\infty, 0) \cup (0, \infty)$. Since $f^{-1} = f$, f^{-1} has the same domain and range.

51. $y_1 = x^2 - 4, x \geqslant 0$; $y_2 = \sqrt{4 + x}$



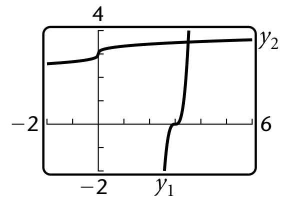
Since it is specified that $x \geq 0$, the domain of f is $[0, \infty)$. The range of f is $[-4, \infty)$. Then the domain of f^{-1} is $[-4, \infty)$ and the range of f^{-1} is $[0, \infty)$.

52. $y_1 = 3 - x^2, x \geqslant 0$;
$y_2 = \sqrt{3 - x}$



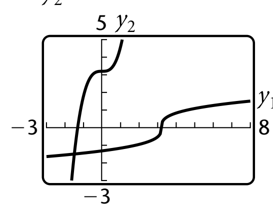
Since it is specified that $x \geq 0$, the domain of f is $[0, \infty)$. The range of f is $(-\infty, 3]$. Then the domain of f^{-1} is $(-\infty, 3]$ and the range of f^{-1} is $[0, \infty)$.

53. $y_1 = (3x - 9)^3$, $y_2 = \dfrac{\sqrt[3]{x} + 9}{3}$



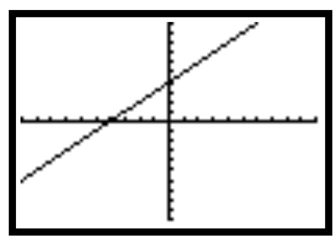
Both the domain and the range of f are the set of all real numbers. Then both the domain and the range of f^{-1} are also the set of all real numbers.

54.
$y_1 = \sqrt[3]{\dfrac{x - 3.2}{1.4}}$,
$y_2 = 1.4x^3 + 3.2$



Both the domain and the range of f are the set of all real numbers. Then both the domain and the range of f^{-1} are also the set of all real numbers.

55. a) The graph of $f(x) = x + 4$ is shown below. It passes the horizontal-line test, so it is one-to-one.



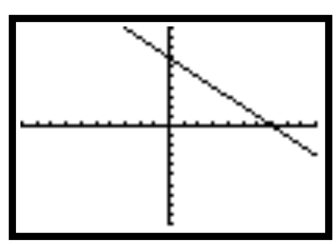
b) Replace $f(x)$ with y: $y = x + 4$

Interchange x and y: $x = y + 4$

Solve for y: $x - 4 = y$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = x - 4$

56. a) The graph of $f(x) = 7 - x$ is shown below. It passes the horizontal-line test, so it is one-to-one.



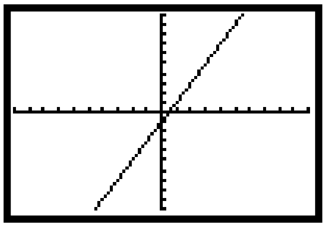
b) Replace $f(x)$ with y: $y = 7 - x$

Interchange x and y: $x = 7 - y$

Solve for y: $y = 7 - x$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = 7 - x$

57. a) The graph of $f(x) = 2x - 1$ is shown below. It passes the horizontal-line test, so it is one-to-one.



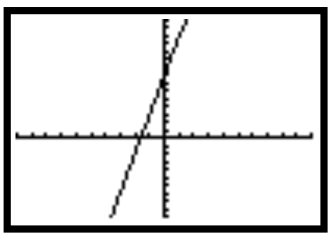
b) Replace $f(x)$ with y: $y = 2x - 1$

Interchange x and y: $x = 2y - 1$

Solve for y: $\frac{x+1}{2} = y$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \frac{x+1}{2}$

58. a) The graph of $f(x) = 5x + 8$ is shown below. It passes the horizontal-line test, so it is one-to-one.



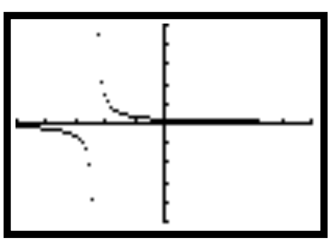
b) Replace $f(x)$ with y: $y = 5x + 8$

Interchange x and y: $x = 5y + 8$

Solve for y: $\frac{x-8}{5} = y$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \frac{x-8}{5}$

59. a) The graph of $f(x) = \frac{4}{x+7}$ is shown below. It passes the horizontal-line test, so the function is one-to-one.



b) Replace $f(x)$ with y: $y = \frac{4}{x+7}$

Interchange x and y: $x = \frac{4}{y+7}$

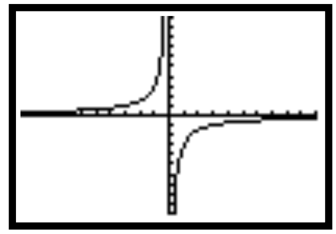
Solve for y: $x(y+7) = 4$

$$y + 7 = \frac{4}{x}$$

$$y = \frac{4}{x} - 7$$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \frac{4}{x} - 7$

60. a) The graph of $f(x) = -\frac{3}{x}$ is shown below. It passes the horizontal-line test, so it is one-to-one.



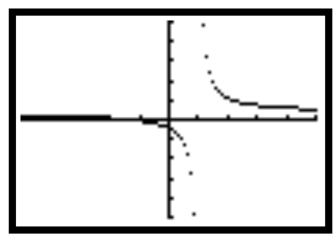
b) Replace $f(x)$ with y: $y = -\frac{3}{x}$

Interchange x and y: $x = -\frac{3}{y}$

Solve for y: $y = -\frac{3}{x}$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = -\frac{3}{x}$

61. a) The graph of $f(x) = \frac{x+4}{x-3}$ is shown below. It passes the horizontal-line test, so the function is one-to-one.



b) Replace $f(x)$ with y: $y = \frac{x+4}{x-3}$

Interchange x and y: $x = \frac{y+4}{y-3}$

Solve for y: $(y-3)x = y + 4$

$$xy - 3x = y + 4$$

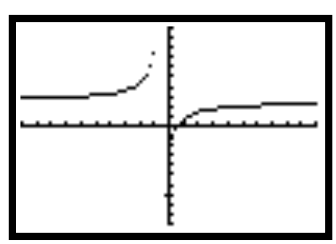
$$xy - y = 3x + 4$$

$$y(x-1) = 3x + 4$$

$$y = \frac{3x+4}{x-1}$$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \frac{3x+4}{x-1}$

62. a) The graph of $f(x) = \frac{5x-3}{2x+1}$ is shown below. It passes the horizontal-line test, so it is one-to-one.



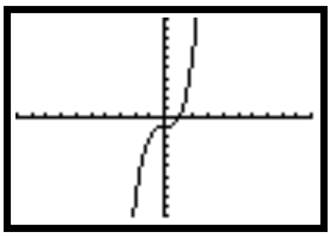
b) Replace $f(x)$ with y: $y = \dfrac{5x-3}{2x+1}$

Interchange x and y: $x = \dfrac{5y-3}{2y+1}$

Solve for y: $\dfrac{x+3}{5-2x} = y$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \dfrac{x+3}{5-2x}$

63. a) The graph of $f(x) = x^3-1$ is shown below. It passes the horizontal-line test, so the function is one-to-one.



b) Replace $f(x)$ with y: $y = x^3 - 1$

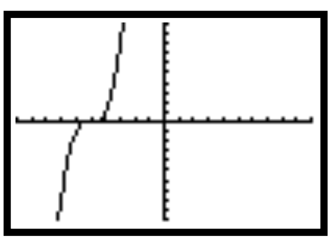
Interchange x and y: $x = y^3 - 1$

Solve for y: $x + 1 = y^3$

$\sqrt[3]{x+1} = y$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \sqrt[3]{x+1}$

64. a) The graph of $f(x) = (x+5)^3$ is shown below. It passes the horizontal-line test, so it is one-to-one.



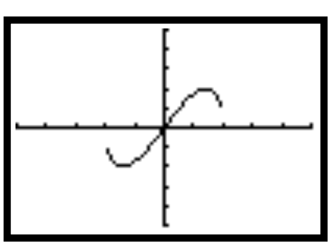
b) Replace $f(x)$ with y: $y = (x+5)^3$

Interchange x and y: $x = (y+5)^3$

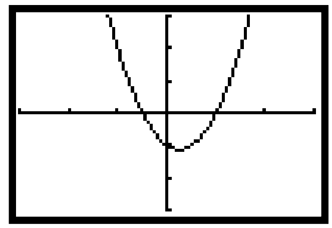
Solve for y: $\sqrt[3]{x} - 5 = y$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \sqrt[3]{x} - 5$

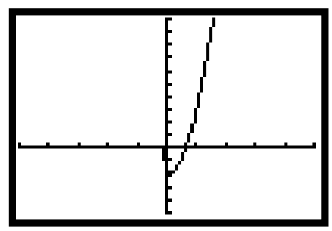
65. a) The graph of $f(x) = x\sqrt{4-x^2}$ is shown below. Since there are many horizontal lines that cross the graph more than once, the function is not one-to-one and thus does not have an inverse that is a function.



66. a) The graph of $f(x) = 2x^2 - x - 1$ is shown below. Since there are many horizontal lines that cross the graph more than once, the function is not one-to-one and thus does not have an inverse that is a function.



67. a) The graph of $f(x) = 5x^2 - 2$, $x \geq 0$ is shown below. It passes the horizontal-line test, so it is one-to-one.



b) Replace $f(x)$ with y: $y = 5x^2 - 2$

Interchange x and y: $x = 5y^2 - 2$

Solve for y: $x + 2 = 5y^2$

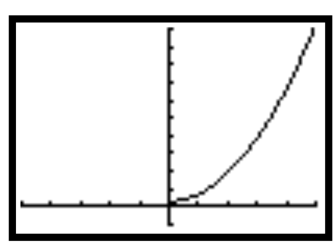
$\dfrac{x+2}{5} = y^2$

$\sqrt{\dfrac{x+2}{5}} = y$

(We take the principal square root, because $x \geq 0$ in the original equation.)

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \sqrt{\dfrac{x+2}{5}}$ for all x in the range of $f(x)$, or $f^{-1}(x) = \sqrt{\dfrac{x+2}{5}}$, $x \geq -2$

68. a) The graph of $f(x) = 4x^2 + 3$, $x \geq 0$ is shown below. It passes the horizontal-line test, so the function is one-to-one.



b) Replace $f(x)$ with y: $y = 4x^2 + 3$

Interchange x and y: $x = 4y^2 + 3$

Solve for y: $x - 3 = 4y^2$

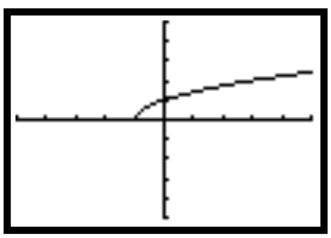
$$\frac{x-3}{4} = y^2$$

$$\frac{\sqrt{x-3}}{2} = y$$

(We take the principal square root since $x \geq 0$ in the original function.)

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \frac{\sqrt{x-3}}{2}$ for all x in the range of $f(x)$, or $f^{-1}(x) = \frac{\sqrt{x-3}}{2}$, $x \geq 3$

69. a) The graph of $f(x) = \sqrt{x+1}$ is shown below. It passes the horizontal-line test, so the function is one-to-one.



b) Replace $f(x)$ with y: $y = \sqrt{x+1}$

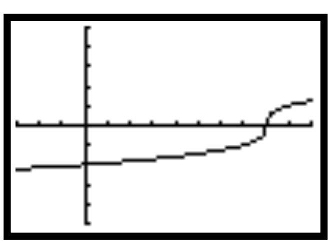
Interchange x and y: $x = \sqrt{y+1}$

Solve for y: $x^2 = y + 1$

$$x^2 - 1 = y$$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = x^2 - 1$ for all x in the range of $f(x)$, or $f^{-1}(x) = x^2 - 1$, $x \geq 0$.

70. a) The graph of $f(x) = \sqrt[3]{x-8}$ is shown below. It passes the horizontal-line test, so the function is one-to-one.



b) Replace $f(x)$ with y: $y = \sqrt[3]{x-8}$

Interchange x and y: $x = \sqrt[3]{y-8}$

Solve for y: $x^3 + 8 = y$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = x^3 + 8$

71. $f(x) = 3x$

The function f multiplies an input by 3. Then to reverse this procedure, f^{-1} would divide each of its inputs by 3. Thus, $f^{-1}(x) = \frac{x}{3}$, or $f^{-1}(x) = \frac{1}{3}x$.

72. $f(x) = \frac{1}{4}x + 7$

The function f multiplies an input by $\frac{1}{4}$ and then adds 7. To reverse this procedure, f^{-1} would subtract 7 from each of its inputs and then multiply by 4. Thus, $f^{-1}(x) = 4(x - 7)$.

73. $f(x) = -x$

The outputs of f are the opposites, or additive inverses, of the inputs. Then the outputs of f^{-1} are the opposites of its inputs. Thus, $f^{-1}(x) = -x$.

74. $f(x) = \sqrt[3]{x} - 5$

The function f takes the cube root of an input and then subtracts 5. To reverse this procedure, f^{-1} would add 5 to each of its inputs and then raise the result to the third power. Thus, $f^{-1}(x) = (x + 5)^3$.

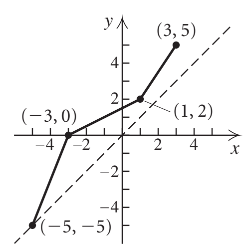
75. $f(x) = \sqrt[3]{x-5}$

The function f subtracts 5 from each input and then takes the cube root of the result. To reverse this procedure, f^{-1} would raise each input to the third power and then add 5 to the result. Thus, $f^{-1}(x) = x^3 + 5$.

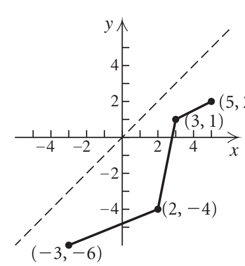
76. $f(x) = x^{-1}$

The outputs of f are the reciprocals of the inputs. Then the outputs of f^{-1} are the reciprocals of its inputs. Thus, $f^{-1}(x) = x^{-1}$.

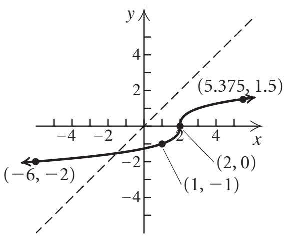
77. We reflect the graph of f across the line $y = x$. The reflections of the labeled points are $(-5, -5)$, $(-3, 0)$, $(1, 2)$, and $(3, 5)$.



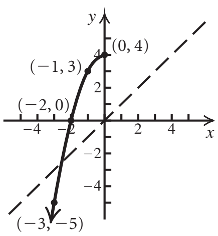
78. We reflect the graph of f across the line $y = x$. The reflections of the labeled points are $(-3, -6)$, $(2, -4)$, $(3, 1)$, and $(5, 2)$.



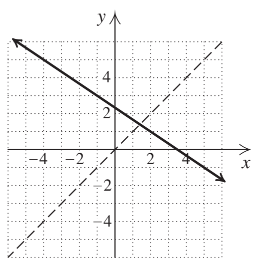
79. We reflect the graph of f across the line $y = x$. The reflections of the labeled points are $(-6, -2)$, $(1, -1)$, $(2, 0)$, and $(5.375, 1.5)$.



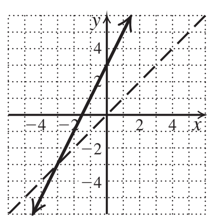
80. We reflect the graph of f across the line $y = x$. The reflections of the labeled points are $(-3,-5)$, $(-2,0)$, $(-1,3)$, and $(0,4)$.



81. We reflect the graph of f across the line $y = x$.



82. We reflect the graph of f across the line $y = x$.



83. We find $(f^{-1}\circ f)(x)$ and $(f\circ f^{-1})(x)$ and check to see that each is x.

$$(f^{-1}\circ f)(x) = f^{-1}(f(x)) = f^{-1}\left(\frac{7}{8}x\right) =$$

$$\frac{8}{7}\left(\frac{7}{8}x\right) = x$$

$$(f\circ f^{-1})(x) = f(f^{-1}(x)) = f\left(\frac{8}{7}x\right) = \frac{7}{8}\left(\frac{8}{7}x\right) = x$$

84. $(f^{-1}\circ f)(x) = 4\left(\dfrac{x+5}{4}\right) - 5 = x+5-5 = x$

$$(f\circ f^{-1})(x) = \frac{4x-5+5}{4} = \frac{4x}{4} = x$$

85. We find $(f^{-1}\circ f)(x)$ and $(f\circ f^{-1})(x)$ and check to see that each is x.

$$(f^{-1}\circ f)(x) = f^{-1}(f(x)) = f^{-1}\left(\frac{1-x}{x}\right) =$$

$$\frac{1}{\frac{1-x}{x}+1} = \frac{1}{\frac{1-x+x}{x}} = \frac{1}{\frac{1}{x}} = x$$

$$(f\circ f^{-1})(x) = f(f^{-1}(x)) = f\left(\frac{1}{x+1}\right) =$$

$$\frac{1-\frac{1}{x+1}}{\frac{1}{x+1}} = \frac{\frac{x+1-1}{x+1}}{\frac{1}{x+1}} = \frac{\frac{x}{x+1}}{\frac{1}{x+1}} = x$$

86. $(f^{-1}\circ f)(x) = (\sqrt[3]{x+4})^3 - 4 = x+4-4 = x$

$(f\circ f^{-1})(x) = \sqrt[3]{x^3-4+4} = \sqrt[3]{x^3} = x$

87. $(f^{-1}\circ f)(x) = f^{-1}(f(x)) = \dfrac{5\left(\frac{2}{5}x+1\right)-5}{2} =$

$$\frac{2x+5-5}{2} = \frac{2x}{2} = x$$

$$(f\circ f^{-1})(x) = f(f^{-1}(x) = \frac{2}{5}\left(\frac{5x-5}{2}\right) + 1 =$$

$$x-1+1 = x$$

88. $(f^{-1}\circ f)(x) = \dfrac{4\left(\frac{x+6}{3x-4}\right)+6}{3\left(\frac{x+6}{3x-4}\right)-1} = \dfrac{\frac{4x+24}{3x-4}+6}{\frac{3x+18}{3x-4}-1} =$

$$\frac{\frac{4x+24+18x-24}{3x-4}}{\frac{3x+18-3x+4}{3x-4}} = \frac{22x}{3x-4}\cdot\frac{3x-4}{22} = \frac{22x}{22} = x$$

$$(f\circ f^{-1})(x) = \frac{\frac{4x+6}{3x-1}+6}{3\left(\frac{4x+6}{3x-1}\right)-4} = \frac{\frac{4x+6+18x-6}{3x-1}}{\frac{12x+18}{3x-1}-4} =$$

$$\frac{\frac{22x}{3x-1}}{\frac{12x+18-12x+4}{3x-1}} = \frac{22x}{3x-1}\cdot\frac{3x-1}{22} = \frac{22x}{22} = x$$

89. Replace $f(x)$ with y: $y = 5x-3$

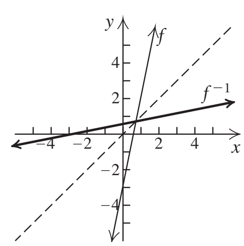
Interchange x and y: $x = 5y-3$

Solve for y: $x+3 = 5y$

$$\frac{x+3}{5} = y$$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \dfrac{x+3}{5}$, or $\dfrac{1}{5}x + \dfrac{3}{5}$

The domain and range of f are $(-\infty,\infty)$, so the domain and range of f^{-1} are also $(-\infty,\infty)$.



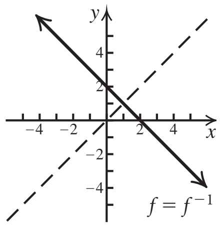
90. Replace $f(x)$ with y: $y = 2 - x$

Interchange x and y: $x = 2 - y$

Solve for y: $y = 2 - x$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = 2 - x$

The domain and range of f are $(-\infty, \infty)$, so the domain and range of f^{-1} are also $(-\infty, \infty)$.



91. Replace $f(x)$ with y: $y = \frac{2}{x}$

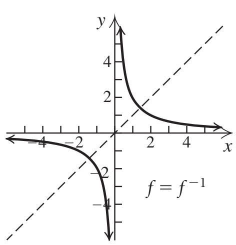
Interchange x and y: $x = \frac{2}{y}$

Solve for y: $xy = 2$

$$y = \frac{2}{x}$$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \frac{2}{x}$

The domain and range of f are $(-\infty, 0) \cup (0, \infty)$, so the domain and range of f^{-1} are also $(-\infty, 0) \cup (0, \infty)$.



92. Replace $f(x)$ with y: $y = -\frac{3}{x+1}$

Interchange x and y: $x = -\frac{3}{y+1}$

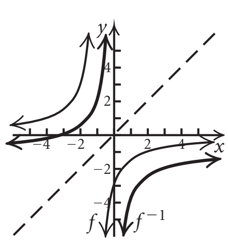
Solve for y: $xy + x = -3$

$$xy = -3 - x$$

$$y = \frac{-3 - x}{x}, or -\frac{3}{x} - 1$$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = -\frac{3}{x} - 1$

The domain of f is $(-\infty, -1) \cup (-1, \infty)$ and the range of f is $(-\infty, 0) \cup (0, \infty)$. Thus the domain of f^{-1} is $(-\infty, 0) \cup (0, \infty)$ and the range of f^{-1} is $(-\infty, -1) \cup (-1, \infty)$.



93. Replace $f(x)$ with y: $y = \frac{1}{3}x^3 - 2$

Interchange x and y: $x = \frac{1}{3}y^3 - 2$

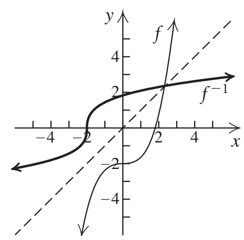
Solve for y: $x + 2 = \frac{1}{3}y^3$

$$3x + 6 = y^3$$

$$\sqrt[3]{3x+6} = y$$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \sqrt[3]{3x+6}$

The domain and range of f are $(-\infty, \infty)$, so the domain and range of f^{-1} are also $(-\infty, \infty)$.



94. Replace $f(x)$ with y: $y = \sqrt[3]{x} - 1$

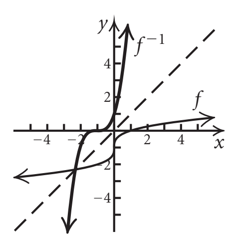
Interchange x and y: $x = \sqrt[3]{y} - 1$

Solve for y: $x + 1 = \sqrt[3]{y}$

$$(x+1)^3 = y$$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = (x+1)^3$

The domain and range of f are $(-\infty, \infty)$, so the domain and range of f^{-1} are also $(-\infty, \infty)$.



95. Replace $f(x)$ with y: $y = \frac{x+1}{x-3}$

Interchange x and y: $x = \frac{y+1}{y-3}$

Solve for y: $xy - 3x = y + 1$

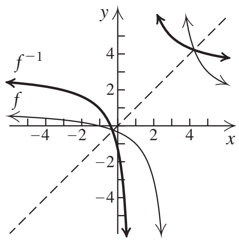
$$xy - y = 3x + 1$$

$$y(x-1) = 3x + 1$$

$$y = \frac{3x+1}{x-1}$$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \frac{3x+1}{x-1}$

The domain of f is $(-\infty, 3) \cup (3, \infty)$ and the range of f is $(-\infty, 1) \cup (1, \infty)$. Thus the domain of f^{-1} is $(-\infty, 1) \cup (1, \infty)$ and the range of f^{-1} is $(-\infty, 3) \cup (3, \infty)$.



96. Replace $f(x)$ with y: $y = \dfrac{x-1}{x+2}$

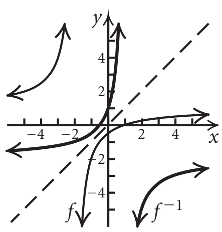
Interchange x and y: $x = \dfrac{y-1}{y+2}$

Solve for y: $xy + 2x = y - 1$

$$2x + 1 = y - xy$$
$$2x + 1 = y(1-x)$$
$$\frac{2x+1}{1-x} = y$$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \dfrac{2x+1}{1-x}$

The domain of f is $(-\infty,-2)\cup(-2,\infty)$ and the range of f is $(-\infty,1)\cup(1,\infty)$. Thus the domain of f^{-1} is $(-\infty,1)\cup(1,\infty)$ and the range of f^{-1} is $(-\infty,-2)\cup(-2,\infty)$.



97. Since $f(f^{-1}(x)) = f^{-1}(f(x)) = x$, then $f(f^{-1}(5)) = 5$ and $f^{-1}(f(a)) = a$.

98. Since $f^{-1}(f(x)) = f(f^{-1}(x)) = x$, then $f^{-1}(f(p)) = p$ and $f(f^{-1}(1253)) = 1253$.

99. a) $s(x) = \dfrac{2x-3}{2}$

$$s(5) = \frac{2\cdot 5 - 3}{2} = \frac{10-3}{2} = \frac{7}{2} = 3\frac{1}{2}$$
$$s\left(7\frac{1}{2}\right) = s(7.5) = \frac{2\cdot 7.5 - 3}{2} = \frac{15-3}{2} = \frac{12}{2} = 6$$
$$s(8) = \frac{2\cdot 8 - 3}{2} = \frac{16-3}{2} = \frac{13}{2} = 6\frac{1}{2}$$

b) The graph of $s(x)$ passes the horizontal-line test and thus has an inverse that is a function.

Replace $f(x)$ with y: $y = \dfrac{2x-3}{2}$

Interchange x and y: $x = \dfrac{2y-3}{2}$

Solve for y: $2x = 2y - 3$

$$2x + 3 = 2y$$
$$\frac{2x+3}{2} = y$$

Replace y with $s^{-1}(x)$: $s^{-1}(x) = \dfrac{2x+3}{2}$

c) $s^{-1}(3) = \dfrac{2\cdot 3 + 3}{2} = \dfrac{6+3}{2} = \dfrac{9}{2} = 4\dfrac{1}{2}$

$$s^{-1}\left(5\frac{1}{2}\right) = s^{-1}(5.5) = \frac{2\cdot 5.5 + 3}{2} = \frac{11+3}{2} = \frac{14}{2} = 7$$
$$s^{-1}(7) = \frac{2\cdot 7 + 3}{2} = \frac{14+3}{2} = \frac{17}{2} = 8\frac{1}{2}$$

100. $C(x) = \dfrac{100+5x}{x}$

Replace $C(x)$ with y: $y = \dfrac{100+5x}{x}$

Interchange x and y: $x = \dfrac{100+5y}{y}$

Solve for y: $y = \dfrac{100}{x-5}$

Replace y with $C^{-1}(x)$: $C^{-1}(x) = \dfrac{100}{x-5}$

$C^{-1}(x)$ gives the number of people in the group, where x is the cost per person, in dollars.

101. a) $D(r) = \dfrac{11r+5}{10}$

$$D(0) = \frac{11\cdot 0 + 5}{10} = \frac{5}{10} = 0.5 \text{ ft}$$
$$D(10) = \frac{11\cdot 10 + 5}{10} = \frac{115}{10} = 11.5 \text{ ft}$$
$$D(20) = \frac{11\cdot 20 + 5}{10} = \frac{225}{10} = 22.5 \text{ ft}$$
$$D(50) = \frac{11\cdot 50 + 5}{10} = \frac{555}{10} = 55.5 \text{ ft}$$
$$D(65) = \frac{11\cdot 65 + 5}{10} = \frac{720}{10} = 72 \text{ ft}$$

b) Replace $D(r)$ with y: $y = \dfrac{11r+5}{10}$

Interchange r and y: $r = \dfrac{11y+5}{10}$

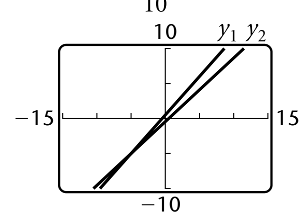
Solve for y: $10r = 11y + 5$

$$10r - 5 = 11y$$
$$\frac{10r-5}{11} = y$$

Replace y with $D^{-1}(r)$: $D^{-1}(r) = \dfrac{10r-5}{11}$

$D^{-1}(r)$ represents the speed, in miles per hour, that the car is traveling when the reaction distance is r feet.

c) $y_1 = \dfrac{11x+5}{10}$, $y_2 = \dfrac{10x-5}{11}$



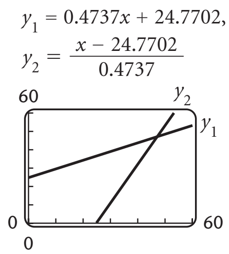
102. a) In 2007, $x = 2007 - 1990 = 17$.

$N(17) = 0.4737(17) + 24.7702 = 32.8231$ lb

In 2010, $x = 2010 - 1990 = 20$.

$N(20) = 0.4737(20) + 24.7702 = 34.2442$ lb

b)



c) $N^{-1}(x)$ represents the number of years after 1990 when x pounds of cheese are consumed per person per year.

103. For an even function f, $f(x) = f(-x)$ so we have $f(x) = f(-x)$ but $x \neq -x$ (for $x \neq 0$). Thus f is not one-to-one and hence it does not have an inverse.

104. C and F are inverses.

105. The functions for which the coefficient of x^2 is negative have a maximum value. These are (b), (d), (f), and (h).

106. The graphs of the functions for which the coefficient of x^2 is positive open up. These are (a), (c), (e), and (g).

107. Since $|2| > 1$ the graph of $f(x) = 2x^2$ can be obtained by stretching the graph of $f(x) = x^2$ vertically. Since $0 < \left|\frac{1}{4}\right| < 1$, the graph of $f(x) = \frac{1}{4}x^2$ can be obtained by shrinking the graph of $y = x^2$ vertically. Thus the graph of $f(x) = 2x^2$, or (a) is narrower.

108. Since $|-5| > 0$ and $0 < \left|\frac{2}{3}\right| < 1$, the graph of (d) is narrower.

109. We can write (f) as $f(x) = -2[x - (-3)]^2 + 1$. Thus the graph of (f) has vertex $(-3, 1)$.

110. For the functions that can be written in the form $f(x) = a(x - 0)^2 + k$, or $f(x) = ax^2 + k$, the line of symmetry is $x = 0$. These are (a), (b), (c), and (d).

111. Graph $y_1 = f(x)$ and $y_2 = g(x)$ and observe that the graphs are reflections of each other across the line $y = x$. Thus, the functions are inverses of each other.

112. Graph $y_1 = f(x)$ and $y_2 = g(x)$ and observe that the graphs are not reflections of each other across the line $y = x$. Thus, the functions are not inverses of each other.

113. Graph $y_1 = f(x)$ and $y_2 = g(x)$ and observe that the graphs are not reflections of each other across the line $y = x$. Thus, the functions are not inverses of each other.

114. Graph $y_1 = f(x)$ and $y_2 = g(x)$ and observe that the graphs are reflections of each other across the line $y = x$. Thus, the functions are inverses of each other.

115. The graph of $f(x) = x^2 - 3$ is a parabola with vertex $(0, -3)$. If we consider x-values such that $x \geq 0$, then the graph is the right-hand side of the parabola and it passes the horizontal line test. We find the inverse of $f(x) = x^2 - 3$, $x \geq 0$.

Replace $f(x)$ with y: $y = x^2 - 3$

Interchange x and y: $x = y^2 - 3$

Solve for y: $x + 3 = y^2$

$\sqrt{x+3} = y$

(We take the principal square root, because $x \geq 0$ in the original equation.)

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \sqrt{x+3}$ for all x in the range of $f(x)$, or $f^{-1}(x) = \sqrt{x+3}$, $x \geq -3$.

Answers may vary. There are other restrictions that also make $f(x)$ one-to-one.

116. No; the graph of f does not pass the horizontal line test.

117. Answers may vary. $f(x) = \frac{3}{x}$, $f(x) = 1 - x$, $f(x) = x$.

118. First find $f^{-1}(x)$.

Replace $f^{-1}(x)$ with y: $y = ax + b$

Interchange x and y: $x = ay + b$

Solve for y: $x - b = ay$

$\frac{x-b}{a} = y$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \frac{x-b}{a} = \frac{1}{a}x - \frac{b}{a}$

Now we find the values of a and b for which $ax + b = \frac{1}{a}x - \frac{b}{a}$. We see that $a = \frac{1}{a}$ for $a = \pm 1$. If $a = 1$, we have $x + b = x - b$, so $b = 0$. If $a = -1$, we have $-x + b = -x + b$, so b can be any real number.

Exercise Set 5.2

1. $e^4 \approx 54.5982$

2. $e^{10} \approx 22,026.4658$

3. $e^{-2.458} \approx 0.0856$

4. $\left(\frac{1}{e^3}\right)^2 \approx 0.0025$

5. $f(x) = -2^x - 1$

$f(0) = -2^0 - 1 = -1 - 1 = -2$

The only graph with y-intercept $(0, -2)$ is (f).

6. $f(x) = -\left(\frac{1}{2}\right)^x$

$f(0) = -\left(\frac{1}{2}\right)^0 = -1$

Since the y-intercept is $(0, -1)$, the correct graph is (a) or (c). Check another point on the graph.

$f(-1) = -\left(\frac{1}{2}\right)^{-1} = -2$, so the point $(-1, -2)$ is on the graph. Thus (c) is the correct choice.

7. $f(x) = e^x + 3$

This is the graph of $f(x) = e^x$ shifted up 3 units. Then (e) is the correct choice.

8. $f(x) = e^{x+1}$

This is the graph of $f(x) = e^x$ shifted left 1 unit. Then (b) is the correct choice.

9. $f(x) = 3^{-x} - 2$

$f(0) = 3^{-0} - 2 = 1 - 2 = -1$

Since the y-intercept is $(0, -1)$, the correct graph is (a) or (c). Check another point on the graph. $f(-1) = 3^{-(-1)} - 2 = 3 - 2 = 1$, so $(-1, 1)$ is on the graph. Thus (a) is the correct choice.

10. $f(x) = 1 - e^x$

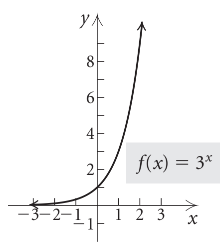
$f(0) = 1 - e^0 = 1 - 1 = 0$

The only graph with y-intercept $(0, 0)$ is (d).

11. Graph $f(x) = 3^x$.

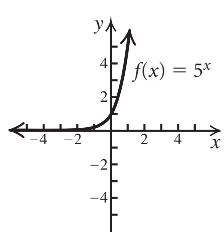
Compute some function values, plot the corresponding points, and connect them with a smooth curve.

x	$y = f(x)$	(x, y)
-3	$\frac{1}{27}$	$\left(-3, \frac{1}{27}\right)$
-2	$\frac{1}{9}$	$\left(-2, \frac{1}{9}\right)$
-1	$\frac{1}{3}$	$\left(-1, \frac{1}{3}\right)$
0	1	$(0, 1)$
1	3	$(1, 3)$
2	9	$(2, 9)$
3	27	$(3, 27)$



12. Graph $f(x) = 5^x$.

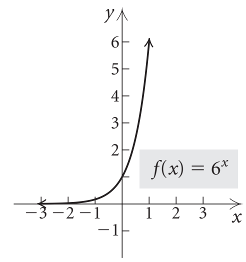
x	$y = f(x)$	(x, y)
-3	$\frac{1}{125}$	$\left(-3, \frac{1}{125}\right)$
-2	$\frac{1}{25}$	$\left(-2, \frac{1}{25}\right)$
-1	$\frac{1}{5}$	$\left(-1, \frac{1}{5}\right)$
0	1	$(0, 1)$
1	5	$(1, 5)$
2	25	$(2, 25)$
3	125	$(3, 125)$



13. Graph $f(x) = 6^x$.

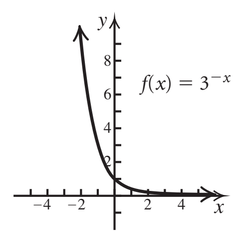
Compute some function values, plot the corresponding points, and connect them with a smooth curve.

x	$y = f(x)$	(x, y)
-3	$\frac{1}{216}$	$\left(-3, \frac{1}{216}\right)$
-2	$\frac{1}{36}$	$\left(-2, \frac{1}{36}\right)$
-1	$\frac{1}{6}$	$\left(-1, \frac{1}{6}\right)$
0	1	$(0, 1)$
1	6	$(1, 6)$
2	36	$(2, 36)$
3	216	$(3, 216)$



14. Graph $f(x) = 3^{-x}$.

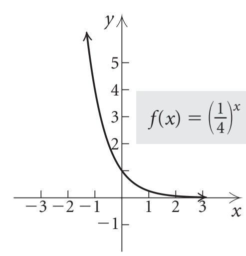
x	$y = f(x)$	(x, y)
-3	27	$(-3, 27)$
-2	9	$(-2, 9)$
-1	3	$(-1, 3)$
0	1	$(0, 1)$
1	$\frac{1}{3}$	$\left(1, \frac{1}{3}\right)$
2	$\frac{1}{9}$	$\left(2, \frac{1}{9}\right)$
3	$\frac{1}{27}$	$\left(3, \frac{1}{27}\right)$



15. Graph $f(x) = \left(\frac{1}{4}\right)^x$.

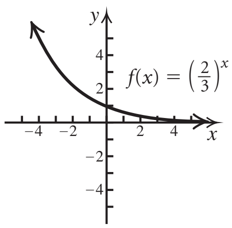
Compute some function values, plot the corresponding points, and connect them with a smooth curve.

x	$y = f(x)$	(x, y)
-3	64	$(-3, 64)$
-2	16	$(-2, 16)$
-1	4	$(-1, 4)$
0	1	$(0, 1)$
1	$\frac{1}{4}$	$\left(1, \frac{1}{4}\right)$
2	$\frac{1}{16}$	$\left(2, \frac{1}{16}\right)$
3	$\frac{1}{64}$	$\left(3, \frac{1}{64}\right)$



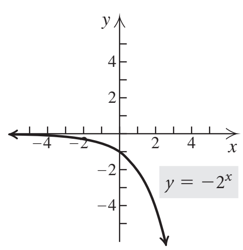
16. Graph $f(x) = \left(\dfrac{2}{3}\right)^x$.

x	$y = f(x)$	(x, y)
-3	$\frac{27}{8}$	$\left(-3, \frac{27}{8}\right)$
-2	$\frac{9}{4}$	$\left(-2, \frac{9}{4}\right)$
-1	$\frac{3}{2}$	$\left(-1, \frac{3}{2}\right)$
0	1	$(0, 1)$
1	$\frac{2}{3}$	$\left(1, \frac{2}{3}\right)$
2	$\frac{4}{9}$	$\left(2, \frac{4}{9}\right)$
3	$\frac{8}{27}$	$\left(3, \frac{8}{27}\right)$



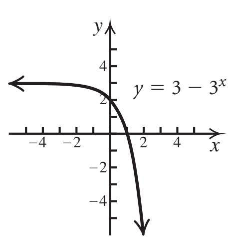
17. Graph $y = -2^x$.

x	y	(x, y)
-3	$-\frac{1}{8}$	$\left(-3, -\frac{1}{8}\right)$
-2	$-\frac{1}{4}$	$\left(-2, -\frac{1}{4}\right)$
-1	$-\frac{1}{2}$	$\left(-1, -\frac{1}{2}\right)$
0	-1	$(0, -1)$
1	-2	$(1, -2)$
2	-4	$(2, -4)$
3	-8	$(3, -8)$



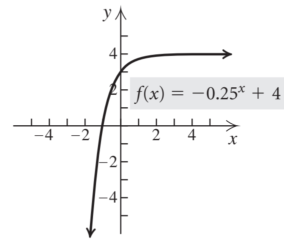
18. Graph $y = 3 - 3^x$.

x	y	(x, y)
-3	$\frac{80}{27}$	$\left(-3, \frac{80}{27}\right)$
-2	$\frac{26}{9}$	$\left(-2, \frac{26}{9}\right)$
-1	$\frac{8}{3}$	$\left(-1, \frac{8}{3}\right)$
0	2	$(0, 2)$
1	0	$(1, 0)$
2	-6	$(2, -6)$
3	-24	$(3, -24)$



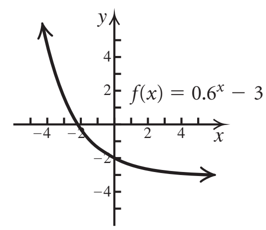
19. Graph $f(x) = -0.25^x + 4$.

x	$y = f(x)$	(x, y)
-3	-60	$(-3, -60)$
-2	-12	$(-2, -12)$
-1	0	$(-1, 0)$
0	3	$(0, 3)$
1	3.75	$(1, 3.75)$
2	3.94	$(2, 3.94)$
3	3.98	$(3, 3.98)$



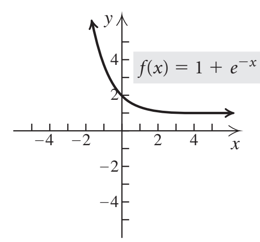
20. Graph $f(x) = 0.6^x - 3$.

x	$y = f(x)$	(x, y)
-3	1.63	$(-3, 1.63)$
-2	-0.22	$(-2, -0.22)$
-1	-1.33	$(-1, -1.33)$
0	-2	$(0, -2)$
1	-2.4	$(1, -2.4)$
2	-2.64	$(2, -2.64)$
3	-2.78	$(3, -2.78)$



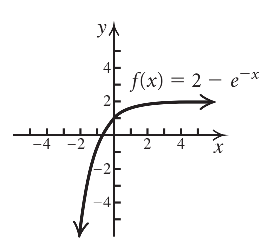
21. Graph $f(x) = 1 + e^{-x}$.

x	$y = f(x)$	(x, y)
-3	21.1	$(-3, 21.1)$
-2	8.4	$(-2, 8.4)$
-1	3.7	$(-1, 3.7)$
0	2	$(0, 2)$
1	1.4	$(1, 1.4)$
2	1.1	$(2, 1.1)$
3	1.0	$(3, 1.0)$



22. Graph $f(x) = 2 - e^{-x}$.

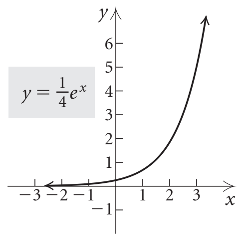
x	$y = f(x)$	(x, y)
-3	-18.1	$(-3, -18.1)$
-2	-5.4	$(-2, -5.4)$
-1	-0.8	$(-1, -0.8)$
0	1	$(0, 1)$
1	1.6	$(1, 1.6)$
2	1.9	$(2, 1.9)$
3	2.0	$(3, 2.0)$



23. Graph $y = \dfrac{1}{4}e^x$.

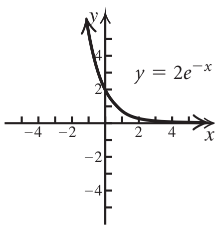
Choose values for x and compute the corresponding y-values. Plot the points (x, y) and connect them with a smooth curve.

x	y	(x,y)
-3	0.0124	$(-3, 0.0124)$
-2	0.0338	$(-2, 0.0338)$
-1	0.0920	$(-1, 0.0920)$
0	0.25	$(0, 0.25)$
1	0.6796	$(1, 0.6796)$
2	1.8473	$(2, 1.8473)$
3	5.0214	$(3, 5.0214)$



24. Graph $y = 2e^{-x}$.

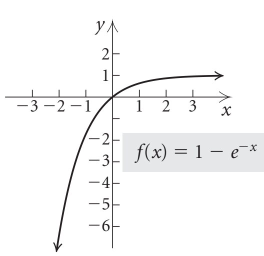
x	y	(x,y)
-3	40.1711	$(-3, 40.1711)$
-2	14.7781	$(-2, 14.7781)$
-1	5.4366	$(-1, 5.4366)$
0	2	$(0, 2)$
1	0.7358	$(1, 0.7358)$
2	0.2707	$(2, 0.2707)$
3	0.0996	$(3, 0.0996)$



25. Graph $f(x) = 1 - e^{-x}$.

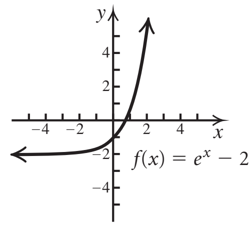
Compute some function values, plot the corresponding points, and connect them with a smooth curve.

x	y	(x,y)
-3	-19.0855	$(-3, -19.0855)$
-2	-6.3891	$(-2, -6.3891)$
-1	-1.7183	$(-1, -1.7183)$
0	0	$(0, 0)$
1	0.6321	$(1, 0.6321)$
2	0.8647	$(2, 0.8647)$
3	0.9502	$(3, 0.9502)$

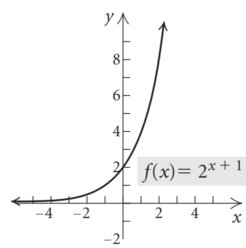


26. Graph $f(x) = e^x - 2$.

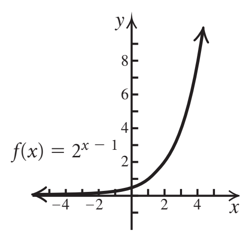
x	y	(x,y)
-3	-1.9502	$(-3, -1.9502)$
-2	-1.8647	$(-2, -1.8647)$
-1	-1.6321	$(-1, -1.6321)$
0	-1	$(0, -1)$
1	0.7183	$(1, 0.7183)$
2	5.3891	$(2, 5.3891)$
3	18.0855	$(3, 18.0855)$



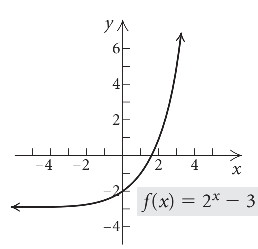
27. Shift the graph of $y = 2^x$ left 1 unit.



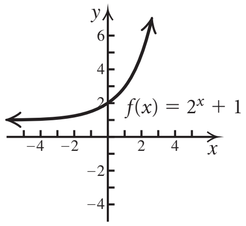
28. Shift the graph of $y = 2^x$ right 1 unit.



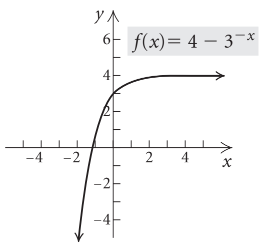
29. Shift the graph of $y = 2^x$ down 3 units.



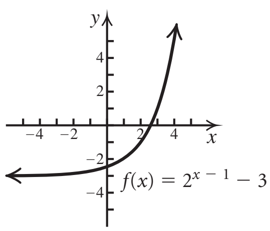
30. Shift the graph of $y = 2^x$ up 1 unit.



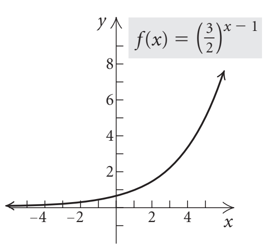
31. Reflect the graph of $y = 3^x$ across the y-axis, then across the x-axis, and then shift it up 4 units.



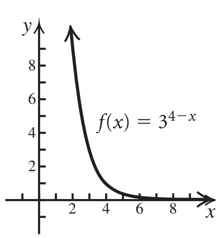
32. Shift the graph of $y = 2^x$ right 1 unit and down 3 units.



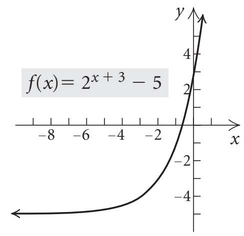
33. Shift the graph of $y = \left(\frac{3}{2}\right)^x$ right 1 unit.



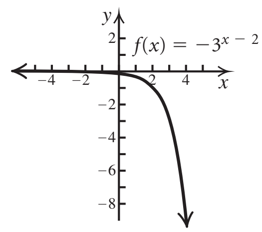
34. Reflect the graph of $y = 3^x$ across the y-axis and then shift it right 4 units.



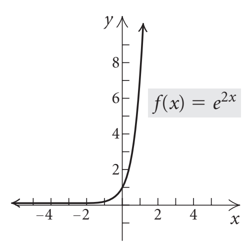
35. Shift the graph of $y = 2^x$ left 3 units and down 5 units.



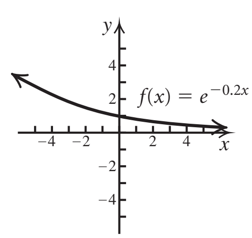
36. Shift the graph of $y = 3^x$ right 2 units and reflect it across the x-axis.



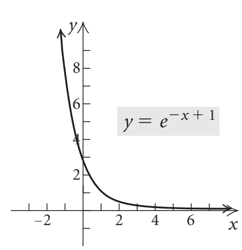
37. Shrink the graph of $y = e^x$ horizontally.



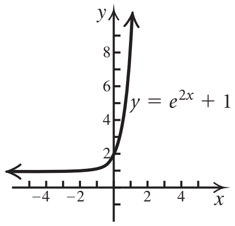
38. Stretch the graph of $y = e^x$ horizontally and reflect it across the y-axis.



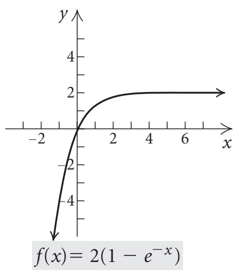
39. Shift the graph of $y = e^x$ left 1 unit and reflect it across the y-axis.



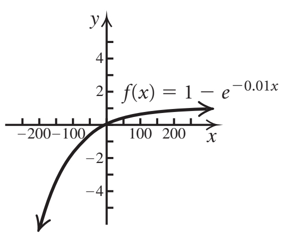
40. Shrink the graph of $y = e^x$ horizontally and shift it up 1 unit.



41. Reflect the graph of $y = e^x$ across the y-axis and then across the x-axis; shift it up 1 unit and then stretch it vertically.



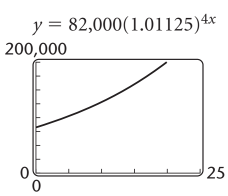
42. Stretch the graph of $y = e^x$ horizontally and reflect it across the y-axis; then reflect it across the x-axis and shift it up 1 unit.



43. a) We use the formula $A = P\left(1 + \dfrac{r}{n}\right)^{nt}$ and substitute 82,000 for P, 0.045 for r, and 4 for n.

$$A(t) = 82,000\left(1 + \frac{0.045}{4}\right)^{4t} = 82,000(1.01125)^{4t}$$

b)



c) $A(0) = 82,000(1.01125)^{4\cdot 0} = \$82,000$

$A(2) = 82,000(1.01125)^{4\cdot 2} \approx \$89,677.22$

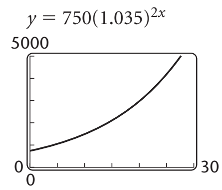
$A(5) = 82,000(1.01125)^{4\cdot 5} \approx \$102,561.54$

$A(10) = 82,000(1.01125)^{4\cdot 10} \approx \$128,278.90$

d) Using the Intersect feature, we find that the first coordinate of the point of intersection of $y_1 = 82,000(1.01125)^{4x}$ and $y_2 = 100,000$ is about 4.43, so the amount in the account will reach \$100,000 in about 4.43 years. This is about 4 years, 5 months, and 5 days.

44. a) $A(t) = 750\left(1 + \dfrac{0.07}{2}\right)^{2t} = 750(1.035)^{2t}$

b)



c) $A(1) = 750(1.035)^{2\cdot 1} \approx \$803.42$

$A(6) = 750(1.035)^{2\cdot 6} \approx \$1133.30$

$A(10) = 750(1.035)^{2\cdot 10} \approx \$1492.34$

$A(15) = 750(1.035)^{2\cdot 15} \approx \$2105.10$

$A(25) = 750(1.035)^{2\cdot 25} \approx \$4188.70$

d) Using the Intersect feature, we find that the first coordinate of the point of intersection of $y_1 = 750(1.035)^{2x}$ and $y_2 = 3000$ is about 20.15, so the amount in the account will reach \$3000 in about 20.15 yr. This is about 20 yr, 1 month, and 24 days.

45. We use the formula $A = P\left(1 + \dfrac{r}{n}\right)^{nt}$ and substitute 3000 for P, 0.05 for r, and 4 for n.

$$A(t) = 3000\left(1 + \frac{0.05}{4}\right)^{4t} = 3000(1.0125)^{4t}$$

On Jacob's sixteenth birthday, $t = 16 - 6 = 10$.

$A(10) = 3000(1.0125)^{4\cdot 10} = 4930.86$

When the CD matures \$4930.86 will be available.

46. a) $A(t) = 10,000\left(1 + \dfrac{0.064}{2}\right)^{2t} = 10,000(1.032)^{2t}$

b) $A(0) = 10,000(1.032)^{2\cdot 0} = \$10,000$

$A(4) = 10,000(1.032)^{2\cdot 4} \approx \$12,865.82$

$A(8) = 10,000(1.032)^{2\cdot 8} \approx \$16,552.94$

$A(10) = 10,000(1.032)^{2\cdot 10} \approx \$18,775.61$

$A(18) = 10,000(1.032)^{2\cdot 18} \approx \$31,079.15$

47. We use the formula $A = P\left(1 + \dfrac{r}{n}\right)^{nt}$ and substitute 3000 for P, 0.04 for r, 2 for n, and 2 for t.

$$A = 3000\left(1 + \frac{0.04}{2}\right)^{2\cdot 2} \approx \$3247.30$$

48. $A = 12,500\left(1 + \dfrac{0.03}{4}\right)^{4\cdot 3} \approx \$13,672.59$

49. We use the formula $A = P\left(1+\frac{r}{n}\right)^{nt}$ and substitute 120,000 for P, 0.025 for r, 1 for n, and 10 for t.

$$A = 120,000\left(1+\frac{0.025}{1}\right)^{1\cdot 10} \approx \$153,610.15$$

50. $A = 120,000\left(1+\frac{0.025}{4}\right)^{4\cdot 10} \approx \$153,963.22$

51. We use the formula $A = P\left(1+\frac{r}{n}\right)^{nt}$ and substitute 53,500 for P, 0.055 for r, 4 for n, and 6.5 for t.

$$A = 53,500\left(1+\frac{0.055}{4}\right)^{4(6.5)} \approx \$76,305.59$$

52. $A = 6250\left(1+\frac{0.0675}{2}\right)^{2(4.5)} \approx \$8425.97$

53. We use the formula $A = P\left(1+\frac{r}{n}\right)^{nt}$ and substitute 17,400 for P, 0.081 for r, 365 for n, and 5 for t.

$$A = 17,400\left(1+\frac{0.081}{365}\right)^{365\cdot 5} \approx \$26,086.69$$

54. $A = 900\left(1+\frac{0.073}{365}\right)^{365(7.25)} \approx \$1527.81$

55. $C(t) = 7.8867(1.0854)^t$

In 2004, $t = 2004 - 2001 = 3$.

$C(3) = 7.8867(1.0854)^3 \approx \$10.1$ billion

In 2009, $t = 2009 - 2001 = 8$.

$C(8) = 7.8867(1.0854)^8 \approx \$15.2$ billion

56. $G(x) = 0.0882(1.0414)^x$

In 1960, $t = 1960 - 1925 = 35$.

$G(35) = 0.0882(1.0414)^{35} \approx 0.36$ lb

In 1990, $t = 1990 - 1925 = 65$.

$G(65) = 0.0882(1.0414)^{65} \approx 1.23$ lb

57. a) $C(t) = 1.0283(1.1483)^t$

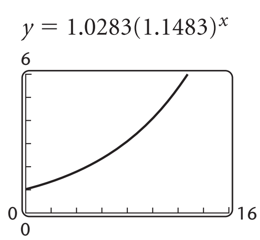
In 2008, $t = 2008 - 1996 = 12$.

$C(12) = 1.0283(1.1483)^{12} \approx 5.4$ billion gallons

In 2010, $t = 2010 - 1996 = 14$.

$C(14) = 1.0283(1.1483)^{14} \approx 7.1$ billion gallons

b)



c) Using the Intersect feature, we find that the first coordinate of the point of intersection of $y_1 = 1.0283(1.1483)^x$ and $y_2 = 5.0$ is about 11.4. Thus, ethanol production will reach 5.0 billion gal about 11.4 yr after 1996.

58. a) $N(10) = 3000(2)^{10/20} \approx 4243$;

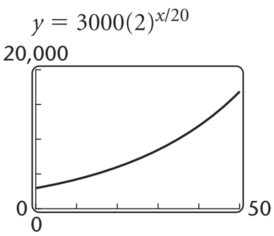
$N(20) = 3000(2)^{20/20} = 6000$;

$N(30) = 3000(2)^{30/20} \approx 8485$;

$N(40) = 3000(2)^{40/20} = 12,000$;

$N(60) = 3000(2)^{60/20} = 24,000$

b)



c) Using the Intersect feature, we find that the first coordinate of the point of intersection of $y_1 = 3000(2)^{x/20}$ and $y_2 = 100,000,000$ is about 300. Thus it takes about 300 min, or 5 hr, for an intestinal infection to be possible.

We could also graph $y = 3000(2)^{x/20} -$ 100,000,000 and use the Zero method to find this result.

59. $G(x) = 433.6(1.5)^x$

In 2009, $x = 2009 - 2004 = 5$.

$G(5) = 433.6(1.5)^5 \approx 3293$ GB

In 2014, $x = 2014 - 2004 = 10$.

$G(10) = 433.6(1.5)^{10} \approx 25,004$ GB

60. $D(20) \approx 166,052$

$D(45) \approx 458,909$

$D(60) \approx 844,531$

$D(70) \approx 1,268,260$

61. In 2007, $x = 2007 - 1996 = 11$.

$E(11) = 139.76(1.194)^{11} \approx \$982.7$ billion

$I(11) = 130.67(1.187)^{11} \approx \$861.3$ billion

In 2012, $x = 2012 - 1996 = 16$.

$E(16) = 139.76(1.194)^{16} \approx \$2384.8$ billion

$I(16) = 130.67(1.187)^{16} \approx \$2029.5$ billion

In 2020, $x = 2020 - 1996 = 24$.

$E(24) = 139.76(1.194)^{24} \approx \$9851.1$ billion

$I(24) = 130.67(1.187)^{24} \approx \$7998.2$ billion

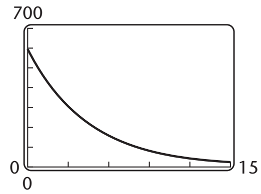
62. In 2010, $t = 2010 - 1964 = 46$.

$K(46) = 35.37(0.99)^{46} \approx 22\%$

In 2015, $t = 2015 - 1964 = 51$.

$K(51) = 35.37(0.99)^{51} \approx 21\%$

63. a) $y = 595(0.8)^x$



b) $V(0) = 595(0.8)^0 = \$595$

$V(1) = 595(0.8)^1 = \$476$

$V(2) = 595(0.8)^2 = \$380.80$

$V(5) = 595(0.8)^5 \approx \$194.97$

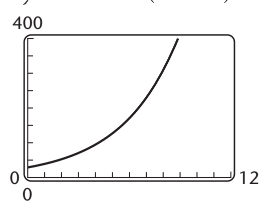
$V(10) = 595(0.8)^{10} \approx \$63.89$

c) Using the Intersect feature, we find that the first coordinate of the point of intersection of $y_1 = 595(0.8)^x$ and $y_2 = 200$ is about 4.89. Thus, the machine will be replaced after about 4.89 years. This is approximately 4 yr, 10 months, and 19 days.

64. $f(16) = 5728.98(1.1214)^{16} \approx 35,830$ visas

$f(19) = 5728.98(1.1214)^{19} \approx 50,527$ visas

65. a) $y = 29.0626(1.3438)^x$



b) $S(t) = 29.0626(1.3438)^t$

In 2000, $t = 2000 - 1999 = 1$.

$S(1) = 29.0626(1.3438)^1 \approx \$39.1$ billion

In 2005, $t = 2005 - 1999 = 6$.

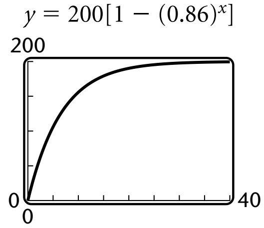
$S(6) = 29.0626(1.3438)^6 \approx \$171.1$ billion

In 2009, $t = 2009 - 1999 = 10$.

$S(10) = 29.0626(1.3438)^{10} \approx \$558.1$ billion

c) Using the Intersect feature, we find that the first coordinate of the point of intersection of $y_1 = 29.0626(1.3438)^x$ and $y_2 = 350.2$ is about 8.4. Thus, online sales will be \$350.2 billion about 8.4 yr after 1999.

66. a) $y = 200[1 - (0.86)^x]$



b) $S(10) = 200[1 - (0.86)^{10}] \approx 155.7$ words per minute

$S(20) = 200[1 - (0.86)^{20}] \approx 190.2$ words per minute

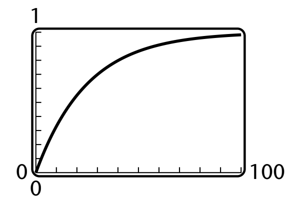
$S(40) = 200[1 - (0.86)^{40}] \approx 199.5$ words per minute

$S(100) \approx 200[1 - (0.86)^{100}] \approx 199.9999$ words per minute

c) Using the Intersect feature, we find that the first coordinate of the point of intersection of $y_1 = 200[1 - (0.86)^x]$ and $y_2 = 100$ is about 4.6. Thus, Sarah's speed is 100 words per minute after she practices for about 4.6 hr.

d) Using the Table feature we find that the graph has a horizontal asymptote, $y = 200$. This means that Sarah could eventually type close to 200 words per minute but never 200 words per minute or more.

67. a) $y = 100(1 - e^{-0.04x})$

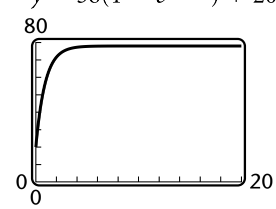


b) $f(25) = 100(1 - e^{-0.04(25)}) \approx 63\%$.

c)
$$\begin{aligned} 90 &= 100(1 - e^{-0.04t}) \\ 0.9 &= 1 - e^{-0.04t} \\ -0.1 &= -e^{-0.04t} \\ 0.1 &= e^{-0.04t} \\ \ln 0.1 &= \ln e^{-0.04t} \\ \ln 0.1 &= -0.04t \\ \frac{\ln 0.01}{-0.04} &= t \\ 58 &\approx t \end{aligned}$$

After about 58 days, 90% of the target market will have bought the product.

68. a) $y = 58(1 - e^{-1.1x}) + 20$



b) $V(1) = \$58(1 - e^{-1.1(1)}) + \$20 \approx \$58.69$

$V(2) = \$58(1 - e^{-1.1(2)}) + \$20 \approx \$71.57$

$V(4) = \$58(1 - e^{-1.1(4)}) + \$20 \approx \$77.29$

$V(6) = \$58(1 - e^{-1.1(6)}) + \$20 \approx \$77.92$

$V(12) = \$58(1 - e^{-1.1(12)}) + \$20 \approx \$78.00$

c) Using the Intersect feature, we find that the first coordinate of the point of intersection of $y_1 = 58(1 - e^{-1.1x}) + 20$ and $y_2 = 75$ is about 2.7. Thus, the value of the stock will be \$75 after about 2.7 months.

69. Graph (c) is the graph of $y = 3^x - 3^{-x}$.

70. Graph (j) is the graph of $y = 3^{-(x+1)^2}$.

71. Graph (a) is the graph of $f(x) = -2.3^x$.

72. Graph (d) is the graph of $f(x) = 30,000(1.4)^x$.

73. Graph (l) is the graph of $y = 2^{-|x|}$.

74. Graph (n) is the graph of $y = 2^{-(x-1)}$.

75. Graph (g) is the graph of $f(x) = (0.58)^x - 1$.

76. Graph (b) is the graph of $y = 2^x + 2^{-x}$.

77. Graph (i) is the graph of $g(x) = e^{|x|}$.

78. Graph (h) is the graph of $f(x) = |2^x - 1|$.

79. Graph (k) is the graph of $y = 2^{-x^2}$.

80. Graph (e) is the graph of $y = |2^{x^2} - 8|$.

81. Graph (m) is the graph of $g(x) = \dfrac{e^x - e^{-x}}{2}$.

82. Graph (f) is the graph of $f(x) = \dfrac{e^x + e^{-x}}{2}$.

83. Graph $y_1 = |1 - 3^x|$ and $y_2 = 4 + 3^{-x^2}$. Use the Intersect feature to find their point of intersection, $(1.481, 4.090)$.

84. Graph $y_1 = 4^x + 4^{-x}$ and $y_2 = 8 - 2x - x^2$. Use the Intersect feature to find their points of intersection, $(-1.551, 8.697)$ and $(1.078, 4.682)$.

85. Graph $y_1 = 2e^x - 3$ and $y_2 = \dfrac{e^x}{x}$. Use the Intersect feature to find their points of intersection, $(-0.402, -1.662)$ and $(1.051, 2.722)$.

86. Graph $y_1 = \dfrac{1}{e^x + 1}$ and $y_2 = 0.3x + \dfrac{7}{9}$. Use the Intersect feature to find their point of intersection, $(-0.510, 0.625)$.

87. Graph $y_1 = 5.3^x - 4.2^x$ and $y_2 = 1073$. Use the Intersect feature to find the first coordinate of their point of intersection. The solution is 4.448.

88. Graph $y_1 = e^x$ and $y_2 = x^3$. Use the Intersect feature to find the first coordinates of their points of intersection. The solutions are 1.857 and 4.536.

89. Graph $y_1 = 2^x$ and $y_2 = 1$. Use the Intersect feature to find their point of intersection, $(0, 1)$. Note that the graph of y_1 lies above the graph of y_2 for all points to the right of this point. Thus the solution set is $(0, \infty)$.

90. Graph $y_1 = 3^x$ and $y_2 = 1$. Use the Intersect feature to find their point of intersection, $(0, 1)$. Note that the graph of y_1 lies on or below the graph of y_2 for all points to the left of this point. Thus the solution set is $(-\infty, 0]$.

91. Graph $y_1 = 2^x + 3^x$ and $y_2 = x^2 + x^3$. Use the Intersect feature to find the first coordinates of their points of intersection. The solutions are 2.294 and 3.228.

92. Graph $y_1 = 31,245e^{-3x}$ and $y_2 = 523,467$. Use the Intersect feature to find the first coordinate of their point of intersection. The solution is -0.940.

93. Some differences are as follows: The range of f is $(-\infty, \infty)$ whereas the range of g is $(0, \infty)$; f has no asymptotes but g has a horizontal asymptote, the x-axis; the y-intercept of f is $(0, 0)$ and the y-intercept of g is $(0, 1)$.

94. The most interest will be earned the eighth year, because the principle is greatest during that year.

95. They are inverses.

96. They are inverses.

97.
$$\begin{aligned}(1-4i)(7+6i) &= 7 + 6i - 28i - 24i^2\\ &= 7 + 6i - 28i + 24\\ &= 31 - 22i\end{aligned}$$

98.
$$\begin{aligned}\frac{2-i}{3+1} &= \frac{2-i}{3+i}\cdot\frac{3-i}{3-i}\\ &= \frac{6-5i+i^2}{9-i^2}\\ &= \frac{6-5i-1}{9+1}\\ &= \frac{5-5i}{10}\\ &= \frac{1}{2} - \frac{1}{2}i\end{aligned}$$

99.
$$\begin{aligned}2x^2 - 13x - 7 &= 0 \quad \text{Setting } f(x) = 0\\ (2x+1)(x-7) &= 0\end{aligned}$$
$$2x + 1 = 0 \quad \text{or} \quad x - 7 = 0$$
$$2x = -1 \quad \text{or} \quad x = 7$$
$$x = -\frac{1}{2} \quad \text{or} \quad x = 7$$

The zeros of the function are $-\dfrac{1}{2}$ and 7, and the x-intercepts are $\left(-\dfrac{1}{2}, 0\right)$ and $(7, 0)$.

100. $h(x) = x^3 - 3x^2 + 3x - 1$

The possible real-number solutions are of the form p/q where $p = \pm 1$ and $q = \pm 1$. Then the possibilities for p/q are 1 and -1. We try 1.

$$\begin{array}{r|rrrr} 1 & 1 & -3 & 3 & -1\\ & & 1 & -2 & 1\\ \hline & 1 & -2 & 1 & 0\end{array}$$

$h(x) = (x-1)(x^2 - 2x + 1)$

Now find the zeros of $h(x)$.

$$(x-1)(x^2 - 2x + 1) = 0$$
$$(x-1)(x-1)(x-1) = 0$$
$$x - 1 = 0 \text{ or } x - 1 = 0 \text{ or } x - 1 = 0$$
$$x = 1 \text{ or } x = 1 \text{ or } x = 1$$

The zero of the function is 1 and the x-intercept is $(1, 0)$.

101.
$$\begin{aligned}x^4 - x^2 &= 0 \quad \text{Setting } h(x) = 0\\ x^2(x^2 - 1) &= 0\\ x^2(x+1)(x-1) &= 0\end{aligned}$$
$$x^2 = 0 \quad \text{or} \quad x + 1 = 0 \quad \text{or} \quad x - 1 = 0$$
$$x = 0 \quad \text{or} \quad x = -1 \quad \text{or} \quad x = 1$$

The zeros of the function are 0, -1, and 1, and the x-intercepts are $(0, 0)$, $(-1, 0)$, and $(1, 0)$.

102. $x^3+x^2-12x=0$

$x(x^2+x-12)=0$

$x(x+4)(x-3)=0$

$x=0 \;\; or \;\; x+4=0 \;\; or \;\; x-3=0$

$x=0 \;\; or \;\; x=-4 \;\; or \;\; x=3$

The zeros of the function are 0, -4, and 3, and the x-intercepts are $(0,0)$, $(-4,0)$, and $(3,0)$.

103. $x^3+6x^2-16x=0$

$x(x^2+6x-16)=0$

$x(x+8)(x-2)=0$

$x=0 \;\; or \;\; x+8=0 \;\; or \;\; x-2=0$

$x=0 \;\; or \;\; x=-8 \;\; or \;\; x=2$

The solutions are 0, -8, and 2.

104. $3x^2-6=5x$

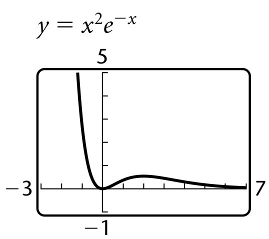
$3x^2-5x-6=0$

$$x=\frac{-(-5)\pm\sqrt{(-5)^2-4\cdot 3\cdot(-6)}}{2\cdot 3}$$

$$=\frac{5\pm\sqrt{97}}{6}$$

105. $7^\pi \approx 451.8078726$ and $\pi^7 \approx 3020.293228$, so π^7 is larger. $70^{80} \approx 4.054\times 10^{147}$ and $80^{70} \approx 1.646\times 10^{133}$, so 70^{80} is larger.

106. a)

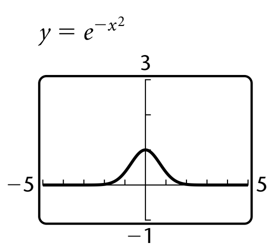


b) Use the Zero feature to find the first coordinate of the x-intercept. The only zero of the function is 0.

c) Relative minimum: 0 at $x=0$;

relative maximum: 0.541 at $x=2$

107. a)



b) There are no x-intercepts, so the function has no zeros.

c) Relative minimum: none;

relative maximum: 1 at $x=0$

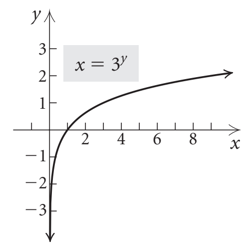
108. Scrolling through the table set in AUTO mode, we see that function values approach 1 as x increases. Similarly, we could set the table in ASK mode and enter increasing values of x. In either case, we see that the horizontal asymptote is $y=1$.

Exercise Set 5.3

1. Graph $x=3^y$.

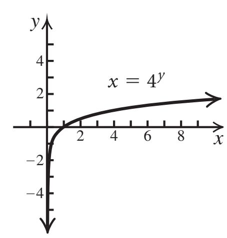
Choose values for y and compute the corresponding x-values. Plot the points (x,y) and connect them with a smooth curve.

x	y	(x,y)
$\frac{1}{27}$	-3	$\left(\frac{1}{27},-3\right)$
$\frac{1}{9}$	-2	$\left(\frac{1}{9},-2\right)$
$\frac{1}{3}$	-1	$\left(\frac{1}{3},-1\right)$
1	0	$(1,0)$
3	1	$(3,1)$
9	2	$(9,2)$
27	3	$(27,3)$



2. Graph $x=4^y$.

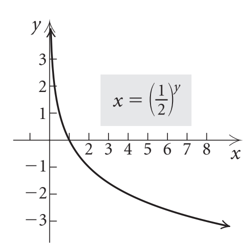
x	y	(x,y)
$\frac{1}{64}$	-3	$\left(\frac{1}{64},-3\right)$
$\frac{1}{16}$	-2	$\left(\frac{1}{16},-2\right)$
$\frac{1}{4}$	-1	$\left(\frac{1}{4},-1\right)$
1	0	$(1,0)$
4	1	$(4,1)$
16	2	$(16,2)$
64	3	$(64,3)$



3. Graph $x=\left(\frac{1}{2}\right)^y$.

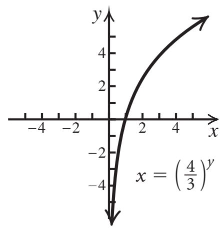
Choose values for y and compute the corresponding x-values. Plot the points (x,y) and connect them with a smooth curve.

x	y	(x,y)
8	-3	$(8,-3)$
4	-2	$(4,-2)$
2	-1	$(2,-1)$
1	0	$(1,0)$
$\frac{1}{2}$	1	$\left(\frac{1}{2},1\right)$
$\frac{1}{4}$	2	$\left(\frac{1}{4},2\right)$
$\frac{1}{8}$	3	$\left(\frac{1}{8},3\right)$



4. Graph $x = \left(\frac{4}{3}\right)^y$.

x	y	(x, y)
$\frac{27}{64}$	-3	$\left(\frac{27}{64}, -3\right)$
$\frac{9}{16}$	-2	$\left(\frac{9}{16}, -2\right)$
$\frac{3}{4}$	-1	$\left(\frac{3}{4}, -1\right)$
1	0	$(1, 0)$
$\frac{4}{3}$	1	$\left(\frac{4}{3}, 1\right)$
$\frac{16}{9}$	2	$\left(\frac{16}{9}, 2\right)$
$\frac{64}{27}$	3	$\left(\frac{64}{27}, 3\right)$



5. Graph $y = \log_3 x$.

The equation $y = \log_3 x$ is equivalent to $x = 3^y$. We can find ordered pairs that are solutions by choosing values for y and computing the corresponding x-values.

For $y = -2$, $x = 3^{-2} = \frac{1}{9}$.

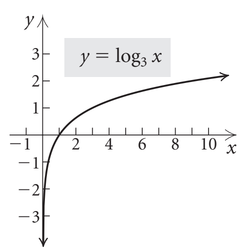
For $y = -1$, $x = 3^{-1} = \frac{1}{3}$.

For $y = 0$, $x = 3^0 = 1$.

For $y = 1$, $x = 3^1 = 3$.

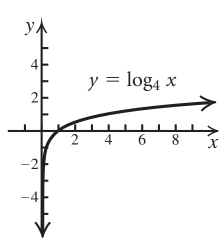
For $y = 2$, $x = 3^2 = 9$.

x, or 3^y	y
$\frac{1}{9}$	-2
$\frac{1}{3}$	-1
1	0
3	1
9	2



6. $y = \log_4 x$ is equivalent to $x = 4^y$.

x, or 4^y	y
$\frac{1}{16}$	-2
$\frac{1}{4}$	-1
1	0
4	1
16	2



7. Graph $f(x) = \log x$.

Think of $f(x)$ as y. The equation $y = \log x$ is equivalent to $x = 10^y$. We can find ordered pairs that are solutions by choosing values for y and computing the corresponding x-values.

For $y = -2$, $x = 10^{-2} = 0.01$.

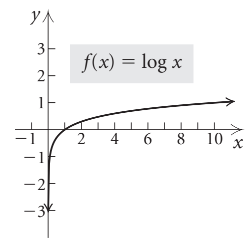
For $y = -1$, $x = 10^{-1} = 0.1$.

For $y = 0$, $x = 10^0 = 1$.

For $y = 1$, $x = 10^1 = 10$.

For $y = 2$, $x = 10^2 = 100$.

x, or 10^y	y
0.01	-2
0.1	-1
1	0
10	1
100	2



8. See Example 10.

9. $\log_2 16 = 4$ because the exponent to which we raise 2 to get 16 is 4.

10. $\log_3 9 = 2$, because the exponent to which we raise 3 to get 9 is 2.

11. $\log_5 125 = 3$, because the exponent to which we raise 5 to get 125 is 3.

12. $\log_2 64 = 6$, because the exponent to which we raise 2 to get 64 is 6.

13. $\log 0.001 = -3$, because the exponent to which we raise 10 to get 0.001 is -3.

14. $\log 100 = 2$, because the exponent to which we raise 10 to get 100 is 2.

15. $\log_2 \frac{1}{4} = -2$, because the exponent to which we raise 2 to get $\frac{1}{4}$ is -2.

16. $\log_8 2 = \frac{1}{3}$, because the exponent to which we raise 8 to get 2 is $\frac{1}{3}$.

17. $\ln 1 = 0$, because the exponent to which we raise e to get 1 is 0.

18. $\ln e = 1$, because the exponent to which we raise e to get e is 1.

19. $\log 10 = 1$, because the exponent to which we raise 10 to get 10 is 1.

20. $\log 1 = 0$, because the exponent to which we raise 10 to get 1 is 0.

21. $\log_5 5^4 = 4$, because the exponent to which we raise 5 to get 5^4 is 4.

22. $\log\sqrt{10} = \log 10^{1/2} = \frac{1}{2}$, because the exponent to which we raise 10 to get $10^{1/2}$ is $\frac{1}{2}$.

23. $\log_3 \sqrt[4]{3} = \log_3 3^{1/4} = \frac{1}{4}$, because the exponent to which we raise 3 to get $3^{1/4}$ is $\frac{1}{4}$.

24. $\log 10^{8/5} = \frac{8}{5}$, because the exponent to which we raise 10 to get $10^{8/5}$ is $\frac{8}{5}$.

25. $\log 10^{-7} = -7$, because the exponent to which we raise 10 to get 10^{-7} is -7.

26. $\log_5 1 = 0$, because the exponent to which we raise 5 to get 1 is 0.

27. $\log_{49} 7 = \frac{1}{2}$, because the exponent to which we raise 49 to get 7 is $\frac{1}{2}$. $(49^{1/2} = \sqrt{49} = 7)$

28. $\log_3 3^{-2} = -2$, because the exponent to which we raise 3 to get 3^{-2} is -2.

29. $\ln e^{3/4} = \frac{3}{4}$, because the exponent to which we raise e to get $e^{3/4}$ is $\frac{3}{4}$.

30. $\log_2 \sqrt{2} = \log_2 2^{1/2} = \frac{1}{2}$, because the exponent to which we raise 2 to get $2^{1/2}$ is $\frac{1}{2}$.

31. $\log_4 1 = 0$, because the exponent to which we raise 4 to get 1 is 0.

32. $\ln e^{-5} = -5$, because the exponent to which we raise e to get e^{-5} is -5.

33. $\ln\sqrt{e} = \ln e^{1/2} = \frac{1}{2}$, because the exponent to which we raise e to get $e^{1/2}$ is $\frac{1}{2}$.

34. $\log_{64} 4 = \frac{1}{3}$, because the exponent to which we raise 64 to get 4 is $\frac{1}{3}$. $(64^{1/3} = \sqrt[3]{64} = 4)$

35.

The exponent is the logarithm.

$10^3 = 1000 \Rightarrow 3 = \log_{10} 1000$

The base remains the same.

We could also say $3 = \log 1000$.

36. $5^{-3} = \frac{1}{125} \Rightarrow \log_5 \frac{1}{125} = -3$

37.

The exponent is the logarithm.

$8^{1/3} = 2 \Rightarrow \log_8 2 = \frac{1}{3}$

The base remains the same.

38. $10^{0.3010} = 2 \Rightarrow \log_{10} 2 = 0.0310$

We could also say $\log 2 = 0.0310$.

39. $e^3 = t \Rightarrow \log_e t = 3$, or $\ln\ t = 3$

40. $Q^t = x \Rightarrow \log_Q x = t$

41. $e^2 = 7.3891 \Rightarrow \log_e 7.3891 = 2$, or $\ln\ 7.3891 = 2$

42. $e^{-1} = 0.3679 \Rightarrow \log_e 0.3679 = -1$, or $\ln\ 0.3679 = -1$

43. $p^k = 3 \Rightarrow \log_p 3 = k$

44. $e^{-t} = 4000 \Rightarrow \log_e 4000 = -t$, or $\ln\ 4000 = -t$

45.

The logarithm is the exponent.

$\log_5 5 = 1 \Rightarrow 5^1 = 5$

The base remains the same.

46. $t = \log_4 7 \Rightarrow 7 = 4^t$

47. $\log 0.01 = -2$ is equivalent to $\log_{10} 0.01 = -2$.

The logarithm is the exponent.

$\log_{10} 0.01 = -2 \Rightarrow 10^{-2} = 0.01$

The base remains the same.

48. $\log 7 = 0.845 \Rightarrow 10^{0.845} = 7$

49. $\ln 30 = 3.4012 \Rightarrow e^{3.4012} = 30$

50. $\ln 0.38 = -0.9676 \Rightarrow e^{-0.9676} = 0.38$

51. $\log_a M = -x \Rightarrow a^{-x} = M$

52. $\log_t Q = k \Rightarrow t^k = Q$

53. $\log_a T^3 = x \Rightarrow a^x = T^3$

54. $\ln W^5 = t \Rightarrow e^t = W^5$

55. $\log 3 \approx 0.4771$

56. $\log 8 \approx 0.9031$

57. $\log 532 \approx 2.7259$

58. $\log 93,100 \approx 4.9689$

59. $\log 0.57 \approx -0.2441$

60. $\log 0.082 \approx -1.0862$

61. $\log(-2)$ does not exist. (The calculator gives an error message.)

62. $\ln 50 \approx 3.9120$

63. $\ln 2 \approx 0.6931$

64. $\ln(-4)$ does not exist. (The calculator gives an error message.)

65. $\ln 809.3 \approx 6.6962$

66. $\ln 0.00037 \approx -7.9020$

67. $\ln(-1.32)$ does not exist. (The calculator gives an error message.)

68. $\ln 0$ does not exist. (The calculator gives an error message.)

69. Let $a = 10$, $b = 4$, and $M = 100$ and substitute in the change-of-base formula.

$$\log_4 100 = \frac{\log_{10} 100}{\log_{10} 4} \approx 3.3219$$

70. $\log_3 20 = \dfrac{\log 20}{\log 3} \approx 2.7268$

71. Let $a = 10$, $b = 100$, and $M = 0.3$ and substitute in the change-of-base formula.

$$\log_{100} 0.3 = \frac{\log_{10} 0.3}{\log_{10} 100} \approx -0.2614$$

72. $\log_\pi 100 = \dfrac{\log 100}{\log \pi} \approx 4.0229$

73. Let $a = 10$, $b = 200$, and $M = 50$ and substitute in the change-of-base formula.

$$\log_{200} 50 = \frac{\log_{10} 50}{\log_{10} 200} \approx 0.7384$$

74. $\log_{5.3} 1700 = \dfrac{\log 1700}{\log 5.3} \approx 4.4602$

75. Let $a = e$, $b = 3$, and $M = 12$ and substitute in the change-of-base formula.

$$\log_3 12 = \frac{\ln 12}{\ln 3} \approx 2.2619$$

76. $\log_4 25 = \dfrac{\ln 25}{\ln 4} \approx 2.3219$

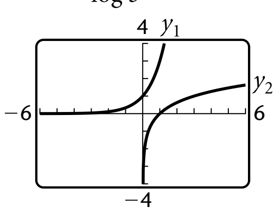
77. Let $a = e$, $b = 100$, and $M = 15$ and substitute in the change-of-base formula.

$$\log_{100} 15 = \frac{\ln 15}{\ln 100} \approx 0.5880$$

78. $\log_9 100 = \dfrac{\ln 100}{\ln 9} \approx 2.0959$

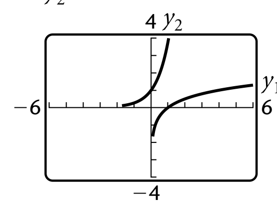
79. Graph $y_1 = 3^x$ and $y_2 = \log_3 x = \dfrac{\log x}{\log 3}$ in the same window, or graph $y = 3^x$ and use the graphing calculator feature that automatically graphs inverses to graph $y = \log_3 x$. On a hand-drawn graph, we can graph $y = 3^x$ and then reflect this graph across the line $y = x$ to get the graph of $y = \log_3 x$.

$y_1 = 3^x$,
$y_2 = \dfrac{\log x}{\log 3}$



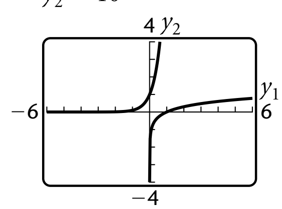
80. Graph $y_1 = \log_4 x = \dfrac{\log x}{\log 4}$ and $y_2 = 4^x$ in the same window, or graph $y = 4^x$ and use the graphing calculator feature that automatically graphs inverses to graph $y = \log_4 x$. (In the second alternative, we choose to graph $y = 4^x$ first since this can be done without using the change-of-base formula.) On a hand-drawn graph, we can graph $y = \log_4 x$ and then reflect this graph across the line $y = x$ to get the graph of $y = 4^x$.

$y_1 = \dfrac{\log x}{\log 4}$,
$y_2 = 4^x$



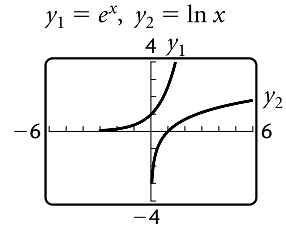
81. Graph $y_1 = \log x$ and $y_2 = 10^x$ in the same window, or graph $y = \log x$ and use the graphing calculator feature that automatically graphs inverses to graph $y = 10^x$. On a hand-drawn graph, we can graph $y = \log x$ and then reflect this graph across the line $y = x$ to get the graph of $y = 10^x$.

$y_1 = \log x$,
$y_2 = 10^x$



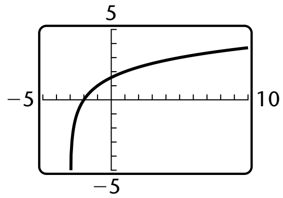
82. Graph $y_1 = e^x$ and $y_2 = \ln x$ in the same window, or graph $y = e^x$ and use the graphing calculator feature that automatically graphs inverses to graph $y = \ln x$. On a hand-drawn graph, we can graph $y = e^x$ and then reflect this graph across the line $y = x$ to get the graph of $y = \ln x$.

$y_1 = e^x$, $y_2 = \ln x$



83. Shift the graph of $y = \log_2 x$ left 3 units. To use the graphing calculator, we must first change the base. Here we change from base 2 to base 10. We get $y = \dfrac{\log(x+3)}{\log 2}$, using $\log_b M = \dfrac{\log_a M}{\log_a b}$ with $b = 2$, $M = x+3$, and $a = 10$.

$$y = \frac{\log (x+3)}{\log 2}$$

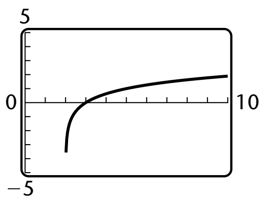


Domain: $(-3, \infty)$

Vertical asymptote: $x = -3$

84. Shift the graph of $y = \log_3 x$ right 2 units. Change from base 3 to base 10: $y = \dfrac{\log(x-2)}{\log 3}$.

$$y = \frac{\log (x-2)}{\log 3}$$

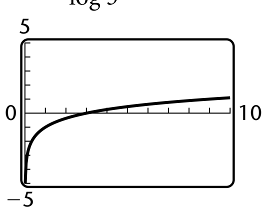


Domain: $(2, \infty)$

Vertical asymptote: $x = 2$

85. Shift the graph of $y = \log_3 x$ down 1 unit. To use the graphing calculator, we must first change the base. Here we change from base 3 to base 10. We get $y = \dfrac{\log x}{\log 3} - 1$, using $\log_b M = \dfrac{\log_a M}{\log_a b}$ with $b = 3$, $M = x$, and $a = 10$.

$$y = \frac{\log x}{\log 3} - 1$$

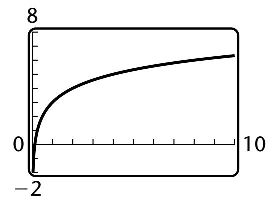


Domain: $(0, \infty)$

Vertical asymptote: $x = 0$

86. Shift the graph of $y = \log_2 x$ up 3 units. Change from base 2 to base 10: $y = 3 + \dfrac{\log x}{\log 2}$.

$$y = 3 + \frac{\log x}{\log 2}$$

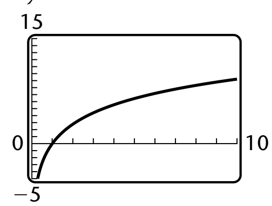


Domain: $(0, \infty)$

Vertical asymptote: $x = 0$

87. Stretch the graph of $y = \ln x$ vertically.

$$y = 4 \ln x$$

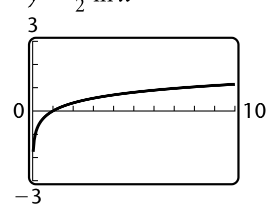


Domain: $(0, \infty)$

Vertical asymptote: $x = 0$

88. Shrink the graph of $y = \ln x$ vertically.

$$y = \frac{1}{2} \ln x$$

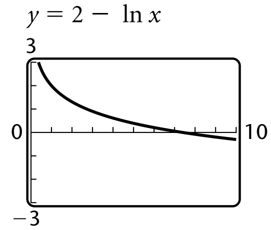


Domain: $(0, \infty)$

Vertical asymptote: $x = 0$

89. Reflect the graph of $y = \ln x$ across the x-axis and then shift it up 2 units.

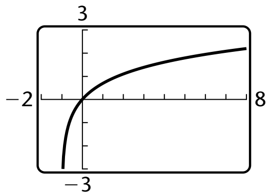
$$y = 2 - \ln x$$



Domain: $(0, \infty)$

Vertical asymptote: $x = 0$

90. Shift the graph of $y = \ln x$ left 1 unit.



Domain: $(-1, \infty)$

Vertical asymptote: $x = -1$

91. a) We substitute 784.242 for P, since P is in thousands.

$$w(784.242) = 0.37 \ln 784.242 + 0.05 \approx 2.5 \text{ ft/sec}$$

b) We substitute 484.246 for P, since P is in thousands.

$$w(484.246) = 0.37 \ln 484.246 + 0.05 \approx 2.3 \text{ ft/sec}$$

c) We substitute 276.963 for P, since P is in thousands.

$$w(276.963) = 0.37 \ln 276.963 + 0.05 \approx 2.1 \text{ ft/sec}$$

d) We substitute 2862.244 for P, since P is in thousands.

$$w(2862.244) = 0.37 \ln 2862.244 + 0.05 \approx 3.0 \text{ ft/sec}$$

e) We substitute 533.492 for P, since P is in thousands.

$$w(533.492) = 0.37 \ln 533.492 + 0.05 \approx 2.4 \text{ ft/sec}$$

f) We substitute 304.973 for P, since P is in thousands.

$$w(304.973) = 0.37 \ln 304.973 + 0.05 \approx 2.2 \text{ ft/sec}$$

g) We substitute 8104.079 for P, since P is in thousands.

$$w(8104.079) = 0.37 \ln 8104.079 + 0.05 \approx 3.4 \text{ ft/sec}$$

h) We substitute 122.206 for P, since P is in thousands.

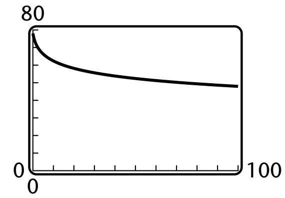
$$w(122.206) = 0.37 \ln 122.206 + 0.05 \approx 1.8 \text{ ft/sec}$$

92. a)
$$\begin{aligned} S(0) &= 78 - 15\log(0+1) \\ &= 78 - 15\log 1 \\ &= 78 - 15 \cdot 0 \\ &= 78\% \end{aligned}$$

b)
$$\begin{aligned} S(4) &= 78 - 15\log(4+1) \\ &= 78 - 15\log 5 \\ &\approx 78 - 15(0.698970) \\ &\approx 67.5\% \end{aligned}$$

$$\begin{aligned} S(24) &= 78 - 15\log(24+1) \\ &= 78 - 15\log 25 \\ &\approx 78 - 15(1.397940) \\ &\approx 57\% \end{aligned}$$

c) $y = 78 - 15\log(x+1), x \geqslant 0$



d)
$$\begin{aligned} 50 &= 78 - 15\log(x+1) \\ -28 &= -15\log(x+1) \quad \text{Subtracting 78} \\ \frac{28}{15} &= \log(x+1) \quad \text{Dividing by } -15 \\ x+1 &= 10^{28/15} \quad \text{Using the definition of logarithm} \\ x &= 10^{28/15} - 1 \\ x &\approx 73 \text{ months} \end{aligned}$$

93. a) $R = \log \dfrac{10^{7.85} \cdot I_0}{I_0} = \log 10^{7.85} = 7.85$

b) $R = \log \dfrac{10^{8.25} \cdot I_0}{I_0} = \log 10^{8.25} = 8.25$

c) $R = \log \dfrac{10^{9.6} \cdot I_0}{I_0} = \log 10^{9.6} = 9.6$

d) $R = \log \dfrac{10^{7.85} \cdot I_0}{I_0} = \log 10^{7.85} = 7.85$

e) $R = \log \dfrac{10^{6.9} \cdot I_0}{I_0} = \log 10^{6.9} = 6.9$

94. a) $\text{pH} = -\log[1.6 \times 10^{-4}] \approx -(-3.8) \approx 3.8$

b) $\text{pH} = -\log[0.0013] \approx -(-2.9) \approx 2.9$

c) $\text{pH} = -\log[6.3 \times 10^{-7}] \approx -(-6.2) \approx 6.2$

d) $\text{pH} = -\log[1.6 \times 10^{-8}] \approx -(-7.8) \approx 7.8$

e) $\text{pH} = -\log[6.3 \times 10^{-5}] \approx -(-4.2) \approx 4.2$

95. a)
$$\begin{aligned} 7 &= -\log[H^+] \\ -7 &= \log[H^+] \\ H^+ &= 10^{-7} \quad \text{Using the definition of logarithm} \end{aligned}$$

b)
$$\begin{aligned} 5.4 &= -\log[H^+] \\ -5.4 &= \log[H^+] \\ H^+ &= 10^{-5.4} \quad \text{Using the definition of logarithm} \\ H^+ &\approx 4.0 \times 10^{-6} \end{aligned}$$

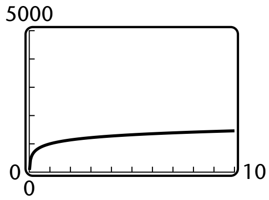
c)
$$\begin{aligned} 3.2 &= -\log[H^+] \\ -3.2 &= \log[H^+] \\ H^+ &= 10^{-3.2} \quad \text{Using the definition of logarithm} \\ H^+ &\approx 6.3 \times 10^{-4} \end{aligned}$$

d)
$$\begin{aligned} 4.8 &= -\log[H^+] \\ -4.8 &= \log[H^+] \\ H^+ &= 10^{-4.8} \quad \text{Using the definition of logarithm} \\ H^+ &\approx 1.6 \times 10^{-5} \end{aligned}$$

96. a) $N(1) = 1000 + 200 \ln 1 = 1000$ units

b) $N(5) = 1000 + 200 \ln 5 \approx 1332$ units

c) $y = 1000 + 200 \ln x$



d) $$\begin{aligned} 2000 &= 1000 + 200 \ln a \\ 1000 &= 200 \ln a \\ 5 &= \ln a \\ a &= e^5 \\ a &\approx \$148 \text{ thousand} \end{aligned}$$

97. a) $$\begin{aligned} L &= 10 \log \frac{2510 \cdot I_0}{I_0} \\ &= 10 \log 2510 \\ &\approx 34 \text{ decibels} \end{aligned}$$

b) $$\begin{aligned} L &= 10 \log \frac{2,500,000 \cdot I_0}{I_0} \\ &= 10 \log 2,500,000 \\ &\approx 64 \text{ decibels} \end{aligned}$$

c) $$\begin{aligned} L &= 10 \log \frac{10^6 \cdot I_0}{I_0} \\ &= 10 \log 10^6 \\ &= 60 \text{ decibels} \end{aligned}$$

d) $$\begin{aligned} L &= 10 \log \frac{10^9 \cdot I_0}{I_0} \\ &= 10 \log 10^9 \\ &= 90 \text{ decibels} \end{aligned}$$

98. Reflect the graph of $f(x) = \ln x$ across the line $y = x$ to obtain the graph of $h(x) = e^x$. Then shift this graph 2 units right to obtain the graph of $g(x) = e^{x-2}$.

99. If $\log b < 0$, then $b < 1$.

100. $$\begin{aligned} 3x - 10y &= 14 \\ 3x - 14 &= 10y \\ \frac{3}{10}x - \frac{7}{5} &= y \end{aligned}$$

Slope: $\frac{3}{10}$; y-intercept: $\left(0, -\frac{7}{5}\right)$

101. $y = 6 = 0 \cdot x + 6$

Slope: 0; y-intercept $(0, 6)$

102. $x = -4$

Slope: not defined; y-intercept: none

103. $$\begin{array}{r|rrrr} -5 & 1 & -6 & 3 & 10 \\ & & -5 & 55 & -290 \\ \hline & 1 & -11 & 58 & -280 \end{array}$$

The remainder is -280, so $f(-5) = -280$.

104. $$\begin{array}{r|rrrrr} -1 & 1 & -2 & 0 & 1 & -6 \\ & & -1 & 3 & -3 & 2 \\ \hline & 1 & -3 & 3 & -2 & -4 \end{array}$$

$f(-1) = -4$

105. $$\begin{aligned} f(x) &= (x - \sqrt{7})(x + \sqrt{7})(x - 0) \\ &= (x^2 - 7)(x) \\ &= x^3 - 7x \end{aligned}$$

106. $$\begin{aligned} f(x) &= (x - 4i)(x + 4i)(x - 1) \\ &= (x^2 + 16)(x - 1) \\ &= x^3 - x^2 + 16x - 16 \end{aligned}$$

107. Using the change-of-base formula, we get

$$\frac{\log_5 8}{\log_2 8} = \log_2 8 = 3.$$

108. Using the change-of-base formula, we get

$$\frac{\log_3 64}{\log_3 16} = \log_{16} 64.$$

Let $\log_{16} 64 = x$. Then we have

$$\begin{aligned} 16^x &= 64 \quad \text{Using the definition of logarithm} \\ (2^4)^x &= 2^6 \\ 2^{4x} &= 2^6, \text{ so} \\ 4x &= 6 \\ x &= \frac{6}{4} = \frac{3}{2} \end{aligned}$$

Thus, $\frac{\log_3 64}{\log_3 16} = \frac{3}{2}$.

109. $f(x) = \log_5 x^3$

x^3 must be positive. Since $x^3 > 0$ for $x > 0$, the domain is $(0, \infty)$.

110. $f(x) = \log_4 x^2$

x^2 must be positive, so the domain is $(-\infty, 0) \cup (0, \infty)$.

111. $f(x) = \ln |x|$

$|x|$ must be positive. Since $|x| > 0$ for $x \neq 0$, the domain is $(-\infty, 0) \cup (0, \infty)$.

112. $f(x) = \log(3x - 4)$

$3x - 4$ must be positive. We have

$$\begin{aligned} 3x - 4 &> 0 \\ x &> \frac{4}{3} \end{aligned}$$

The domain is $\left(\frac{4}{3}, \infty\right)$.

113. Graph $y = \log_2(2x + 5) = \frac{\log(2x+5)}{\log 2}$. Observe that outputs are negative for inputs between $-\frac{5}{2}$ and -2. Thus, the solution set is $\left(-\frac{5}{2}, -2\right)$.

114. Graph $y_1 = \log_2(x-3) = \dfrac{\log(x-3)}{\log 2}$ and $y_2 = 4$.

Observe that the graph of y_1 lies on or above the graph of y_2 for all inputs greater than or equal to 19. Thus, the solution set is $[19, \infty)$.

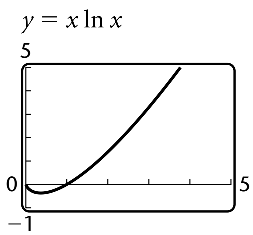
115. Graph (d) is the graph of $f(x) = \ln|x|$.

116. Graph (c) is the graph of $f(x) = |\ln x|$.

117. Graph (b) is the graph of $f(x) = \ln x^2$.

118. Graph (a) is the graph of $g(x) = |\ln(x-1)|$.

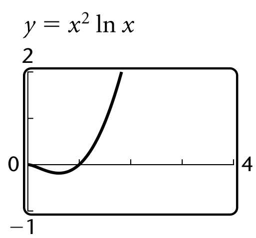
119. a)



b) Use the Zero feature. The zero is 1.

c) Use the Minimum feature. The relative minimum is -0.368 at $x = 0.368$. There is no relative maximum.

120. a)

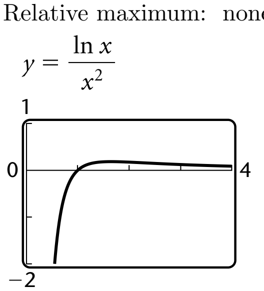


b) 1

c) Relative minimum: -0.184 at $x = 0.607$

Relative maximum: none

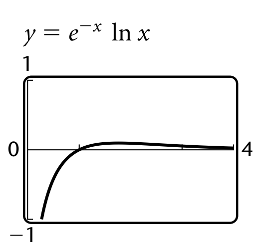
121. a)



b) Use the Zero feature. The zero is 1.

c) Use the Maximum feature. There is no relative minimum. The relative maximum is 0.184 at $x = 1.649$.

122. a)



b) 1

c) Relative minimum: none

Relative maximum: 0.097 at $x = 1.763$

123. Graph $y_1 = 4\ln x$ and $y_2 = \dfrac{4}{e^x + 1}$ and use the Intersect feature to find the point(s) of intersection of the graphs. The point of intersection is $(1.250, 0.891)$.

Exercise Set 5.4

1. Use the product rule.

$\log_3(81 \cdot 27) = \log_3 81 + \log_3 27 = 4 + 3 = 7$

2. $\log_2(8 \cdot 64) = \log_2 8 + \log_2 64 = 3 + 6 = 9$

3. Use the product rule.

$\log_5(5 \cdot 125) = \log_5 5 + \log_5 125 = 1 + 3 = 4$

4. $\log_4(64 \cdot 4) = \log_4 64 + \log_4 4 = 3 + 1 = 4$

5. Use the product rule.

$\log_t 8Y = \log_t 8 + \log_t Y$

6. $\log 0.2x = \log 0.2 + \log x$

7. Use the product rule.

$\ln xy = \ln x + \ln y$

8. $\ln ab = \ln a + \ln b$

9. Use the power rule.

$\log_b t^3 = 3\log_b t$

10. $\log_a x^4 = 4\log_a x$

11. Use the power rule.

$\log y^8 = 8\log y$

12. $\ln y^5 = 5 \ln y$

13. Use the power rule.

$\log_c K^{-6} = -6\log_c K$

14. $\log_b Q^{-8} = -8\log_b Q$

15. Use the power rule.

$\ln \sqrt[3]{4} = \ln 4^{1/3} = \dfrac{1}{3}\ln 4$

16. $\ln \sqrt{a} = \ln a^{1/2} = \dfrac{1}{2}\ln a$

17. Use the quotient rule.

$\log_t \dfrac{M}{8} = \log_t M - \log_t 8$

18. $\log_a \dfrac{76}{13} = \log_a 76 - \log_a 13$

19. Use the quotient rule.

$\log \dfrac{x}{y} = \log x - \log y$

20. $\ln \dfrac{a}{b} = \ln a - \ln b$

21. Use the quotient rule.

$\ln \frac{r}{s} = \ln r - \ln s$

22. $\log_b \frac{3}{w} = \log_b 3 - \log_b w$

23. $\log_a 6xy^5z^4$

$= \log_a 6 + \log_a x + \log_a y^5 + \log_a z^4$ Product rule

$= \log_a 6 + \log_a x + 5\log_a y + 4\log_a z$ Power rule

24. $\log_a x^3y^2z$

$= \log_a x^3 + \log_a y^2 + \log_a z$

$= 3\log_a x + 2\log_a y + \log_a z$

25. $\log_b \frac{p^2q^5}{m^4b^9}$

$= \log_b p^2q^5 - \log_b m^4b^9$ Quotient rule

$= \log_b p^2 + \log_b q^5 - (\log_b m^4 + \log_b b^9)$ Product rule

$= \log_b p^2 + \log_b q^5 - \log_b m^4 - \log_b b^9$

$= \log_b p^2 + \log_b q^5 - \log_b m^4 - 9$ $(\log_b b^9 = 9)$

$= 2\log_b p + 5\log_b q - 4\log_b m - 9$ Power rule

26. $\log_b \frac{x^2y}{b^3} = \log_b x^2y - \log_b b^3$

$= \log_b x^2 + \log_b y - \log_b b^3$

$= \log_b x^2 + \log_b y - 3$

$= 2\log_b x + \log_b y - 3$

27. $\ln \frac{2}{3x^3y}$

$= \ln 2 - \ln 3x^3y$ Quotient rule

$= \ln 2 - (\ln 3 + \ln x^3 + \ln y)$ Product rule

$= \ln 2 - \ln 3 - \ln x^3 - \ln y$

$= \ln 2 - \ln 3 - 3\ln x - \ln y$ Power rule

28. $\log \frac{5a}{4b^2} = \log 5a - \log 4b^2$

$= \log 5 + \log a - (\log 4 + \log b^2)$

$= \log 5 + \log a - \log 4 - \log b^2$

$= \log 5 + \log a - \log 4 - 2\log b$

29. $\log \sqrt{r^3t}$

$= \log(r^3t)^{1/2}$

$= \frac{1}{2}\log r^3t$ Power rule

$= \frac{1}{2}(\log r^3 + \log t)$ Product rule

$= \frac{1}{2}(3\log r + \log t)$ Power rule

$= \frac{3}{2}\log r + \frac{1}{2}\log t$

30. $\ln \sqrt[3]{5x^5} = \ln(5x^5)^{1/3}$

$= \frac{1}{3}\ln 5x^5$

$= \frac{1}{3}(\ln 5 + \ln x^5)$

$= \frac{1}{3}(\ln 5 + 5\ln x)$

$= \frac{1}{3}\ln 5 + \frac{5}{3}\ln x$

31. $\log_a \sqrt{\frac{x^6}{p^5q^8}}$

$= \frac{1}{2}\log_a \frac{x^6}{p^5q^8}$

$= \frac{1}{2}[\log_a x^6 - \log_a(p^5q^8)]$ Quotient rule

$= \frac{1}{2}[\log_a x^6 - (\log_a p^5 + \log_a q^8)]$ Product rule

$= \frac{1}{2}(\log_a x^6 - \log_a p^5 - \log_a q^8)$

$= \frac{1}{2}(6\log_a x - 5\log_a p - 8\log_a q)$ Power rule

$= 3\log_a x - \frac{5}{2}\log_a p - 4\log_a q$

32. $\log_c \sqrt[3]{\frac{y^3z^2}{x^4}}$

$= \frac{1}{3}\log_c \frac{y^3z^2}{x^4}$

$= \frac{1}{3}(\log_c y^3z^2 - \log_c x^4)$

$= \frac{1}{3}(\log_c y^3 + \log_c z^2 - \log_c x^4)$

$= \frac{1}{3}(3\log_c y + 2\log_c z - 4\log_c x)$

$= \log_c y + \frac{2}{3}\log_c z - \frac{4}{3}\log_c x$

33. $\log_a \sqrt[4]{\frac{m^8n^{12}}{a^3b^5}}$

$= \frac{1}{4}\log_a \frac{m^8n^{12}}{a^3b^5}$ Power rule

$= \frac{1}{4}(\log_a m^8n^{12} - \log_a a^3b^5)$ Quotient rule

$= \frac{1}{4}[\log_a m^8 + \log_a n^{12} - (\log_a a^3 + \log_a b^5)]$ Product rule

$= \frac{1}{4}(\log_a m^8 + \log_a n^{12} - \log_a a^3 - \log_a b^5)$

$= \frac{1}{4}(\log_a m^8 + \log_a n^{12} - 3 - \log_a b^5)$ $(\log_a a^3 = 3)$

$= \frac{1}{4}(8\log_a m + 12\log_a n - 3 - 5\log_a b)$ Power rule

$= 2\log_a m + 3\log_a n - \frac{3}{4} - \frac{5}{4}\log_a b$

34. $\log_a \sqrt{\dfrac{a^6b^8}{a^2b^5}} = \log_a \sqrt{a^4b^3}$

$= \dfrac{1}{2}(\log_a a^4 + \log_a b^3)$

$= \dfrac{1}{2}(4 + 3\log_a b)$

$= 2 + \dfrac{3}{2}\log_a b$

35. $\log_a 75 + \log_a 2$

$= \log_a(75 \cdot 2)$ Product rule

$= \log_a 150$

36. $\log 0.01 + \log 1000 = \log(0.01 \cdot 1000) = \log 10 = 1$

37. $\log 10,000 - \log 100$

$= \log \dfrac{10,000}{100}$ Quotient rule

$= \log 100$

$= 2$

38. $\ln 54 - \ln 6 = \ln \dfrac{54}{6} = \ln 9$

39. $\dfrac{1}{2}\log n + 3\log m$

$= \log n^{1/2} + \log m^3$ Power rule

$= \log n^{1/2}m^3$, or Product rule

$\log m^3\sqrt{n}$ $n^{1/2} = \sqrt{n}$

40. $\dfrac{1}{2}\log a - \log 2 = \log a^{1/2} - \log 2$

$= \log \dfrac{a^{1/2}}{2}$, or $\log \dfrac{\sqrt{a}}{2}$

41. $\dfrac{1}{2}\log_a x + 4\log_a y - 3\log_a x$

$= \log_a x^{1/2} + \log_a y^4 - \log_a x^3$ Power rule

$= \log_a x^{1/2}y^4 - \log_a x^3$ Product rule

$= \log_a \dfrac{x^{1/2}y^4}{x^3}$ Quotient rule

$= \log_a x^{-5/2}y^4$, or $\log_a \dfrac{y^4}{x^{5/2}}$ Simplifying

42. $\dfrac{2}{5}\log_a x - \dfrac{1}{3}\log_a y = \log_a x^{2/5} - \log_a y^{1/3} =$

$\log_a \dfrac{x^{2/5}}{y^{1/3}}$

43. $\ln x^2 - 2\ln\sqrt{x}$

$= \ln x^2 - \ln(\sqrt{x})^2$ Power rule

$= \ln x^2 - \ln x$ $[(\sqrt{x})^2 = x]$

$= \ln \dfrac{x^2}{x}$ Quotient rule

$= \ln x$

44. $\ln 2x + 3(\ln x - \ln y) = \ln 2x + 3\ln \dfrac{x}{y}$

$= \ln 2x + \ln\left(\dfrac{x}{y}\right)^3$

$= \ln 2x\left(\dfrac{x}{y}\right)^3$

$= \ln \dfrac{2x^4}{y^3}$

45. $\ln(x^2-4) - \ln(x+2)$

$= \ln \dfrac{x^2-4}{x+2}$ Quotient rule

$= \ln \dfrac{(x+2)(x-2)}{x+2}$ Factoring

$= \ln(x-2)$ Removing a factor of 1

46. $\log(x^3-8) - \log(x-2)$

$= \log \dfrac{x^3-8}{x-2}$

$= \log \dfrac{(x-2)(x^2+2x+4)}{x-2}$

$= \log(x^2+2x+4)$

47. $\log(x^2-5x-14) - \log(x^2-4)$

$= \log \dfrac{x^2-5x-14}{x^2-4}$ Quotient rule

$= \log \dfrac{(x+2)(x-7)}{(x+2)(x-2)}$ Factoring

$= \log \dfrac{x-7}{x-2}$ Removing a factor of 1

48. $\log_a \dfrac{a}{\sqrt{x}} - \log_a \sqrt{ax} = \log_a \dfrac{a}{\sqrt{x}\sqrt{ax}}$

$= \log_a \dfrac{\sqrt{a}}{x}$

$= \log_a \sqrt{a} - \log_a x$

$= \dfrac{1}{2}\log_a a - \log_a x$

$= \dfrac{1}{2} - \log_a x$

49. $\ln x - 3[\ln(x-5) + \ln(x+5)]$

$= \ln x - 3\ln[(x-5)(x+5)]$ Product rule

$= \ln x - 3\ln(x^2-25)$

$= \ln x - \ln(x^2-25)^3$ Power rule

$= \ln \dfrac{x}{(x^2-25)^3}$ Quotient rule

50. $\dfrac{2}{3}[\ln(x^2-9) - \ln(x+3)] + \ln(x+y)$

$= \dfrac{2}{3}\ln \dfrac{x^2-9}{x+3} + \ln(x+y)$

$= \dfrac{2}{3}\ln \dfrac{(x+3)(x-3)}{x+3} + \ln(x+y)$

$= \dfrac{2}{3}\ln(x-3) + \ln(x+y)$

$= \ln(x-3)^{2/3} + \ln(x+y)$

$= \ln[(x-3)^{2/3}(x+y)]$

51. $\frac{3}{2}\ln 4x^6 - \frac{4}{5}\ln 2y^{10}$

$= \frac{3}{2}\ln 2^2x^6 - \frac{4}{5}\ln 2y^{10}$ Writing 4 as 2^2

$= \ln(2^2x^6)^{3/2} - \ln(2y^{10})^{4/5}$ Power rule

$= \ln(2^3x^9) - \ln(2^{4/5}y^8)$

$= \ln\frac{2^3x^9}{2^{4/5}y^8}$ Quotient rule

$= \ln\frac{2^{11/5}x^9}{y^8}$

52. $120(\ln\sqrt[5]{x^3} + \ln\sqrt[3]{y^2} - \ln\sqrt[4]{16z^5})$

$= 120\left(\ln\frac{\sqrt[5]{x^3}\sqrt[3]{y^2}}{\sqrt[4]{16z^5}}\right)$

$= 120\left(\ln\frac{x^{3/5}y^{2/3}}{2z^{5/4}}\right)$

$= \ln\left(\frac{x^{3/5}y^{2/3}}{2z^{5/4}}\right)^{120}$

$= \ln\frac{x^{72}y^{80}}{2^{120}z^{150}}$

53. $\log_a\frac{2}{11} = \log_a 2 - \log_a 11$ Quotient rule

$\approx 0.301 - 1.041$

≈ -0.74

54. $\log_a 14 = \log_a(2\cdot 7)$

$= \log_a 2 + \log_a 7$

$\approx 0.301 + 0.845$

≈ 1.146

55. $\log_a 98 = \log_a(7^2\cdot 2)$

$= \log_a 7^2 + \log_a 2$ Product rule

$= 2\log_a 7 + \log_a 2$ Power rule

$\approx 2(0.845) + 0.301$

≈ 1.991

56. $\log_a\frac{1}{7} = \log_a 1 - \log_a 7$

$\approx 0 - 0.845$

≈ -0.845

57. $\frac{\log_a 2}{\log_a 7} \approx \frac{0.301}{0.845} \approx 0.356$

58. $\log_a 9$ cannot be found using the given information.

59. $\log_b 125 = \log_b 5^3$

$= 3\log_b 5$ Power rule

$\approx 3(1.609)$

≈ 4.827

60. $\log_b\frac{5}{3} = \log_b 5 - \log_b 3$

$\approx 1.609 - 1.099$

≈ 0.51

61. $\log_b\frac{1}{6} = \log_b 1 - \log_b 6$ Quotient rule

$= \log_b 1 - \log_b(2\cdot 3)$

$= \log_b 1 - (\log_b 2 + \log_b 3)$ Product rule

$= \log_b 1 - \log_b 2 - \log_b 3$

$\approx 0 - 0.693 - 1.099$

≈ -1.792

62. $\log_b 30 = \log_b(2\cdot 3\cdot 5)$

$= \log_b 2 + \log_b 3 + \log_b 5$

$\approx 0.693 + 1.099 + 1.609$

≈ 3.401

63. $\log_b\frac{3}{b} = \log_b 3 - \log_b b$ Quotient rule

$\approx 1.099 - 1$

≈ 0.099

64. $\log_b 15b = \log_b(3\cdot 5\cdot b)$

$= \log_b 3 + \log_b 5 + \log_b b$

$\approx 1.099 + 1.609 + 1$

≈ 3.708

65. $\log_p p^3 = 3$ $(\log_a a^x = x)$

66. $\log_t t^{2713} = 2713$

67. $\log_e e^{|x-4|} = |x-4|$ $(\log_a a^x = x)$

68. $\log_q q^{\sqrt{3}} = \sqrt{3}$

69. $3^{\log_3 4x} = 4x$ $(a^{\log_a x} = x)$

70. $5^{\log_5(4x-3)} = 4x - 3$

71. $10^{\log w} = w$ $(a^{\log_a x} = x)$

72. $e^{\ln x^3} = x^3$

73. $\ln e^{8t} = 8t$ $(\log_a a^x = x)$

74. $\log 10^{-k} = -k$

75. $\log_b\sqrt{b} = \log_b b^{1/2}$

$= \frac{1}{2}\log_b b$ Power rule

$= \frac{1}{2}\cdot 1$ $(\log_b b = 1)$

$= \frac{1}{2}$

76. $\log_b\sqrt{b^3} = \log_b b^{3/2}$

$= \frac{3}{2}\log_b b$

$= \frac{3}{2}\cdot 1$

$= \frac{3}{2}$

77. $f(x) = a^x$, $g(x) = \log_a x$

Since f and g are inverses, we know that $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$. Now $(f \circ g)(x) = f(g(x)) = f(\log_a x) = a^{\log_a x}$, so we know that $a^{\log_a x} = x$. Also $(g \circ f)(x) = g(f(x)) = g(a^x) = \log_a a^x$, so we know that $\log_a a^x = x$. These results are alternate proofs of the Logarithm of a Base to a Power property and the Base to a Logarithmic Power property.

78. $\log_a ab^3 \neq (\log_a a)(\log_a b^3)$. If the first step had been correct, then so would the second step. The correct procedure follows.

$\log_a ab^3 = \log_a a + \log_a b^3 = 1 + 3\log_a b$

79. The degree of $f(x) = 5 - x^2 + x^4$ is 4, so the function is quartic.

80. The variable in $f(x) = 2^x$ is in the exponent, so $f(x)$ is an exponential function.

81. $f(x) = -\frac{3}{4}$ is of the form $f(x) = mx + b$ $\left(\text{with } m = 0 \text{ and } b = -\frac{3}{4}\right)$, so it is a linear function. In fact, it is a constant function.

82. The variable in $f(x) = 4^x - 8$ is in the exponent, so $f(x)$ is an exponential function.

83. $f(x) = -\frac{3}{x}$ is of the form $f(x) = \frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomials and $q(x)$ is not the zero polynomial, so $f(x)$ is a rational function.

84. $f(x) = \log x + 6$ is a logarithmic function.

85. The degree of $f(x) = -\frac{1}{3}x^3 - 4x^2 + 6x + 42$ is 3, so the function is cubic.

86. $f(x) = \frac{x^2 - 1}{x^2 + x - 6}$ is of the form $f(x) = \frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomials and $q(x)$ is not the zero polynomial, so $f(x)$ is a rational function.

87. $f(x) = \frac{1}{2}x + 3$ is of the form $f(x) = mx + b$, so it is a linear function.

88. The degree of $f(x) = 2x^2 - 6x + 3$ is 2, so the function is quadratic.

89.
$$\begin{aligned} 5^{\log_5 8} &= 2x \\ 8 &= 2x \quad (a^{\log_a x} = x) \\ 4 &= x \end{aligned}$$

The solution is 4.

90.
$$\begin{aligned} \ln e^{3x-5} &= -8 \\ 3x - 5 &= -8 \\ 3x &= -3 \\ x &= -1 \end{aligned}$$

The solution is -1.

91.
$$\begin{aligned} &\log_a(x^2 + xy + y^2) + \log_a(x - y) \\ &= \log_a[(x^2 + xy + y^2)(x - y)] \quad \text{Product rule} \\ &= \log_a(x^3 - y^3) \quad \text{Multiplying} \end{aligned}$$

92.
$$\begin{aligned} &\log_a(a^{10} - b^{10}) - \log_a(a + b) \\ &= \log_a \frac{a^{10} - b^{10}}{a + b}, \text{ or} \\ &\log_a(a^9 - a^8b + a^7b^2 - a^6b^3 + a^5b^4 - a^4b^5 + \\ &a^3b^6 - a^2b^7 + ab^8 - b^9) \end{aligned}$$

93.
$$\begin{aligned} &\log_a \frac{x - y}{\sqrt{x^2 - y^2}} \\ &= \log_a \frac{x - y}{(x^2 - y^2)^{1/2}} \\ &= \log_a(x - y) - \log_a(x^2 - y^2)^{1/2} \quad \text{Quotient rule} \\ &= \log_a(x - y) - \frac{1}{2}\log_a(x^2 - y^2) \quad \text{Power rule} \\ &= \log_a(x - y) - \frac{1}{2}\log_a[(x + y)(x - y)] \\ &= \log_a(x - y) - \frac{1}{2}[\log_a(x + y) + \log_a(x - y)] \\ &\qquad \text{Product rule} \\ &= \log_a(x - y) - \frac{1}{2}\log_a(x + y) - \frac{1}{2}\log_a(x - y) \\ &= \frac{1}{2}\log_a(x - y) - \frac{1}{2}\log_a(x + y) \end{aligned}$$

94.
$$\begin{aligned} &\log_a \sqrt{9 - x^2} \\ &= \log_a(9 - x^2)^{1/2} \\ &= \frac{1}{2}\log_a(9 - x^2) \\ &= \frac{1}{2}\log_a[(3 + x)(3 - x)] \\ &= \frac{1}{2}[\log_a(3 + x) + \log_a(3 - x)] \\ &= \frac{1}{2}\log_a(3 + x) + \frac{1}{2}\log_a(3 - x) \end{aligned}$$

95.
$$\log_a \frac{\sqrt[4]{y^2z^5}}{\sqrt[4]{x^3z^{-2}}}$$
$$= \log_a \sqrt[4]{\frac{y^2z^5}{x^3z^{-2}}}$$
$$= \log_a \sqrt[4]{\frac{y^2z^7}{x^3}}$$
$$= \log_a \left(\frac{y^2z^7}{x^3}\right)^{1/4}$$
$$= \frac{1}{4}\log_a \left(\frac{y^2z^7}{x^3}\right) \quad \text{Power rule}$$
$$= \frac{1}{4}(\log_a y^2z^7 - \log_a x^3) \quad \text{Quotient rule}$$
$$= \frac{1}{4}(\log_a y^2 + \log_a z^7 - \log_a x^3) \quad \text{Product rule}$$
$$= \frac{1}{4}(2\log_a y + 7\log_a z - 3\log_a x) \quad \text{Power rule}$$
$$= \frac{1}{4}(2\cdot 3 + 7\cdot 4 - 3\cdot 2)$$
$$= \frac{1}{4}\cdot 28$$
$$= 7$$

96. $\log_a M + \log_a N = \log_a (M+N)$

Let $a = 10$, $M = 1$, and $N = 10$. Then $\log_{10} 1 + \log_{10} 10 = 0 + 1 = 1$, but $\log_{10}(1+10) = \log_{10} 11 \approx 1.0414$. Thus, the statement is false.

97. $\log_a M - \log_a N = \log_a \dfrac{M}{N}$

This is the quotient rule, so it is true.

98. $\dfrac{\log_a M}{\log_a N} = \log_a M - \log_a N$

Let $M = a^2$ and $N = a$. Then $\dfrac{\log_a a^2}{\log_a a} = \dfrac{2}{1} = 2$, but $\log_a a^2 - \log_a a = 2 - 1 = 1$. Thus, the statement is false.

99. $\dfrac{\log_a M}{x} = \dfrac{1}{x}\log_a M = \log_a M^{1/x}$. The statement is true by the power rule.

100. $\log_a x^3 = 3\log_a x$ is true by the power rule.

101. $\log_a 8x = \log_a 8 + \log_a x = \log_a x + \log_a 8$. The statement is true by the product rule and the commutative property of addition.

102.
$$\log_N (M\cdot N)^x = x\log_N (M\cdot N)$$
$$= x(\log_N M + \log_N N)$$
$$= x(\log_N M + 1)$$
$$= x\log_N M + x$$

The statement is true.

103. $\log_a \left(\dfrac{1}{x}\right) = \log_a x^{-1} = -1\cdot\log_a x = -1\cdot 2 = -2$

104. $\log_a x = 2$

$a^2 = x$

Let $\log_{1/a} x = n$ and solve for n.

$\log_{1/a} a^2 = n$ Substituting a^2 for x

$$\left(\frac{1}{a}\right)^n = a^2$$
$$(a^{-1})^n = a^2$$
$$a^{-n} = a^2$$
$$-n = 2$$
$$n = -2$$

Thus, $\log_{1/a} x = -2$ when $\log_a x = 2$.

105. We use the change-of-base formula.

$$\log_{10} 11\cdot\log_{11} 12\cdot\log_{12} 13\cdots \log_{998} 999\cdot\log_{999} 1000$$
$$= \log_{10} 11\cdot\frac{\log_{10} 12}{\log_{10} 11}\cdot\frac{\log_{10} 13}{\log_{10} 12}\cdots \frac{\log_{10} 999}{\log_{10} 998}\cdot\frac{\log_{10} 1000}{\log_{10} 999}$$
$$= \frac{\log_{10} 11}{\log_{10} 11}\cdot\frac{\log_{10} 12}{\log_{10} 12}\cdots\frac{\log_{10} 999}{\log_{10} 999}\cdot\log_{10} 1000$$
$$= \log_{10} 1000$$
$$= 3$$

106.
$$\log_a x + \log_a y - mz = 0$$
$$\log_a x + \log_a y = mz$$
$$\log_a xy = mz$$
$$a^{mz} = xy$$

107.
$$\ln a - \ln b + xy = 0$$
$$\ln a + \ln b = -xy$$
$$\ln ab = -xy$$

Then, using the definition of a logarithm, we have $e^{-xy} = ab$.

108. $\log_a \dfrac{1}{x} = \log_a 1 - \log_a x = -\log_a x$.

Let $-\log_a x = y$. Then $\log_a x = -y$ and $x = a^{-y} = a^{-1\cdot y} = \left(\dfrac{1}{a}\right)^y$, so $\log_{1/a} x = y$. Thus, $\log_a \left(\dfrac{1}{x}\right) = -\log_a x = \log_{1/a} x$.

109.
$$\log_a \left(\frac{x+\sqrt{x^2-5}}{5}\right)$$
$$= \log_a \left(\frac{x+\sqrt{x^2-5}}{5}\cdot\frac{x-\sqrt{x^2-5}}{x-\sqrt{x^2-5}}\right)$$
$$= \log_a \left(\frac{5}{5(x-\sqrt{x^2-5})}\right) = \log_a \left(\frac{1}{x-\sqrt{x^2-5}}\right)$$
$$= \log_a 1 - \log_a (x-\sqrt{x^2-5})$$
$$= -\log_a (x-\sqrt{x^2-5})$$

Exercise Set 5.5

1. $3^x = 81$
$3^x = 3^4$
$x = 4$ The exponents are the same.

The solution is 4.

2. $2^x = 32$
$2^x = 2^5$
$x = 5$
The solution is 5.

3. $2^{2x} = 8$
$2^{2x} = 2^3$
$2x = 3$ The exponents are the same.
$x = \frac{3}{2}$
The solution is $\frac{3}{2}$.

4. $3^{7x} = 27$
$3^{7x} = 3^3$
$7x = 3$
$x = \frac{3}{7}$
The solution is $\frac{3}{7}$.

5. $2^x = 33$
$\log 2^x = \log 33$ Taking the common logarithm on both sides
$x \log 2 = \log 33$ Power rule
$x = \frac{\log 33}{\log 2}$
$x \approx \frac{1.5185}{0.3010}$
$x \approx 5.044$
The solution is 5.044.

6. $2^x = 40$
$\log 2^x = \log 40$
$x \log 2 = \log 40$
$x = \frac{\log 40}{\log 2}$
$x \approx \frac{1.6021}{0.3010}$
$x \approx 5.322$
The solution is 5.322.

7. $5^{4x-7} = 125$
$5^{4x-7} = 5^3$
$4x - 7 = 3$
$4x = 10$
$x = \frac{10}{4} = \frac{5}{2}$
The solution is $\frac{5}{2}$.

8. $4^{3x-5} = 16$
$4^{3x-5} = 4^2$
$3x - 5 = 2$
$3x = 7$
$x = \frac{7}{3}$
The solution is $\frac{7}{3}$.

9. $27 = 3^{5x} \cdot 9^{x^2}$
$3^3 = 3^{5x} \cdot (3^2)^{x^2}$
$3^3 = 3^{5x} \cdot 3^{2x^2}$
$3^3 = 3^{5x+2x^2}$
$3 = 5x + 2x^2$
$0 = 2x^2 + 5x - 3$
$0 = (2x - 1)(x + 3)$
$x = \frac{1}{2}$ *or* $x = -3$
The solutions are -3 and $\frac{1}{2}$.

10. $3^{x^2+4x} = \frac{1}{27}$
$3^{x^2+4x} = 3^{-3}$
$x^2 + 4x = -3$
$x^2 + 4x + 3 = 0$
$(x + 3)(x + 1) = 0$
$x = -3$ *or* $x = -1$
The solutions are -3 and -1.

11. $84^x = 70$
$\log 84^x = \log 70$
$x \log 84 = \log 70$
$x = \frac{\log 70}{\log 84}$
$x \approx \frac{1.8451}{1.9243}$
$x \approx 0.959$
The solution is 0.959.

12. $28^x = 10^{-3x}$
$\log 28^x = \log 10^{-3x}$
$x \log 28 = -3x$
$x \log 28 + 3x = 0$
$x(\log 28 + 3) = 0$
$x = 0$
The solution is 0.

13.
$$\begin{aligned} 10^{-x} &= 5^{2x} \\ \log\ 10^{-x} &= \log\ 5^{2x} \\ -x &= 2x\log\ 5 \\ 0 &= x + 2x\log\ 5 \\ 0 &= x(1+2\log\ 5) \\ 0 &= x \qquad \text{Dividing by } 1+2\log\ 5 \end{aligned}$$
The solution is 0.

14.
$$\begin{aligned} 15^x &= 30 \\ \log 15^x &= \log 30 \\ x\log 15 &= \log 30 \\ x &= \frac{\log 30}{\log 15} \\ x &\approx 1.256 \end{aligned}$$
The solution is 1.256.

15.
$$\begin{aligned} e^{-c} &= 5^{2c} \\ \ln\ e^{-c} &= \ln 5^{2c} \\ -c &= 2c\ \ln\ 5 \\ 0 &= c + 2c\ \ln\ 5 \\ 0 &= c(1+2\ \ln\ 5) \\ 0 &= c \qquad \text{Dividing by } 1+2\ \ln\ 5 \end{aligned}$$
The solution is 0.

16.
$$\begin{aligned} e^{4t} &= 200 \\ \ln\ e^{4t} &= \ln\ 200 \\ 4t &= \ln\ 200 \\ t &= \frac{\ln\ 200}{4} \\ t &\approx 1.325 \end{aligned}$$
The solution is 1.325.

17.
$$\begin{aligned} e^t &= 1000 \\ \ln e^t &= \ln 1000 \\ t &= \ln 1000 \quad \text{Using } \log_a a^x = x \\ t &\approx 6.908 \end{aligned}$$
The solution is 6.908.

18.
$$\begin{aligned} e^{-t} &= 0.04 \\ \ln e^{-t} &= \ln 0.04 \\ -t &= \ln 0.04 \\ t &= -\ln 0.04 \approx 3.219 \end{aligned}$$
The solution is 3.219.

19.
$$\begin{aligned} e^{-0.03t} &= 0.08 \\ \ln e^{-0.03t} &= \ln 0.08 \\ -0.03t &= \ln 0.08 \\ t &= \frac{\ln 0.08}{-0.03} \\ t &\approx \frac{-2.5257}{-0.03} \\ t &\approx 84.191 \end{aligned}$$
The solution is 84.191.

20.
$$\begin{aligned} 1000e^{0.09t} &= 5000 \\ e^{0.09t} &= 5 \\ \ln e^{0.09t} &= \ln 5 \\ 0.09t &= \ln 5 \\ t &= \frac{\ln 5}{0.09} \\ t &\approx 17.883 \end{aligned}$$
The solution is 17.883.

21.
$$\begin{aligned} 3^x &= 2^{x-1} \\ \ln 3^x &= \ln 2^{x-1} \\ x\ln 3 &= (x-1)\ln 2 \\ x\ln 3 &= x\ln 2 - \ln 2 \\ \ln 2 &= x\ln 2 - x\ln 3 \\ \ln 2 &= x(\ln 2 - \ln 3) \\ \frac{\ln 2}{\ln 2 - \ln 3} &= x \\ \frac{0.6931}{0.6931 - 1.0986} &\approx x \\ -1.710 &\approx x \end{aligned}$$
The solution is -1.710.

22.
$$\begin{aligned} 5^{x+2} &= 4^{1-x} \\ \log 5^{x+2} &= \log 4^{1-x} \\ (x+2)\log 5 &= (1-x)\log 4 \\ x\log 5 + 2\log 5 &= \log 4 - x\log 4 \\ x\log 5 + x\log 4 &= \log 4 - 2\log 5 \\ x(\log 5 + \log 4) &= \log 4 - 2\log 5 \\ x &= \frac{\log 4 - 2\log 5}{\log 5 + \log 4} \\ x &\approx -0.612 \end{aligned}$$
The solution is -0.612.

23.
$$\begin{aligned} (3.9)^x &= 48 \\ \log(3.9)^x &= \log 48 \\ x\log 3.9 &= \log 48 \\ x &= \frac{\log 48}{\log 3.9} \\ x &\approx \frac{1.6812}{0.5911} \\ x &\approx 2.844 \end{aligned}$$
The solution is 2.844.

24.
$$\begin{aligned} 250 - (1.87)^x &= 0 \\ 250 &= (1.87)^x \\ \log 250 &= \log(1.87)^x \\ \log 250 &= x\log 1.87 \\ \frac{\log 250}{\log 1.87} &= x \\ 8.821 &\approx x \end{aligned}$$
The solution is 8.821.

25.
$$e^x + e^{-x} = 5$$
$$e^{2x} + 1 = 5e^x \quad \text{Multiplying by } e^x$$
$$e^{2x} - 5e^x + 1 = 0 \quad \text{This equation is quadratic in } e^x.$$
$$e^x = \frac{5 \pm \sqrt{21}}{2}$$
$$x = \ln\left(\frac{5 \pm \sqrt{21}}{2}\right) \approx \pm 1.567$$
The solutions are -1.567 and 1.567.

26.
$$e^x - 6e^{-x} = 1$$
$$e^{2x} - 6 = e^x$$
$$e^{2x} - e^x - 6 = 0$$
$$(e^x - 3)(e^x + 2) = 0$$
$$e^x = 3 \quad \textit{or} \quad e^x = -2$$
$$\ln e^x = \ln 3 \quad \text{No solution}$$
$$x = \ln 3$$
$$x \approx 1.099$$
The solution is 1.099.

27.
$$3^{2x-1} = 5^x$$
$$\log 3^{2x-1} = \log 5^x$$
$$(2x-1)\log 3 = x \log 5$$
$$2x \log 3 - \log 3 = x \log 5$$
$$-\log 3 = x \log 5 - 2x \log 3$$
$$-\log 3 = x(\log 5 - 2 \log 3)$$
$$\frac{-\log 3}{\log 5 - 2\log 3} = x$$
$$1.869 \approx x$$
The solution is 1.869.

28.
$$2^{x+1} = 5^{2x}$$
$$\ln 2^{x+1} = \ln 5^{2x}$$
$$(x+1)\ln 2 = 2x \ln 5$$
$$x \ln 2 + \ln 2 = 2x \ln 5$$
$$\ln 2 = 2x \ln 5 - x \ln 2$$
$$\ln 2 = x(2 \ln 5 - \ln 2)$$
$$\frac{\ln 2}{2 \ln 5 - \ln 2} = x$$
$$0.274 \approx x$$
The solution is 0.274.

29. $\log_5 x = 4$
$$x = 5^4 \quad \text{Writing an equivalent exponential equation}$$
$$x = 625$$
The solution is 625.

30. $\log_2 x = -3$
$$x = 2^{-3}$$
$$x = \frac{1}{8}$$
The solution is $\frac{1}{8}$.

31. $\log x = -4$ The base is 10.
$$x = 10^{-4}, \text{ or } 0.0001$$
The solution is 0.0001.

32. $\log x = 1$
$$x = 10^1 = 10$$
The solution is 10.

33. $\ln x = 1$ The base is e.
$$x = e^1 = e$$
The solution is e.

34. $\ln x = -2$
$$x = e^{-2}, \text{ or } \frac{1}{e^2}$$
The solution is e^{-2}, or $\frac{1}{e^2}$.

35. $\log_2(10 + 3x) = 5$
$$2^5 = 10 + 3x$$
$$32 = 10 + 3x$$
$$22 = 3x$$
$$\frac{22}{3} = x$$
The answer checks. The solution is $\frac{22}{3}$.

36. $\log_5(8 - 7x) = 3$
$$5^3 = 8 - 7x$$
$$125 = 8 - 7x$$
$$117 = -7x$$
$$-\frac{117}{7} = x$$
The answer checks. The solution is $-\frac{117}{7}$.

37. $\log x + \log(x - 9) = 1$ The base is 10.
$$\log_{10}[x(x-9)] = 1$$
$$x(x-9) = 10^1$$
$$x^2 - 9x = 10$$
$$x^2 - 9x - 10 = 0$$
$$(x-10)(x+1) = 0$$
$$x = 10 \quad \text{or} \quad x = -1$$
Check: For 10:

$\log x + \log(x-9) = 1$		
$\log 10 + \log(10 - 9)$	? 1	
$\log 10 + \log 1$		
$1 + 0$		
1	1	TRUE

For -1:

$\log x + \log(x-9) = 1$	
$\log(-1) + \log(-1 - 9)$	? 1

The number -1 does not check, because negative numbers do not have logarithms. The solution is 10.

38. $\log_2(x+1)+\log_2(x-1)=3$

$$\begin{aligned} \log_2[(x+1)(x-1)] &= 3 \\ (x+1)(x-1) &= 2^3 \\ x^2-1 &= 8 \\ x^2 &= 9 \\ x &= \pm 3 \end{aligned}$$

The number 3 checks, but -3 does not. The solution is 3.

39. $\log_2(x+20)-\log_2(x+2)=\log_2 x$

$$\begin{aligned} \log_2 \frac{x+20}{x+2} &= \log_2 x \\ \frac{x+20}{x+2} &= x \quad \text{Using the property of logarithmic equality} \\ x+20 &= x^2+2x \quad \text{Multiplying by } x+2 \\ 0 &= x^2+x-20 \\ 0 &= (x+5)(x-4) \end{aligned}$$

$x+5=0$ *or* $x-4=0$

$x=-5$ *or* $x=4$

Check: For -5:

$\log_2(x+20)-\log_2(x+2)=\log_2 x$		
$\log_2(-5+20)-\log_2(-5+2)$	?	$\log_2(-5)$

The number -5 does not check, because negative numbers do not have logarithms.

For 4:

$\log_2(x+20)-\log_2(x+2)=\log_2 x$		
$\log_2(4+20)-\log_2(4+2)$	?	$\log_2 4$
$\log_2 24-\log_2 6$		
$\log_2 \frac{24}{6}$		
$\log_2 4$		$\log_2 4$ TRUE

The solution is 4.

40. $\log(x+5)-\log(x-3)=\log 2$

$$\begin{aligned} \log \frac{x+5}{x-3} &= \log 2 \\ \frac{x+5}{x-3} &= 2 \\ x+5 &= 2x-6 \\ 11 &= x \end{aligned}$$

The answer checks. The solution is 11.

41. $\log_8(x+1)-\log_8 x=2$

$$\begin{aligned} \log_8\left(\frac{x+1}{x}\right) &= 2 \quad \text{Quotient rule} \\ \frac{x+1}{x} &= 8^2 \\ \frac{x+1}{x} &= 64 \\ x+1 &= 64x \\ 1 &= 63x \\ \frac{1}{63} &= x \end{aligned}$$

The answer checks. The solution is $\frac{1}{63}$.

42. $\log x-\log(x+3)=-1$

$$\begin{aligned} \log_{10}\frac{x}{x+3} &= -1 \\ \frac{x}{x+3} &= 10^{-1} \\ \frac{x}{x+3} &= \frac{1}{10} \\ 10x &= x+3 \\ 9x &= 3 \\ x &= \frac{1}{3} \end{aligned}$$

The answer checks. The solution is $\frac{1}{3}$.

43. $\log x+\log(x+4)=\log 12$

$$\begin{aligned} \log x(x+4) &= \log 12 \\ x(x+4) &= 12 \quad \text{Using the property of logarithmic equality} \\ x^2+4x &= 12 \\ x^2+4x-12 &= 0 \\ (x+6)(x-2) &= 0 \end{aligned}$$

$x+6=0$ *or* $x-2=0$

$x=-6$ *or* $x=2$

Check: For -6:

$\log x+\log(x+4)=\log 12$		
$\log(-6)+\log(-6+4)$	?	$\log 12$

The number -6 does not check, because negative numbers do not have logarithms.

For 2:

$\log x+\log(x+4)=\log 12$		
$\log 2+\log(2+4)$	?	$\log 12$
$\log 2+\log 6$		
$\log(2\cdot 6)$		
$\log 12$		$\log 12$ TRUE

The solution is 2.

44. $\ln x - \ln(x-4) = \ln 3$

$$\ln \frac{x}{x-4} = \ln 3$$
$$\frac{x}{x-4} = 3$$
$$x = 3x - 12$$
$$12 = 2x$$
$$6 = x$$

The answer checks. The solution is 6.

45. $\log_4(x+3) + \log_4(x-3) = 2$

$$\log_4[(x+3)(x-3)] = 2 \quad \text{Product rule}$$
$$(x+3)(x-3) = 4^2$$
$$x^2 - 9 = 16$$
$$x^2 = 25$$
$$x = \pm 5$$

The number 5 checks, but -5 does not. The solution is 5.

46. $\ln(x+1) - \ln x = \ln 4$

$$\ln \frac{x+1}{x} = \ln 4$$
$$\frac{x+1}{x} = 4$$
$$x + 1 = 4x$$
$$1 = 3x$$
$$\frac{1}{3} = x$$

The answer checks. The solution is $\frac{1}{3}$.

47. $\log(2x+1) - \log(x-2) = 1$

$$\log\left(\frac{2x+1}{x-2}\right) = 1 \quad \text{Quotient rule}$$
$$\frac{2x+1}{x-2} = 10^1 = 10$$
$$2x + 1 = 10x - 20$$

Multiplying by $x-2$

$$21 = 8x$$
$$\frac{21}{8} = x$$

The answer checks. The solution is $\frac{21}{8}$.

48. $\log_5(x+4) + \log_5(x-4) = 2$

$$\log_5[(x+4)(x-4)] = 2$$
$$x^2 - 16 = 25$$
$$x^2 = 41$$
$$x = \pm\sqrt{41}$$

Only $\sqrt{41}$ checks. The solution is $\sqrt{41}$.

49. $2e^x = 5 - e^{-x}$

$$2e^x - 5 + e^{-x} = 0$$
$$e^x(2e^x - 5 + e^{-x}) = e^x \cdot 0 \quad \text{Multiplying by } e^x$$
$$2e^{2x} - 5e^x + 1 = 0$$

Let $u = e^x$.

$$2u^2 - 5u + 1 = 0 \quad \text{Substituting}$$

$a = 2$, $b = -5$, $c = 1$

$$u = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$u = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 2 \cdot 1}}{2 \cdot 2}$$
$$u = \frac{5 \pm \sqrt{17}}{4}$$

Replace u with e^x.

$$e^x = \frac{5-\sqrt{17}}{4} \quad or \quad e^x = \frac{5+\sqrt{17}}{4}$$
$$\ln e^x = \ln\left(\frac{5-\sqrt{17}}{4}\right) \quad or \quad \ln e^x = \ln\left(\frac{5+\sqrt{17}}{4}\right)$$
$$x \approx -1.518 \quad or \quad x \approx 0.825$$

The solutions are -1.518 and 0.825.

50. $e^x + e^{-x} = 4$

$$e^x - 4 + e^{-x} = 0$$
$$e^{2x} - 4e^x + 1 = 0 \quad \text{Multiplying by } e^x$$

Let $u = e^x$.

$$u = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} = \frac{4 \pm \sqrt{12}}{2}$$
$$e^x = \frac{4-\sqrt{12}}{2} \quad or \quad e^x = \frac{4+\sqrt{12}}{2}$$
$$\ln e^x = \ln\left(\frac{4-\sqrt{12}}{2}\right) \quad or \quad \ln e^x = \ln\left(\frac{4+\sqrt{12}}{2}\right)$$
$$x \approx -1.317 \quad or \quad x \approx 1.317$$

The solutions are -1.317 and 1.317.

51. $\ln(x+8) + \ln(x-1) = 2\ln x$

$$\ln(x+8)(x-1) = \ln x^2$$
$$(x+8)(x-1) = x^2 \quad \text{Using the property of logarithmic equality}$$
$$x^2 + 7x - 8 = x^2$$
$$7x - 8 = 0$$
$$7x = 8$$
$$x = \frac{8}{7}$$

The answer checks. The solution is $\frac{8}{7}$.

52. $\log_3 x + \log_3(x+1) = \log_3 2 + \log_3(x+3)$

$$\log_3 x(x+1) = \log_3 2(x+3)$$
$$x(x+1) = 2(x+3)$$
$$x^2 + x = 2x + 6$$
$$x^2 - x - 6 = 0$$
$$(x-3)(x+2) = 0$$

$x = 3$ *or* $x = -2$

The number 3 checks, but -2 does not. The solution is 3.

53. $e^{7.2x} = 14.009$

Graph $y_1 = e^{7.2x}$ and $y_2 = 14.009$ and find the first coordinate of the point of intersection using the Intersect feature. The solution is 0.367.

54. $0.082e^{0.05x} = 0.034$

Graph $y_1 = 0.082e^{0.05x}$ and $y_2 = 0.034$ and find the first coordinate of the point of intersection using the Intersect feature. The solution is -17.607.

55. $2^x - 5 = 3x + 1$

Graph $y_1 = 2^x - 5$ and $y_2 = 3x + 1$ and find the first coordinates of the points of intersection using the Intersect feature. The solutions are -1.911 and 4.222.

56. $4x - 3^x = -6$

Graph $y_1 = 4x - 3^x$ and $y_2 = -6$ and find the first coordinates of the points of intersection using the Intersect feature. The solutions are -1.449 and 2.531.

57. $xe^{3x} - 1 = 3$

Graph $y_1 = xe^{3x} - 1$ and $y_2 = 3$ and find the first coordinate of the point of intersection using the Intersect feature. The solution is 0.621.

58. $5e^{5x} + 10 = 3x + 40$

Graph $y_1 = 5e^{5x} + 10$ and $y_2 = 3x + 40$ and find the first coordinates of the points of intersection using the Intersect feature. The solutions are -10 and 0.366.

59. $4\ln(x + 3.4) = 2.5$

Graph $y_1 = 4\ln(x + 3.4)$ and $y_2 = 2.5$ and find the first coordinate of the point of intersection using the Intersect feature. The solution is -1.532.

60. $\ln x^2 = -x^2$

Graph $y_1 = \ln x^2$ and $y_2 = -x^2$ and find the first coordinates of the points of intersection using the Intersect feature. The solutions are -0.753 and 0.753.

61. $\log_8 x + \log_8(x + 2) = 2$

Graph $y_1 = \dfrac{\log x}{\log 8} + \dfrac{\log(x+2)}{\log 8}$ and $y_2 = 2$ and find the first coordinate of the point of intersection using the intersect feature. The solution is 7.062.

62. $\log_3 x + 7 = 4 - \log_5 x$

Graph $y_1 = \dfrac{\log x}{\log 3} + 7$ and $y_2 = 4 - \dfrac{\log x}{\log 5}$ and find the first coordinate of the point of intersection using the Intersect feature. The solution is 0.141.

63. $\log_5(x + 7) - \log_5(2x - 3) = 1$

Graph $y_1 = \dfrac{\log(x+7)}{\log 5} - \dfrac{\log(2x-3)}{\log 5}$ and $y_2 = 1$ and find the first coordinate of the point of intersection using the Intersect feature. The solution is 2.444.

64. Graph $y_1 = \ln 3x$ and $y_2 = 3x - 8$ and use the Intersect feature to find the points of intersection. They are $(0.0001, -7.9997)$ and $(3.445, 2.336)$.

65. Solving the first equation for y, we get $y = \dfrac{12.4 - 2.3x}{3.8}$. Graph $y_1 = \dfrac{12.4 - 2.3x}{3.8}$ and $y_2 = 1.1\ln(x - 2.05)$ and use the Intersect feature to find the point of intersection. It is $(4.093, 0.786)$.

66. Graph $y_1 = 2.3\ln(x + 10.7)$ and $y_2 = 10e^{-0.07x^2}$ and use the Intersect feature to find the points of intersection. They are $(-9.694, 0.014)$, $(-3.334, 4.593)$, and $(2.714, 5.971)$.

67. Graph $y_1 = 2.3\ln(x + 10.7)$ and $y_2 = 10e^{-0.007x^2}$ and use the Intersect feature to find the point of intersection. It is $(7.586, 6.684)$.

68. The final result would have been the same, but to find t we would have computed $\dfrac{\log 2500}{0.08 \log e}$.

It seems best to take the natural logarithm on both sides since the final computation for t is simpler.

69. Use the graph of $y = \ln x$ to estimate the x-value that corresponds to the value on the right-hand side of the equation.

70. $f(x) = -x^2 + 6x - 8$

a) $-\dfrac{b}{2a} = -\dfrac{6}{2(-1)} = 3$

$f(3) = -3^2 + 6 \cdot 3 - 8 = 1$

The vertex is $(3, 1)$.

b) $x = 3$

c) Maximum: 1 at $x = 3$

71. $g(x) = x^2 - 6$

a) $-\dfrac{b}{2a} = -\dfrac{0}{2 \cdot 1} = 0$

$g(0) = 0^2 - 6 = -6$

The vertex is $(0, -6)$.

b) The axis of symmetry is $x = 0$.

c) Since the coefficient of the x^2-term is positive, the function has a minimum value. It is the second coordinate of the vertex, -6, and it occurs when $x = 0$.

72. $H(x) = 3x^2 - 12x + 16$

a) $-\dfrac{b}{2a} = -\dfrac{-12}{2 \cdot 3} = 2$

$H(2) = 3 \cdot 2^2 - 12 \cdot 2 + 16 = 4$

The vertex is $(2, 4)$.

b) $x = 2$

c) Minimum: 4 at $x = 2$

73. $G(x) = -2x^2 - 4x - 7$

a) $-\dfrac{b}{2a} = -\dfrac{-4}{2(-2)} = -1$

$G(-1) = -2(-1)^2 - 4(-1) - 7 = -5$

The vertex is $(-1, -5)$.

b) The axis of symmetry is $x = -1$.

c) Since the coefficient of the x^2-term is negative, the function has a maximum value. It is the second coordinate of the vertex, -5, and it occurs when $x = -1$.

74. $\dfrac{5^x - 5^{-x}}{5^x + 5^{-x}} = 8$

$5^x - 5^{-x} = 8 \cdot 5^x + 8 \cdot 5^{-x}$

$-9 \cdot 5^{-x} = 7 \cdot 5^x$

$-9 = 7 \cdot 5^{2x}$ Multiplying by 5^x

$-\dfrac{9}{7} = 5^{2x}$

The number 5 raised to any power is non-negative. Thus, the equation has no solution.

75. $\dfrac{e^x + e^{-x}}{e^x - e^{-x}} = 3$

$e^x + e^{-x} = 3e^x - 3e^{-x}$ Multiplying by $e^x - e^{-x}$

$4e^{-x} = 2e^x$ Subtracting e^x and adding e^{-x}

$2e^{-x} = e^x$

$2 = e^{2x}$ Multiplying by e^x

$\ln 2 = \ln e^{2x}$

$\ln 2 = 2x$

$\dfrac{\ln 2}{2} = x$

$0.347 \approx x$

The solution is 0.347.

76. $\ln(\ln x) = 2$

$\ln x = e^2$

$x = e^{e^2} \approx 1618.178$

The answer checks. The solution is e^{e^2}, or 1618.178.

77. $\ln(\log x) = 0$

$\log x = e^0$

$\log x = 1$

$x = 10^1 = 10$

The answer checks. The solution is 10.

78. $\ln \sqrt[4]{x} = \sqrt{\ln x}$

$\dfrac{1}{4} \ln x = \sqrt{\ln x}$

$\dfrac{1}{16}(\ln x)^2 = \ln x$ Squaring both sides

$\dfrac{1}{16}(\ln x)^2 - \ln x = 0$

Let $u = \ln x$ and substitute.

$\dfrac{1}{16}u^2 - u = 0$

$u\left(\dfrac{1}{16}u - 1\right) = 0$

$u = 0$ *or* $\dfrac{1}{16}u - 1 = 0$

$u = 0$ *or* $u = 16$

$\ln x = 0$ *or* $\ln x = 16$

$x = e^0$ *or* $x = e^{16}$

$x = 1$ *or* $x = e^{16} \approx 8,886,110.521$

Both answers check. The solutions are 1 and e^{16}, or 1 and 8,886,110.521.

79. $\sqrt{\ln x} = \ln \sqrt{x}$

$\sqrt{\ln x} = \dfrac{1}{2} \ln x$ Power rule

$\ln x = \dfrac{1}{4}(\ln x)^2$ Squaring both sides

$0 = \dfrac{1}{4}(\ln x)^2 - \ln x$

Let $u = \ln x$ and substitute.

$\dfrac{1}{4}u^2 - u = 0$

$u\left(\dfrac{1}{4}u - 1\right) = 0$

$u = 0$ *or* $\dfrac{1}{4}u - 1 = 0$

$u = 0$ *or* $\dfrac{1}{4}u = 1$

$u = 0$ *or* $u = 4$

$\ln x = 0$ *or* $\ln x = 4$

$x = e^0 = 1$ *or* $x = e^4 \approx 54.598$

Both answers check. The solutions are 1 and e^4, or 1 and 54.598.

80. $\log_3(\log_4 x) = 0$

$\log_4 x = 3^0$

$\log_4 x = 1$

$x = 4^1$

$x = 4$

The answer checks. The solution is 4.

81. $(\log_3 x)^2 - \log_3 x^2 = 3$

$(\log_3 x)^2 - 2\log_3 x - 3 = 0$

Let $u = \log_3 x$ and substitute:

$u^2 - 2u - 3 = 0$

$(u - 3)(u + 1) = 0$

$u = 3$ *or* $u = -1$

$\log_3 x = 3$ *or* $\log_3 x = -1$

$x = 3^3$ *or* $x = 3^{-1}$

$x = 27$ *or* $x = \dfrac{1}{3}$

Both answers check. The solutions are $\dfrac{1}{3}$ and 27.

82. $(\log x)^2 - \log x^2 = 3$

$(\log x)^2 - 2\log x - 3 = 0$

Let $u = \log x$ and substitute.

$u^2 - 2u - 3 = 0$

$(u + 1)(u - 3) = 0$

$u = -1$ *or* $u = 3$

$\log x = -1$ *or* $\log x = 3$

$x = \dfrac{1}{10}$ *or* $x = 1000$

Both answers check. The solutions are $\dfrac{1}{10}$ and 1000.

83. $\ln x^2 = (\ln x)^2$

$2\ln x = (\ln x)^2$

$0 = (\ln x)^2 - 2\ln x$

Let $u = \ln x$ and substitute.

$0 = u^2 - 2u$

$0 = u(u-2)$

$u = 0$ *or* $u = 2$

$\ln x = 0$ *or* $\ln x = 2$

$x = 1$ *or* $x = e^2 \approx 7.389$

Both answers check. The solutions are 1 and e^2, or 1 and 7.389.

84. $e^{2x} - 9 \cdot e^x + 14 = 0$

$(e^x - 2)(e^x - 7) = 0$

$e^x = 2$ *or* $e^x = 7$

$\ln e^x = \ln 2$ *or* $\ln e^x = \ln 7$

$x = \ln 2$ *or* $x = \ln 7$

$x \approx 0.693$ *or* $x \approx 1.946$

The solutions are 0.693 and 1.946.

85. $5^{2x} - 3 \cdot 5^x + 2 = 0$

$(5^x - 1)(5^x - 2) = 0$ This equation is quadratic in 5^x.

$5^x = 1$ *or* $5^x = 2$

$\log 5^x = \log 1$ *or* $\log 5^x = \log 2$

$x \log 5 = 0$ *or* $x\log 5 = \log 2$

$x = 0$ *or* $x = \dfrac{\log 2}{\log 5} \approx 0.431$

The solutions are 0 and 0.431.

86. $x\left(\ln \dfrac{1}{6}\right) = \ln 6$

$x(\ln 1 - \ln 6) = \ln 6$

$-x\ln 6 = \ln 6$ $(\ln 1 = 0)$

$x = -1$

The solution is -1.

87. $\log_3 |x| = 2$

$|x| = 3^2$

$|x| = 9$

$x = -9$ *or* $x = 9$

Both answers check. The solutions are -9 and 9.

88. $x^{\log x} = \dfrac{x^3}{100}$

$\log x^{\log x} = \log \dfrac{x^3}{100}$

$\log x \cdot \log x = \log x^3 - \log 100$

$(\log x)^2 = 3\log x - 2$

$(\log x)^2 - 3\log x + 2 = 0$

Let $u = \log x$ and substitute.

$u^2 - 3u + 2 = 0$

$(u-1)(u-2) = 0$

$u = 1$ *or* $u = 2$

$\log x = 1$ *or* $\log x = 2$

$x = 10$ *or* $x = 10^2 = 100$

Both answers check. The solutions are 10 and 100.

89. $\ln x^{\ln x} = 4$

$\ln x \cdot \ln x = 4$

$(\ln x)^2 = 4$

$\ln x = \pm 2$

$\ln x = -2$ *or* $\ln x = 2$

$x = e^{-2}$ *or* $x = e^2$

$x \approx 0.135$ *or* $x \approx 7.389$

Both answers check. The solutions are e^{-2} and e^2, or 0.135 and 7.389.

90. $\dfrac{(e^{3x+1})^2}{e^4} = e^{10x}$

$\dfrac{e^{6x+2}}{e^4} = e^{10x}$

$e^{6x-2} = e^{10x}$

$6x - 2 = 10x$

$-2 = 4x$

$-\dfrac{1}{2} = x$

The solution is $-\dfrac{1}{2}$.

91. $\dfrac{\sqrt{(e^{2x} \cdot e^{-5x})^{-4}}}{e^x \div e^{-x}} = e^7$

$\dfrac{\sqrt{e^{12x}}}{e^{x-(-x)}} = e^7$

$\dfrac{e^{6x}}{e^{2x}} = e^7$

$e^{4x} = e^7$

$4x = 7$

$x = \dfrac{7}{4}$

The solution is $\dfrac{7}{4}$.

92. $e^x < \dfrac{4}{5}$

$\ln e^x < \ln 0.8$

$x < -0.223$

The solution set is $(-\infty, -0.223)$.

93. $|\log_5 x| + 3\log_5 |x| = 4$

Note that we must have $x > 0$. First consider the case when $0 < x < 1$. When $0 < x < 1$, then $\log_5 x < 0$, so $|\log_5 x| = -\log_5 x$ and $|x| = x$. Thus we have:

$$\begin{aligned} -\log_5 x + 3\log_5 x &= 4 \\ 2\log_5 x &= 4 \\ \log_5 x^2 &= 4 \\ x^2 &= 5^4 \\ x &= 5^2 \\ x &= 25 \quad \text{(Recall that } x > 0.) \end{aligned}$$

25 cannot be a solution since we assumed $0 < x < 1$.

Now consider the case when $x > 1$. In this case $\log_5 x > 0$, so $|\log_5 x| = \log_5 x$ and $|x| = x$. Thus we have:

$$\begin{aligned} \log_5 x + 3\log_5 x &= 4 \\ 4\log_5 x &= 4 \\ \log_5 x &= 1 \\ x &= 5 \end{aligned}$$

This answer checks. The solution is 5.

94. $|2^{x^2} - 8| = 3$

$$\begin{aligned} 2^{x^2} - 8 &= -3 \quad \textit{or} \quad 2^{x^2} - 8 = 3 \\ 2^{x^2} &= 5 \quad \textit{or} \quad 2^{x^2} = 11 \\ \log 2^{x^2} &= \log 5 \quad \textit{or} \quad \log 2^{x^2} = \log 11 \\ x^2 \log 2 &= \log 5 \quad \textit{or} \quad x^2 \log 2 = \log 11 \\ x^2 &= \frac{\log 5}{\log 2} \quad \textit{or} \quad x^2 = \frac{\log 11}{\log 2} \\ x &= \pm 1.524 \quad \textit{or} \quad x = \pm 1.860 \end{aligned}$$

The solutions are -1.860, -1.524, 1.524, and 1.860.

95. $a = \log_8 225$, so $8^a = 225 = 15^2$.

$b = \log_2 15$, so $2^b = 15$.

Then
$$\begin{aligned} 8^a &= (2^b)^2 \\ (2^3)^a &= 2^{2b} \\ 2^{3a} &= 2^{2b} \\ 3a &= 2b \\ a &= \frac{2}{3}b. \end{aligned}$$

96. $\log_5 125 = 3$ and $\log_{125} 5 = \dfrac{1}{3}$, so $a = (\log_{125} 5)^{\log_5 125}$ is equivalent to $a = \left(\dfrac{1}{3}\right)^3 = \dfrac{1}{27}$. Then $\log_3 a = \log_3 \dfrac{1}{27} = -3$.

97. $\log_2[\log_3(\log_4 x)] = 0$ yields $x = 64$.

$\log_3[\log_2(\log_4 y)] = 0$ yields $y = 16$.

$\log_4[\log_3(\log_2 z)] = 0$ yields $z = 8$.

Then $x + y + z = 64 + 16 + 8 = 88$.

98. $f(x) = e^x - e^{-x}$

Replace $f(x)$ with y: $y = e^x - e^{-x}$

Interchange x and y: $x = e^y - e^{-y}$

Solve for y: $xe^y = e^{2y} - 1$ Multiplying by e^y

$$0 = e^{2y} - xe^y - 1$$

Using the quadratic formula with $a = 1$, $b = -x$, and $c = -1$ and taking the positive square root (since $e^y > 0$), we get $e^y = \dfrac{x + \sqrt{x^2 + 4}}{2}$. Then we have

$$\begin{aligned} \ln e^y &= \ln\left(\frac{x + \sqrt{x^2 + 4}}{2}\right) \\ y &= \ln\left(\frac{x + \sqrt{x^2 + 4}}{2}\right) \end{aligned}$$

Replace y with $f^{-1}(x)$:

$$f^{-1}(x) = \ln\left(\frac{x + \sqrt{x^2 + 4}}{2}\right).$$

Exercise Set 5.6

1. a) Substitute 6.5 for P_0 and 0.0114 for k in $P(t) = P_0 e^{kt}$. We have:

$P(t) = 6.5e^{0.0114t}$, where $P(t)$ is in billions and t is the number of years after 2006.

b) In 2009, $t = 2009 - 2006 = 3$.

$P(3) = 6.5e^{0.0114(3)} \approx 6.7$ billion

In 2015, $t = 2015 - 2006 = 9$.

$P(9) = 6.5e^{0.0114(9)} \approx 7.2$ billion

c) Substitute 8 for $P(t)$ and solve for t.

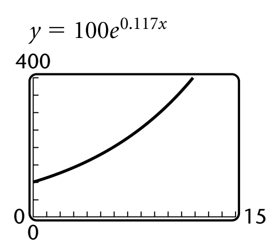
$$\begin{aligned} 8 &= 6.5e^{0.0114t} \\ \frac{8}{6.5} &= e^{0.0114t} \\ \ln \frac{8}{6.5} &= \ln e^{0.0114t} \\ \ln \frac{8}{6.5} &= 0.0114t \\ \frac{\ln \frac{8}{6.5}}{0.0114} &= t \\ 18.2 &\approx t \end{aligned}$$

The world population will be 8 billion about 18.2 yr after 2006.

d) $T = \dfrac{\ln 2}{0.0114} \approx 60.8$ yr

2. a) $P(t) = 100e^{0.117t}$

b)



c) $P(7) = 100e^{0.117(7)} \approx 227$

Note that 2 weeks $= 2 \cdot 7$ days $= 14$ days.

$P(14) = 100e^{0.117(14)} \approx 514$

d) $t = \dfrac{\ln 2}{0.117} \approx 5.9$ days

3. a) $T = \dfrac{\ln 2}{0.024} \approx 28.9$ yr

b) $k = \dfrac{\ln 2}{63} \approx 1.1\%$ per yr

c) $T = \dfrac{\ln 2}{0.014} \approx 49.5$ yr

d) $T = \dfrac{\ln 2}{0.002} \approx 346.6$ yr

e) $k = \dfrac{\ln 2}{38.5} \approx 1.8\%$ per yr

f) $k = \dfrac{\ln 2}{36.5} \approx 1.9\%$ per yr

g) $k = \dfrac{\ln 2}{0.009} \approx 77.0$ yr

h) $k = \dfrac{\ln 2}{69.3} \approx 1.0\%$ per yr

i) $k = \dfrac{\ln 2}{0.012} \approx 57.8$ yr

j) $k = \dfrac{\ln 2}{0.026} \approx 26.7$ yr

4. a)

$$\begin{aligned} 29.01 &= 0.5e^{k\cdot 6} \\ 58.02 &= e^{6k} \\ \ln 58.02 &= \ln e^{6k} \\ \ln 58.02 &= 6k \\ \frac{\ln 58.02}{6} &= k \\ 0.6768 &\approx k \end{aligned}$$

$D(t) = 0.5e^{0.6768t}$, where $D(t)$ is in millions and t is the number of years after 1998.

b) $D(7) = 0.5e^{0.6768(7)} \approx 57.1$ million

$D(10) = 0.5e^{0.6768(10)} \approx 434.8$ million

$D(13) = 0.5e^{0.6768(13)} \approx 3312$ million

5.

$$\begin{aligned} P(t) &= P_0e^{kt} \\ 24,839,654,400 &= 6,276,883e^{0.013t} \\ \frac{24,839,654,400}{6,276,883} &= e^{0.013t} \\ \ln\left(\frac{24,839,654,400}{6,276,883}\right) &= \ln e^{0.013t} \\ \ln\left(\frac{24,839,654,400}{6,276,883}\right) &= 0.013t \\ \frac{\ln\left(\frac{24,839,654,400}{6,276,883}\right)}{0.013} &= t \\ 637 &\approx t \end{aligned}$$

There will be one person for every square yard of land about 637 yr after 2005.

6. In 2010, $t = 2010 - 1626 = 384$.

$P(384) = 24e^{0.06(384)} \approx \$243,419,914,000$, or

about $243,000,000,000

7. a) Substitute 10,000 for P_0 and 5.4%, or 0.054 for k.

$P(t) = 10,000e^{0.054t}$

b)

$$\begin{aligned} P(1) &= 10,000e^{0.054(1)} \approx \$10,554.85 \\ P(2) &= 10,000e^{0.054(2)} \approx \$11,140.48 \\ P(5) &= 10,000e^{0.054(5)} \approx \$13,099.64 \\ P(10) &= 10,000e^{0.054(10)} \approx \$17,160.07 \end{aligned}$$

c) $T = \dfrac{\ln 2}{0.054} \approx 12.8$ yr

8. a) $T = \dfrac{\ln 2}{0.062} \approx 11.2$ yr

$P(5) = 35,000e^{0.062(5)} \approx \$47,719.88$

b)

$$\begin{aligned} 7130.90 &= 5000e^{5k} \\ 1.4618 &= e^{5k} \\ \ln 1.4618 &= \ln e^{5k} \\ \ln 1.4618 &= 5k \\ \frac{\ln 1.4618}{5} &= k \\ 0.071 &\approx k \\ 7.1\% &\approx k \end{aligned}$$

$T = \dfrac{\ln 2}{0.071} \approx 9.8$ yr

c)

$$\begin{aligned} 11,414.71 &= P_0e^{0.084(5)} \\ \frac{11,414.71}{e^{0.084(5)}} &= P_0 \\ \$7500 &\approx P_0 \end{aligned}$$

$T = \dfrac{\ln 2}{0.084} \approx 8.3$ yr

d) $k = \dfrac{\ln 2}{11} \approx 0.063$, or 6.3%

$$\begin{aligned} 17,539.32 &= P_0e^{0.063(5)} \\ \frac{17,539.32}{e^{0.063(5)}} &= P_0 \\ \$12,800 &\approx P_0 \end{aligned}$$

9. We use the function found in Example 5. If the mummy has lost 46% of its carbon-14 from an initial amount P_0, then $54\%P_0$, or $0.54P_0$ remains. We substitute in the function.

$$\begin{aligned} 0.54P_0 &= P_0e^{-0.00012t} \\ 0.54 &= e^{-0.00012t} \\ \ln 0.54 &= \ln e^{-0.00012t} \\ \ln 0.54 &= -0.00012t \\ \frac{\ln 0.54}{-0.00012} &= t \\ 5135 &\approx t \end{aligned}$$

The mummy is about 5135 years old.

10. Let x = the percent of carbon-14 remaining in the mummies.

For 3300 yr:

$$\begin{aligned} xP_0 &= P_0e^{-0.00012(3300)} \\ x &= e^{-0.00012(3300)} \\ x &\approx 0.673, \text{ or } 67.3\% \end{aligned}$$

If 63.7% of the carbon-14 remains, the mummies have lost 100% − 67.3%, or 32.7%, of their carbon-14.

For 3500 yr:

$$xP_0 = P_0e^{-0.00012(3500)}$$
$$x = e^{-0.00012(3500)}$$
$$x \approx 0.657, \text{ or } 65.7\%$$

If 65.7% of the carbon-14 remains, the mummies have lost 100% − 65.7%, or 34.3%, of their carbon-14.

The mummies have lost about 32.7% to 34.3% of their carbon-14.

11. a) $K = \frac{\ln 2}{3} \approx 0.231$, or 23.1% per min

b) $k = \frac{\ln 2}{22} \approx 0.0315$, or 3.15% per yr

c) $T = \frac{\ln 2}{0.096} \approx 7.2$ days

d) $T = \frac{\ln 2}{0.063} \approx 11$ yr

e) $k = \frac{\ln 2}{25} \approx 0.028$, or 2.8% per yr

f) $k = \frac{\ln 2}{4560} \approx 0.00015$, or 0.015% per yr

g) $k = \frac{\ln 2}{23,105} \approx 0.00003$, or 0.003% per yr

12. a) $t = 2005 - 1950 = 55$

$$N(t) = N_0e^{-kt}$$
$$2,100,990 = 5,650,000e^{-k(55)}$$
$$\frac{2,100,990}{5,650,000} = e^{-55k}$$
$$\ln\left(\frac{2,100,990}{5,650,000}\right) = \ln e^{-55k}$$
$$\ln\left(\frac{2,100,990}{5,650,000}\right) = -55k$$
$$\frac{\ln\left(\frac{2,100,990}{5,650,000}\right)}{-55} = k$$
$$0.018 \approx k$$

$N(t) = 5,650,000e^{-0.018t}$

b) In 2009, $t = 2009 - 1950 = 59$.

$N(59) = 5,650,000e^{-0.018(59)} \approx 1,953,564$ farms

In 2015, $t = 2015 - 1950 = 65$.

$N(65) = 5,650,000e^{-0.018(65)} \approx 1,753,573$ farms

c)

$$1,000,000 = 5,650,000e^{-0.018t}$$
$$\frac{1,000,000}{5,650,000} = e^{-0.018t}$$
$$\ln\left(\frac{1,000,000}{5,650,000}\right) = \ln e^{-0.018t}$$
$$\ln\left(\frac{1,000,000}{5,650,000}\right) = -0.018t$$
$$\frac{\ln\left(\frac{1,000,000}{5,650,000}\right)}{-0.018} = t$$
$$96 \approx t$$

Only 1,000,000 farms will remain about 96 years after 1950, or in 2046.

13. a)

$$N(t) = N_0e^{-kt}$$
$$13,767 = 69,895e^{-k(50)}$$
$$\frac{13,767}{69,895} = e^{-50k}$$
$$\ln\left(\frac{13,767}{69,895}\right) = \ln e^{-50k}$$
$$\ln\left(\frac{13,767}{69,895}\right) = -50k$$
$$\frac{\ln\left(\frac{13,767}{69,895}\right)}{-50} = k$$
$$0.0325 \approx k$$

$N(t) = 69,895e^{-0.0325t}$

b) In 2008, $t = 2008 - 1956 = 52$.

$N(52) = 69,895e^{-0.0325(52)} \approx 12,897$ cases

In 2010, $t = 2010 - 1956 = 54$.

$N(54) = 69,895e^{-0.0325(54)} \approx 12,085$ cases

c)

$$5000 = 69,895e^{-0.0325t}$$
$$\frac{5000}{69,895} = e^{-0.0325t}$$
$$\ln\left(\frac{5000}{69,895}\right) = \ln e^{-0.0325t}$$
$$\ln\left(\frac{5000}{69,895}\right) = -0.0325t$$
$$\frac{\ln\left(\frac{5000}{69,895}\right)}{-0.0325} = t$$
$$81 \approx t$$

There will be 5000 cases of tuberculosis about 81 yr after 1956, or in 2037.

14. a)
$$27,000 = 800e^{k\cdot 39}$$
$$33.75 = e^{39k}$$
$$\ln\ 33.75 = \ln e^{39k}$$
$$\ln\ 33.75 = 39k$$
$$0.0902 \approx k$$

We have $V(t) = 800e^{0.0902t}$, where t is the number of years after 1967.

b) $V(41) = 800e^{0.0902(41)} \approx \$32,300$

c) $T = \dfrac{\ln\ 2}{0.0902} \approx 7.7$ yr

d)
$$40,000 = 800e^{0.0902t}$$
$$50 = e^{0.0902t}$$
$$\ln 50 = \ln e^{0.0902t}$$
$$\ln 50 = 0.0902t$$
$$43 \approx t$$

The value of the car will be \$40,000 about 43 yr after 1967, or in 2010.

15. a) In 2006, $t = 2006 - 1960 = 46$
$$R(t) = R_0e^{kt}$$
$$15,400,000 = 900e^{k(46)}$$
$$\frac{154,000}{9} = e^{46k}$$
$$\ln\left(\frac{154,000}{9}\right) = \ln e^{46k}$$
$$\ln\left(\frac{154,000}{9}\right) = 46k$$
$$\frac{\ln\left(\frac{154,000}{9}\right)}{46} = k$$
$$0.2119 \approx k$$

$R(t) = 900e^{0.2119t}$

b) In 2010, $t = 2010 - 1960 = 50$.

$R(50)=900e^{0.2119(50)}\approx\$35,941,198\approx\$35.9$ million

c) $T = \dfrac{\ln 2}{0.2119} \approx 3.3$ yr

d)
$$25,000,000 = 900e^{0.2119t}$$
$$\frac{250,000}{9} = e^{0.2119t}$$
$$\ln\left(\frac{250,000}{9}\right) = \ln e^{0.2119t}$$
$$\ln\left(\frac{250,000}{9}\right) = 0.2119t$$
$$\frac{\ln\left(\frac{250,000}{9}\right)}{0.2119} = t$$
$$48 \approx t$$

The value of the painting will be \$25 million about 48 yr after 1960, or in 2008.

16. a) $t = 2007 - 1971 = 36$
$$2,800,000 = 1000e^{k(36)}$$
$$2800 = e^{36k}$$
$$\ln 2800 = \ln e^{36k}$$
$$\ln 2800 = 36k$$
$$\frac{\ln 2800}{36} = k$$
$$0.2205 \approx k$$

$W(t) = 1000e^{0.2205(t)}$

b) $t = 2011 - 1971 = 40$

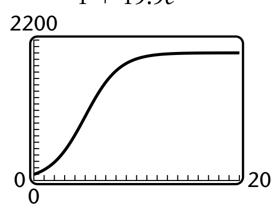
$W(40)=1000e^{0.2205(40)}\approx\$6,768,265\approx\$6.77$ million

c) $T = \dfrac{\ln 2}{0.2205} \approx 3.1$ yr

d)
$$3,000,000 = 1000e^{0.2205t}$$
$$3000 = e^{0.2205t}$$
$$\ln 3000 = \ln e^{0.2205t}$$
$$\ln 3000 = 0.2205t$$
$$\frac{\ln 3000}{0.2205} = t$$
$$36.3 \approx t$$

The value of the card will be \$3 million about 36.3 yr after 1971, or in 2007.

17. a) $y = \dfrac{2000}{1 + 19.9e^{-0.6x}}$



b) $N(0) = \dfrac{3500}{1 + 19.9e^{-0.6(0)}} \approx 167$

c) $N(2) = \dfrac{3500}{1 + 19.9e^{-0.6(2)}} \approx 500$

$N(5) = \dfrac{3500}{1 + 19.9e^{-0.6(5)}} \approx 1758$

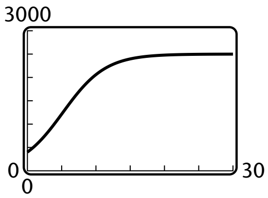
$N(8) = \dfrac{3500}{1 + 19.9e^{-0.6(8)}} \approx 3007$

$N(12) = \dfrac{3500}{1 + 19.9e^{-0.6(12)}} \approx 3449$

$N(16) = \dfrac{3500}{1 + 19.9e^{-0.6(16)}} \approx 3495$

d) As $t \to \infty$, $N(t) \to 3500$; the number of people infected approaches 3500 but never actually reaches it.

18. a) $y = \dfrac{2500}{1 + 5.25e^{-0.32x}}$



b) $P(0) = \dfrac{2500}{1 + 5.25e^{-0.32(0)}} = 400$

$P(1) = \dfrac{2500}{1 + 5.25e^{-0.32(1)}} \approx 520$

$P(5) = \dfrac{2500}{1 + 5.25e^{-0.32(5)}} \approx 1214$

$P(10) = \dfrac{2500}{1 + 5.25e^{-0.32(10)}} \approx 2059$

$P(15) = \dfrac{2500}{1 + 5.25e^{-0.32(15)}} \approx 2396$

$P(20) = \dfrac{2500}{1 + 5.25e^{-0.32(20)}} \approx 2478$

19. To find k we substitute 105 for T_1, 0 for T_0, 5 for t, and 70 for $T(t)$ and solve for k.

$$70 = 0 + (105 - 0)e^{-5k}$$
$$70 = 105e^{-5k}$$
$$\frac{70}{105} = e^{-5k}$$
$$\ln\frac{70}{105} = \ln e^{-5k}$$
$$\ln\frac{70}{105} = -5k$$
$$\frac{\ln\frac{70}{105}}{-5} = k$$
$$0.081 \approx k$$

The function is $T(t) = 105e^{-0.081t}$.

Now we find $T(10)$.

$T(10) = 105e^{-0.081(10)} \approx 46.7\ °\text{F}$

20. To find k we substitute 375 for T_1, 72 for T_0, 3 for t, and 365 for $T(t)$.

$$365 = 72 + (375 - 72)e^{-3k}$$
$$293 = 303e^{-3k}$$
$$\frac{293}{303} = e^{-3k}$$
$$\ln\frac{293}{303} = \ln e^{-3k}$$
$$\ln\frac{293}{303} = -3k$$
$$\frac{\ln\frac{293}{303}}{-3} = k$$
$$0.011 \approx k$$

The function is $T(t) = 72 + 303e^{-0.011t}$.

$T(15) = 72 + 303e^{-0.011(15)} \approx 329°\text{F}$

(Answers may vary slightly due to rounding differences.)

21. To find k we substitute 43 for T_1, 68 for T_0, 12 for t, and 55 for $T(t)$ and solve for k.

$$55 = 68 + (43 - 68)e^{-12k}$$
$$-13 = -25e^{-12k}$$
$$0.52 = e^{-12k}$$
$$\ln 0.52 = \ln e^{-12k}$$
$$\ln 0.52 = -12k$$
$$0.0545 \approx k$$

The function is $T(t) = 68 - 25e^{-0.0545t}$.

Now we find $T(20)$.

$T(20) = 68 - 25e^{-0.0545(20)} \approx 59.6°\text{F}$

22. To find k we substitute 94.6 for T_1, 70 for T_0, 60 for t (1 hr = 60 min), and 93.4 for $T(t)$.

$$93.4 = 70 + |94.6 - 70|e^{-k(60)}$$
$$23.4 = 24.6e^{-60k}$$
$$\frac{23.4}{24.6} = e^{-60k}$$
$$\ln\frac{23.4}{24.6} = \ln e^{-60k}$$
$$\ln\frac{23.4}{24.6} = -60k$$
$$k = \frac{\ln\frac{23.4}{24.6}}{-60} \approx 0.0008$$

The function is $T(t) = 70 + 24.6e^{-0.0008t}$.

We substitute 98.6 for $T(t)$ and solve for t.

$$98.6 = 70 + 24.6e^{-0.0008t}$$
$$28.6 = 24.6e^{-0.0008t}$$
$$\frac{28.6}{24.6} = e^{-0.0008t}$$
$$\ln\frac{28.6}{24.6} = \ln e^{-0.0008t}$$
$$\ln\frac{28.6}{24.6} = -0.0008t$$
$$t = \frac{\ln\frac{28.6}{24.6}}{-0.0008} \approx -188$$

The murder was committed at approximately 188 minutes, or about 3 hours, before 12:00 PM, or at about 9:00 AM. (Answers may vary slightly due to rounding differences.)

23. The data have the pattern of a decreasing exponential function, so function (d) might be used as a model.

24. The data have an S-shaped pattern, so function (f) might be used as a model.

25. The data have a parabolic pattern, so function (a) might be used as a model.

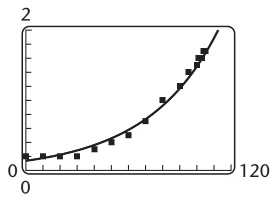
26. The data fit the pattern of a polynomial function with degree greater than two. Thus, function (b) might be used as a model.

27. The data have a logarithmic pattern, so function (e) might be used as a model.

28. The data have the pattern of an increasing exponential function, so function (c) might be used as a model.

29. a) $y = 0.136563665(1.024108508)^x$, where x is the number of years after 1900.

Since $r^2 = 0.9709036309$, the function is a good fit.

b) 

c) In 2007, $x = 2007 - 1900 = 107$.

$y = 0.136563665(1.024108508)^{107} \approx 1.7\%$

In 2015, $x = 2015 - 1900 = 115$.

$y = 0.136563665(1.024108508)^{115} \approx 2.1\%$

In 2020, $x = 2020 - 1900 = 120$.

$y = 0.136563665(1.024108508)^{120} \approx 2.4\%$

30. a) Using the logarithmic regression feature on a graphing calculator, we get $y = 84.94353992 - 0.5412834098 \ln x$.

b) For $x = 8$, $y \approx 83.8\%$.

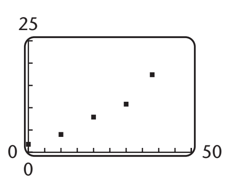
For $x = 10$, $y \approx 83.7\%$.

For $x = 24$, $y \approx 83.2\%$.

For $x = 36$, $y \approx 83.0\%$.

c) $82 = 84.94353992 - 0.5412834098 \ln x$

Graph $y_1 = 84.94353992 - 0.5412834098 \ln x$ and $y_2 = 82$ and find the first coordinate of the point of intersection of the graphs. It is approximately 230, so test scores will fall below 82% after about 230 months.

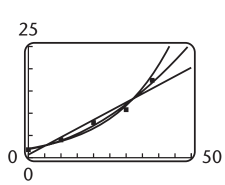
31. a) 

b) Linear: $y = 0.3920710572x + 0.695407279$; $r^2 \approx 0.9465$

Quadratic: $y = 0.0072218083x^2 + 0.1172668587x + 1.973805022$; $r^2 \approx 0.9851$

Exponential: $y = 2.062091236(1.059437743)^x$; $r^2 \approx 0.9775$

The value of r^2 is greatest for the quadratic function, so it provides the best fit.

c) 

d) In 2008, $x = 2008 - 1967 = 41$.

For the linear function, when $x = 41$, $y \approx 16.8\%$.

For the quadratic function, when $x = 41$, $y \approx 18.9\%$.

For the exponential function, when $x = 41$, $y \approx 22.0\%$.

Given the trend shown in the table, the quadratic model seems the most realistic. Answers may vary.

32. a) Using the exponential regression feature on a graphing calculator, we get $y = 22.28866067(1.092917808)^x$, where x is the number of years after 1980 and y is in millions of dollars.

b) In 2008, $x = 2008 - 1980 = 28$.

$y = 22.28866067(1.092917808)^{28} \approx \$268.2$ million

In 2012, $x = 2012 - 1980 = 32$.

$y = 22.28866067(1.092917808)^{32} \approx \$382.7$ million

c) Graph $y_1 = 22.28866067(1.092917808)^x$ and $y_2 = 546$ and find the first coordinate of the point of intersection of the graphs. It is slightly less than 36, so the total cost will first exceed \$546 million for the first convention year that is 36 yr after 1980, or 2016.

33. a) $y = 255.0890581(1.277632801)^x$, where x is the number of years after 1995.

b) In 2005, $x = 2005 - 1995 = 10$.

$y = 255.0890581(1.277632801)^{10} \approx 2956$ surgeries

In 2009, $x = 2009 - 1995 = 14$.

$y = 255.0890581(1.277632801)^{14} \approx 7877$ surgeries

c) Graph $y_1 = 255.0890581(1.277632801)^x$ and $y_2 = 12,000$ and find the first coordinate of the point of intersection of the graphs. It is approximately 16, so there will be 12,000 surgeries about 16 yr after 1995, or in 2011.

34. a) Using the logistic regression feature on a graphing calculator, we get

$$y = \frac{99.98884912}{1 + 489.2438401e^{-0.1299899024x}}.$$

b) For $x = 55$, $y \approx 72.2\%$.

For $x = 100$, $y \approx 99.9\%$.

c) $y = 99.98884912$ is the horizontal asymptote; as more and more ads are run, the percent of people who buy the product approaches 100%.

35. Answers will vary.

36. Measure the atmospheric pressure P at the top of the building. Substitute that value in the equation $P = 14.7e^{-0.00005a}$, and solve for the height, or altitude, a, of the top of the building. Also measure the atmospheric pressure at the base of the building and solve for the altitude of the base. Then subtract to find the height of the building.

37. Multiplication principle for inequalities

38. Product rule

39. Principle of zero products

40. Principle of square roots

41. Power rule

42. Multiplication principle for equations

43.
$$\begin{aligned} 480e^{-0.003p} &= 150e^{0.004p} \\ \frac{480}{150} &= \frac{e^{0.004p}}{e^{-0.003p}} \\ 3.2 &= e^{0.007p} \\ \ln 3.2 &= \ln e^{0.007p} \\ \ln 3.2 &= 0.007p \\ \frac{\ln 3.2}{0.007} &= p \\ \$166.16 &\approx p \end{aligned}$$

44.
$$\begin{aligned} P(4000) &= P_0e^{-0.00012(4000)} \\ &= 0.619P_0, \text{ or } 61.9\%P_0 \end{aligned}$$

Thus, about 61.9% of the carbon-14 remains, so about 38.1% has been lost.

45.
$$\begin{aligned} P(t) &= P_0e^{kt} \\ 50,000 &= P_0e^{0.07(18)} \\ \frac{50,000}{e^{0.07(18)}} &= P_0 \\ \$14,182.70 &\approx P_0 \end{aligned}$$

46. a)
$$\begin{aligned} P &= P_0e^{kt} \\ \frac{P}{e^{kt}} &= P_0, \text{ or} \\ Pe^{-kt} &= P_0 \end{aligned}$$

b) $P_0 = 50,000e^{-0.064(18)} \approx \$15,800.21$

47.
$$\begin{aligned} i &= \frac{V}{R}\left[1 - e^{-(R/L)t}\right] \\ \frac{iR}{V} &= 1 - e^{-(R/L)t} \\ e^{-(R/L)t} &= 1 - \frac{iR}{V} \\ \ln e^{-(R/L)t} &= \ln\left(1 - \frac{iR}{V}\right) \\ -\frac{R}{L}t &= \ln\left(1 - \frac{iR}{V}\right) \\ t &= -\frac{L}{R}\left[\ln\left(1 - \frac{iR}{V}\right)\right] \end{aligned}$$

48. a) At 1 m: $I = I_0e^{-1.4(1)} \approx 0.247I_0$

24.7% of I_0 remains.

At 3 m: $I = I_0e^{-1.4(3)} \approx 0.015I_0$

1.5% of I_0 remains.

At 5 m: $I = I_0e^{-1.4(5)} \approx 0.0009I_0$

0.09% of I_0 remains.

At 50 m: $I = I_0e^{-1.4(50)} \approx (3.98 \times 10^{-31})I_0$

Now, $3.98 \times 10^{-31} = (3.98 \times 10^{-29}) \times 10^{-2}$, so $(3.98 \times 10^{-29})\%$ remains.

b) $I = I_0e^{-1.4(10)} \approx 0.0000008I_0$

Thus, 0.00008% remains.

49.
$$\begin{aligned} y &= ae^x \\ \ln y &= \ln(ae^x) \\ \ln y &= \ln a + \ln e^x \\ \ln y &= \ln a + x \\ Y &= x + \ln a \end{aligned}$$

This function is of the form $y = mx + b$, so it is linear.

50.
$$\begin{aligned} y &= ax^b \\ \ln y &= \ln(ax^b) \\ \ln y &= \ln a + b\ln x \\ Y &= \ln a + bX \end{aligned}$$

This function is of the form $y = mx + b$, so it is linear.

Chapter 5 Review Exercises

1. The statement is true. See page 383 in the text.

2. $f(0) = e^{-0} = 1$, so the y-intercept is $(0, 1)$. The statement is false.

3. The graph of f^{-1} is a reflection of the graph of f across $y = x$, so the statement is false.

4. The statement is true. See page 384 in the text.

5. The domain of all logarithmic functions is $(0, \infty)$, so the statement is false.

6. The statement is true. See pages 395 and 396 in the text.

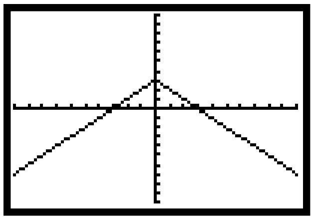
7. We interchange the first and second coordinates of each pair to find the inverse of the relation. It is $\{(-2.7, 1.3), (-3, 8), (3, -5), (-3, 6), (-5, 7)\}$.

8. a) $x = -2y + 3$

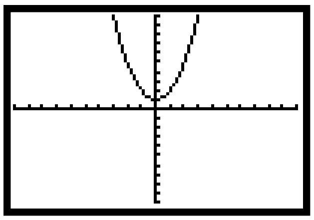
b) $x = 3y^2 + 2y - 1$

c) $0.8y^3 - 5.4x^2 = 3y$

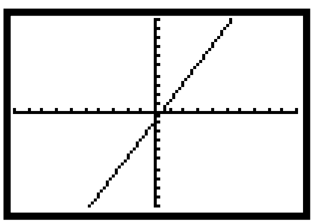
9. The graph of $f(x) = -|x| + 3$ is shown below. The function is not one-to-one, because there are many horizontal lines that cross the graph more than once.



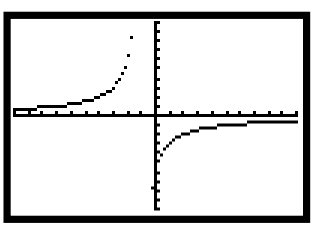
10. The graph of $f(x) = x^2 + 1$ is shown below. The function is not one-to-one, because there are many horizontal lines that cross the graph more than once.



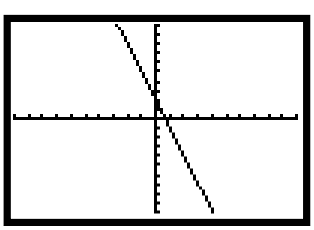
11. The graph of $f(x) = 2x - \frac{3}{4}$ is shown below. The function is one-to-one, because no horizontal line crosses the graph more than once.



12. The graph of $f(x) = -\frac{6}{x+1}$ is shown below. The function is one-to-one, because no horizontal line crosses the graph more than once.



13. a) The graph of $f(x) = 2-3x$ is shown below. It passes the horizontal-line test, so the function is one-to-one.



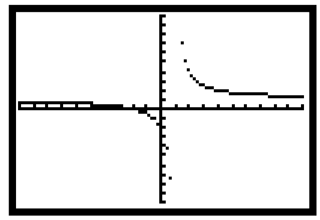
b) Replace $f(x)$ with y: $y = 2 - 3x$

Interchange x and y: $x = 2 - 3y$

Solve for y: $y = \frac{-x+2}{3}$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \frac{-x+2}{3}$

14. a) The graph of $f(x) = \frac{x+2}{x-1}$ is shown below. It passes the horizontal-line test, so the function is one-to-one.



b) Replace $f(x)$ with y: $y = \frac{x+2}{x-1}$

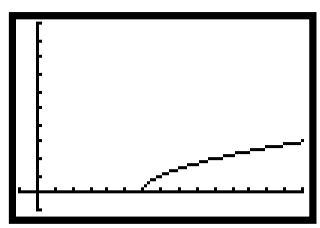
Interchange x and y: $x = \frac{y+2}{y-1}$

Solve for y: $(y-1)x = y + 2$

$$xy - x = y + 2$$
$$xy - y = x + 2$$
$$y(x-1) = x + 2$$
$$y = \frac{x+2}{x-1}$$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \frac{x+2}{x-1}$

15. a) The graph of $f(x) = \sqrt{x-6}$ is shown below. It passes the horizontal-line test, so the function is one-to-one.



b) Replace $f(x)$ with y: $y = \sqrt{x-6}$

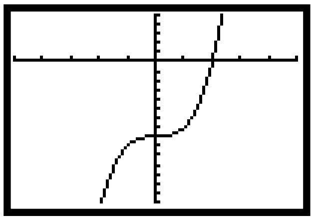
Interchange x and y: $x = \sqrt{y-6}$

Solve for y: $x^2 = y - 6$

$$x^2 + 6 = y$$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = x^2 + 6$, for all x in the range of $f(x)$, or $f^{-1}(x) = x^2 + 6$, $x \geq 0$.

16. a) The graph of $f(x) = x^3 - 8$ is shown below. It passes the horizontal-line test, so the function is one-to-one.



b) Replace $f(x)$ with y: $y = x^3 - 8$

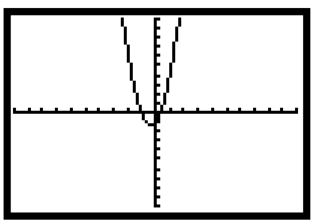
Interchange x and y: $x = y^3 - 8$

Solve for y: $x + 8 = y^3$

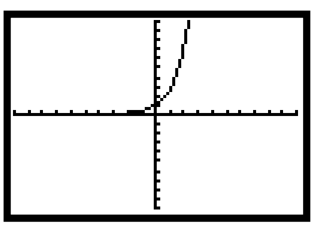
$\sqrt[3]{x+8} = y$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \sqrt[3]{x+8}$

17. a) The graph of $f(x) = 3x^2 + 2x - 1$ is shown below. It is not one-to-one since there are many horizontal lines that cross the graph more than once. The function does not have an inverse that is a function.



18. a) The graph of $f(x) = e^x$ is shown below. It passes the horizontal-line test, so the function is one-to-one.



b) Replace $f(x)$ with y: $y = e^x$

Interchange x and y: $x = e^y$

Solve for y: $y = \ln x$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \ln x$

19. We find $(f^{-1} \circ f)(x)$ and $(f \circ f^{-1})(x)$ and check to see that each is x.

$$(f^{-1} \circ f)(x) = f^{-1}(f(x)) = f^{-1}(6x-5) = \frac{(6x-5)+5}{6} = \frac{6x}{6} = x$$

$$(f \circ f^{-1})(x) = f(f^{-1}(x)) = f\left(\frac{x+5}{6}\right) = 6\left(\frac{x+5}{6}\right) - 5 = x + 5 - 5 = x$$

20. $$(f^{-1} \circ f)(x) = f^{-1}(f(x)) = f^{-1}\left(\frac{x+1}{x}\right) = \frac{1}{\left(\frac{x+1}{x}\right) - 1} = \frac{x}{x+1-x} = \frac{x}{1} = x$$

$$(f \circ f^{-1})(x) = f(f^{-1}(x)) = f\left(\frac{1}{x-1}\right) = \frac{\left(\frac{1}{x-1}\right) + 1}{\frac{1}{x-1}} = \frac{1+(x-1)}{1} = \frac{x}{1} = x$$

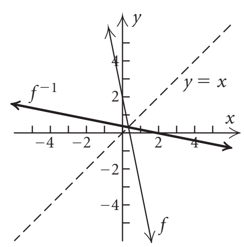
21. Replace $f(x)$ with y: $y = 2 - 5x$

Interchange x and y: $x = 2 - 5y$

Solve for y: $y = \frac{2-x}{5}$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \frac{2-x}{5}$

The domain and range of f are $(-\infty, \infty)$, so the domain and range of f^{-1} are also $(-\infty, \infty)$.



22. Replace $f(x)$ with y: $y = \frac{x-3}{x+2}$

Interchange x and y: $x = \frac{y-3}{y+2}$

Solve for y: $xy + 2x = y - 3$

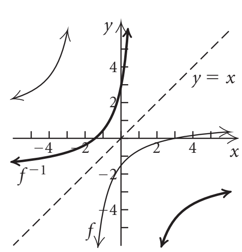
$2x + 3 = y - xy$

$2x + 3 = y(1-x)$

$\frac{2x+3}{1-x} = y$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \frac{2x+3}{1-x}$, or $\frac{-2x-3}{x-1}$

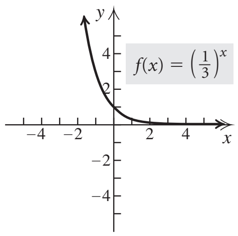
The domain of f is $(-\infty, -2) \cup (-2, \infty)$, and the range of f is $(-\infty, 1) \cup (1, \infty)$. Thus the domain of f^{-1} is $(-\infty, 1) \cup (1, \infty)$ and the range of f^{-1} is $(-\infty, -2) \cup (-2, \infty)$.



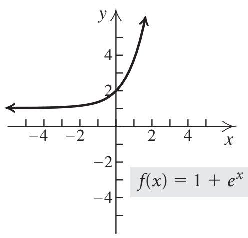
23. Since $f(f^{-1}(x)) = x$, then $f(f^{-1}(657)) = 657$.

24. Since $f(f^{-1}(x)) = x$, then $f(f^{-1}(a)) = a$.

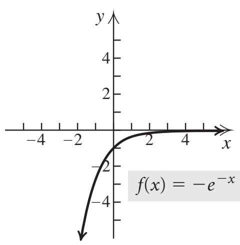
25.



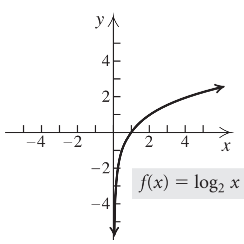
26.



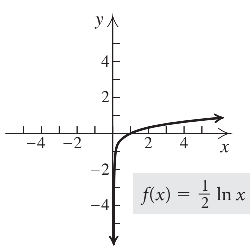
27.



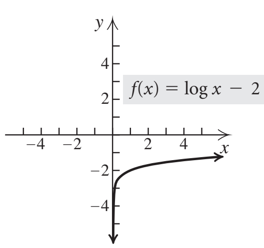
28.



29.



30.



31. $f(x) = e^{x-3}$

This is the graph of $f(x) = e^x$ shifted right 3 units. The correct choice is (c).

32. $f(x) = \log_3 x$

$f(1) = \log_3(1) = 0$

The only graph with x-intercept $(1, 0)$ is (a).

33. $f(x) = -\log_3(x + 1)$

This is the graph of $\log_3 x$ shifted left 1 unit and reflected across the y-axis. The correct choice is (b).

34. $y = \left(\frac{1}{2}\right)^x$

At $x = 0$, $y = \left(\frac{1}{2}\right)^0 = 1$

The only graph with y-intercept $(0, 1)$ is (f).

35. $f(x) = 3(1 - e^{-x})$, $x \geq 0$

This is the graph of $f(x) = e^x$ reflected across the y-axis, reflected across the x-axis, shifted up 1 unit, and stretched by a factor of 3. The correct choice is (e).

36. $f(x) = |\ln(x - 4)|$

This is the graph of $f(x) = \ln x$ shifted right 4 units. The absolute value reflects negative outputs across the x-axis. The line $x = 4$ is a vertical asymptote. The correct choice is (d).

37. $\log_5 125 = 3$ because the exponent to which we raise 5 to get 125 is 3.

38. $\log 100,000 = 5$ because the exponent to which we raise 10 to get 100,000 is 5.

39. $\ln e = 1$ because the exponent to which we raise e to get e is 1.

40. $\ln 1 = 0$ because the exponent to which we raise e to get 1 is 0.

41. $\log 10^{1/4} = \frac{1}{4}$ because the exponent to which we raise 10 to get $10^{1/4}$ is $\frac{1}{4}$.

42. $\log_3 \sqrt{3} = \log_3 3^{1/2} = \frac{1}{2}$ because the exponent to which we raise 3 to get $3^{1/2}$ is $\frac{1}{2}$.

43. $\log 1 = 0$ because the exponent to which we raise 10 to get 1 is 0.

44. $\log 10 = 1$ because the exponent to which we raise 10 to get 10 is 1.

45. $\log_2 \sqrt[3]{2} = \log_2 2^{1/3} = \frac{1}{3}$ because the exponent to which we raise 2 to get $2^{1/3}$ is $\frac{1}{3}$.

46. $\log 0.01 = -2$ because the exponent to which we raise 10 to get 0.01 is -2.

47. $\log_4 x = 2 \Rightarrow 4^2 = x$

48. $\log_a Q = k \Rightarrow a^k = Q$

49. $4^{-3} = \frac{1}{64} \Rightarrow \log_4 \frac{1}{64} = -3$

50. $e^x = 80 \Rightarrow \ln 80 = x$, or $\log_e 80 = x$

51. $\log 11 \approx 1.0414$

52. $\log 0.234 \approx -0.6308$

53. $\ln 3 \approx 1.0986$

54. $\ln 0.027 \approx -3.6119$

55. $\log(-3)$ does not exist. (The calculator gives an error message.)

56. $\ln 0$ does not exist. (The calculator gives an error message.)

57. $\log_5 24 = \dfrac{\log 24}{\log 5} \approx 1.9746$

58. $\log_8 3 = \dfrac{\log 3}{\log 8} \approx 0.5283$

59.
$$\begin{aligned}&3\log_b x - 4\log_b y + \frac{1}{2}\log_b z\\ &= \log_b x^3 - \log_b y^4 + \log_b z^{1/2}\\ &= \log_b \frac{x^3 z^{1/2}}{y^4}, \text{ or } \log_b \frac{x^3\sqrt{z}}{y^4}\end{aligned}$$

60.
$$\begin{aligned}&\ln(x^3-8) - \ln(x^2+2x+4) + \ln(x+2)\\ &= \ln\frac{(x^3-8)(x+2)}{x^2+2x+4}\\ &= \ln\frac{(x-2)(x^2+2x+4)(x+2)}{x^2+2x+4}\\ &= \ln(x-2)(x+2)\\ &= \ln(x^2-4)\end{aligned}$$

61.
$$\begin{aligned}\ln\sqrt[4]{wr^2} &= \ln(wr^2)^{1/4}\\ &= \frac{1}{4}\ln wr^2\\ &= \frac{1}{4}(\ln w + \ln r^2)\\ &= \frac{1}{4}(\ln w + 2\ln r)\\ &= \frac{1}{4}\ln w + \frac{1}{2}\ln r\end{aligned}$$

62.
$$\begin{aligned}\log\sqrt[3]{\frac{M^2}{N}} &= \log\left(\frac{M^2}{N}\right)^{1/3}\\ &= \frac{1}{3}\log\frac{M^2}{N}\\ &= \frac{1}{3}(\log M^2 - \log N)\\ &= \frac{1}{3}(2\log M - \log N)\\ &= \frac{2}{3}\log M - \frac{1}{3}\log N\end{aligned}$$

63.
$$\begin{aligned}\log_a 3 &= \log_a\left(\frac{6}{2}\right)\\ &= \log_a 6 - \log_a 2\\ &\approx 0.778 - 0.301\\ &\approx 0.477\end{aligned}$$

64.
$$\begin{aligned}\log_a 50 &= \log_a(2\cdot 5^2)\\ &= \log_a 2 + \log_a 5^2\\ &= \log_a 2 + 2\log_a 5\\ &\approx 0.301 + 2(0.699)\\ &\approx 1.699\end{aligned}$$

65.
$$\begin{aligned}\log_a \frac{1}{5} &= \log_a 5^{-1}\\ &= -\log_a 5\\ &\approx -0.699\end{aligned}$$

66.
$$\begin{aligned}\log_a \sqrt[3]{5} &= \log_a 5^{1/3}\\ &= \frac{1}{3}\log_a 5\\ &\approx \frac{1}{3}(0.699)\\ &\approx 0.233\end{aligned}$$

67. $\ln e^{-5k} = -5k \quad (\log_a a^x = x)$

68. $\log_5 5^{-6t} = -6t$

69.
$$\begin{aligned}\log_4 x &= 2\\ x &= 4^2 = 16\end{aligned}$$
The solution is 16.

70.
$$\begin{aligned}3^{1-x} &= 9^{2x}\\ 3^{1-x} &= (3^2)^{2x}\\ 3^{1-x} &= 3^{4x}\\ 1-x &= 4x\\ 1 &= 5x\\ \frac{1}{5} &= x\end{aligned}$$
The solution is $\dfrac{1}{5}$.

71.
$$\begin{aligned}e^x &= 80\\ \ln e^x &= \ln 80\\ x &= \ln 80\\ x &\approx 4.382\end{aligned}$$
The solution is 4.382.

72.
$$\begin{aligned}4^{2x-1} - 3 &= 61\\ 4^{2x-1} &= 64\\ 4^{2x-1} &= 4^3\\ 2x-1 &= 3\\ 2x &= 4\\ x &= 2\end{aligned}$$
The solution is 2.

73. $$\begin{aligned} \log_{16} 4 &= x \\ 16^x &= 4 \\ (4^2)^x &= 4^1 \\ 4^{2x} &= 4^1 \\ 2x &= 1 \\ x &= \frac{1}{2} \end{aligned}$$

The solution is $\frac{1}{2}$.

74. $$\begin{aligned} \log_x 125 &= 3 \\ x^3 &= 125 \\ x &= \sqrt[3]{125} \\ x &= 5 \end{aligned}$$

The solution is 5.

75. $$\begin{aligned} \log_2 x + \log_2 (x-2) &= 3 \\ \log_2 x(x-2) &= 3 \\ x(x-2) &= 2^3 \\ x^2 - 2x &= 8 \\ x^2 - 2x - 8 &= 0 \\ (x+2)(x-4) &= 0 \end{aligned}$$

$x + 2 = 0$ *or* $x - 4 = 0$

$x = -2$ *or* $x = 4$

The number 4 checks, but -2 does not. The solution is 4.

76. $$\begin{aligned} \log(x^2 - 1) - \log(x-1) &= 1 \\ \log \frac{x^2 - 1}{x - 1} &= 1 \\ \frac{(x+1)(x-1)}{x-1} &= 10^1 \\ x + 1 &= 10 \\ x &= 9 \end{aligned}$$

The answer checks. The solution is 9.

77. $$\begin{aligned} \log x^2 &= \log x \\ x^2 &= x \\ x^2 - x &= 0 \\ x(x-1) &= 0 \end{aligned}$$

$x = 0$ *or* $x - 1 = 0$

$x = 0$ *or* $x = 1$

The number 1 checks, but 0 does not. The solution is 1.

78. $$\begin{aligned} e^{-x} &= 0.02 \\ \ln e^{-x} &= \ln 0.02 \\ -x &= \ln 0.02 \\ x &= -\ln 0.02 \\ x &\approx 3.912 \end{aligned}$$

The answer checks. The solution is 3.912.

79. a) $A(t) = 16,000(1.0105)^{4t}$

b) $$\begin{aligned} A(0) &= 16,000(1.0105)^{4\cdot 0} = \$16,000 \\ A(6) &= 16,000(1.0105)^{4\cdot 6} \approx \$20,558.51 \\ A(12) &= 16,000(1.0105)^{4\cdot 12} \approx \$26,415.77 \\ A(18) &= 16,000(1.0105)^{4\cdot 18} \approx \$33,941.80 \end{aligned}$$

80. $$\begin{aligned} W(10) &= 1665.945(1.087)^{10} \approx 3837 \text{ cases} \\ W(20) &= 1665.945(1.087)^{20} \approx 8836 \text{ cases} \\ W(28) &= 1665.945(1.087)^{28} \approx 17,222 \text{ cases} \end{aligned}$$

81. $T = \dfrac{\ln 2}{0.086} \approx 8.1$ years

82. $k = \dfrac{\ln 2}{30} \approx 0.023$, or 2.3%

83. $$\begin{aligned} P(t) &= P_0 e^{kt} \\ 0.73P_0 &= P_0 e^{-0.00012t} \\ 0.73 &= e^{-0.00012t} \\ \ln 0.73 &= \ln e^{-0.00012t} \\ \ln 0.73 &= -0.00012t \\ \frac{\ln 0.73}{-0.00012} &= t \\ 2623 &\approx t \end{aligned}$$

The skeleton is about 2623 years old.

84. $\text{pH} = -\log[2.3 \times 10^{-6}] \approx -(-5.6) = 5.6$

85. $R = \log \dfrac{10^{6.3} \cdot I_0}{I_0} = \log 10^{6.3} = 6.3$

86. $$\begin{aligned} L &= 10 \log \frac{1000 \cdot I_0}{I_0} \\ &= 10 \log 1000 \\ &= 10 \cdot 3 \\ &= 30 \text{ decibels} \end{aligned}$$

87. a) We substitute 353.823 for P, since P is in thousands.

$$\begin{aligned} W(353.823) &= 0.37 \ln 353.823 + 0.05 \\ &\approx 2.2 \text{ ft/sec} \end{aligned}$$

b) We substitute 3.4 for W and solve for P.

$$\begin{aligned} 3.4 &= 3.7 \ln P + 0.05 \\ 3.35 &= 0.37 \ln P \\ \frac{3.35}{0.37} &= \ln P \\ e^{3.35/0.37} &= P \\ P &\approx 8553.143 \end{aligned}$$

The population is about 8553.143 thousand, or 8,553,143. (Answers may vary due to rounding differences.)

88. a)
$$492 = 0.035e^{k\cdot 64}$$
$$\frac{492}{0.035} = e^{64k}$$
$$\ln\frac{492}{0.035} = \ln e^{64k}$$
$$\ln\frac{492}{0.035} = 64k$$
$$\frac{\ln\frac{492}{0.035}}{64} = k$$
$$0.1492 \approx k$$

b) We have $S(t) = 0.035e^{0.1492t}$, where $S(t)$ is in billions of dollars and t is the number of years after 1940.

c) $S(25) \approx \$1.459$ billion

$S(55) \approx \$128.2$ billion

$S(75) \approx \$2534$ billion, or $\$2.534$ trillion

d) 1 trillion is 1000 billion.
$$1000 = 0.035e^{0.1492t}$$
$$\frac{1000}{0.035} = e^{0.1492t}$$
$$\ln\frac{1000}{0.035} = \ln e^{0.1492t}$$
$$\ln\frac{1000}{0.035} = 0.1492t$$
$$\frac{\ln\frac{1000}{0.035}}{0.1492} = t$$
$$69 \approx t$$

Cash benefits will reach $1 trillion about 69 yr after 1940, or in 2009.

89. a) $P(t) = 3.039e^{0.013t}$, where $P(t)$ is in millions and t is the number of years after 2005.

b) In 2009, $t = 2009 - 2005 = 4$.

$P(4) = 3.039e^{0.013(4)} \approx 3.201$ million

In 2015, $t = 2015 - 2005 = 10$.

$P(10) = 3.039e^{0.013(10)} \approx 3.461$ million

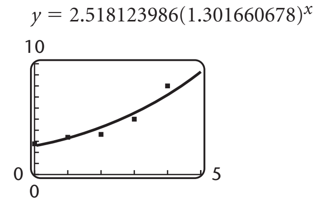
c)
$$10 = 3.039e^{0.013t}$$
$$\frac{10}{3.039} = e^{0.013t}$$
$$\ln\left(\frac{10}{3.039}\right) = \ln e^{0.013t}$$
$$\ln\left(\frac{10}{3.039}\right) = 0.013t$$
$$\frac{\ln\left(\frac{10}{3.039}\right)}{0.013} = t$$
$$92 \approx t$$

The population will be 10 million about 92 yr after 2005.

d) $T = \frac{\ln 2}{0.013} \approx 53.3$ yr

90. a) $y = 2.518123986(1.301660678)^x$, where y is in millions of dollars and x is the number of years after 2001.

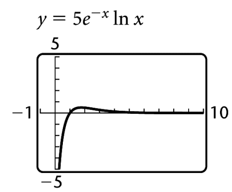
b)



c) For $x = 9$, $y \approx \$27$ million

91. We graph $y_1 = \frac{4+3x}{x-2}$, $y_2 = \frac{x+4}{x-3}$, and $y_3 = x$ and observe that the graphs of y_1 and y_2 are not reflections of each other across the third line, $y = x$. Thus $f(x)$ and $g(x)$ are not inverses.

92. a)



b) Relative maximum: 0.486 at $x = 1.763$; no relative minimum

93. The graph of $f(x) = e^{x-3} + 2$ is a translation of the graph of $y = e^x$ right three units and up 2 units. The horizontal asymptote of $y = e^x$ is $y = 0$, so the horizontal asymptote of $f(x) = e^{x-3} + 2$ is translated up 2 units from $y = 0$. Thus, it is $y = 2$, and answer D is correct.

94. We must have $2x - 3 > 0$, or $x > \frac{3}{2}$, so answer A is correct.

95. The graph of $f(x) = 2^{x-2}$ is the graph of $g(x) = 2^x$ shifted 2 units to the right. Thus D is the correct graph.

96. The graph of $f(x) = \log_2 x$ is the graph of $g(x) = 2^x$ reflected across the line $y = x$. Thus B is the correct graph.

97. By the product rule, $\log_2 x + \log_2 5 = \log_2 5x$, not $\log_2(x+5)$. Also, substituting various numbers for x shows that the two sides of the inequality are indeed unequal. You could also graph each side and show that the graphs do not coincide.

98. The inverse of a function $f(x)$ is written $f^{-1}(x)$, whereas $[f(x)]^{-1}$ means $\frac{1}{f(x)}$.

99. $|\log_4 x| = 3$
$$\log_4 x = -3 \quad or \quad \log_4 x = 3$$
$$x = 4^{-3} \quad or \quad x = 4^3$$
$$x = \frac{1}{64} \quad or \quad x = 64$$

Both answers check. The solutions are $\frac{1}{64}$ and 64.

100. $\log x = \ln x$

Graph $y_1 = \log x$ and $y_2 = \ln x$ and find the first coordinates of the points of intersection of the graph. We see that the only solution is 1.

101. $5^{\sqrt{x}} = 625$

$5^{\sqrt{x}} = 5^4$

$\sqrt{x} = 4$

$x = 16$

102. $f(x) = \log_3(\ln x)$

$\ln x$ must be positive, so $x > 1$. The domain is $(1, \infty)$.

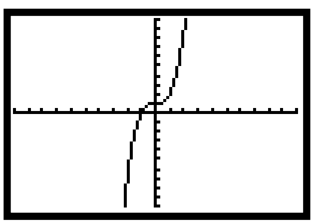
Chapter 5 Test

1. We interchange the first and second coordinates of each pair to find the inverse of the relation. It is $\{(5,-2),(3,4)(-1,0),(-3,-6)\}$.

2. The function is not one-to-one, because there are many horizontal lines that cross the graph more than once.

3. The function is one-to-one, because no horizontal line crosses the graph more than once.

4. a) The graph of $f(x) = x^3+1$ is shown below. It passes the horizontal-line test, so the function is one-to-one.



b) Replace $f(x)$ with y: $y = x^3 + 1$

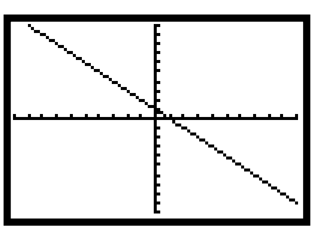
Interchange x and y: $x = y^3 + 1$

Solve for y: $y^3 = x - 1$

$y = \sqrt[3]{x-1}$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \sqrt[3]{x-1}$

5. a) The graph of $f(x) = 1-x$ is shown below. It passes the horizontal-line test, so the function is one-to-one.



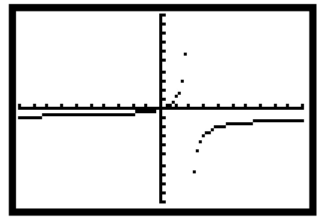
b) Replace $f(x)$ with y: $y = 1 - x$

Interchange x and y: $x = 1 - y$

Solve for y: $y = 1 - x$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = 1 - x$

6. a) The graph of $f(x) = \dfrac{x}{2-x}$ is shown below. It passes the horizontal-line test, so the function is one-to-one.



b) Replace $f(x)$ with y: $y = \dfrac{x}{2-x}$

Interchange x and y: $x = \dfrac{y}{2-y}$

Solve for y: $(2-y)x = y$

$2x - xy = y$

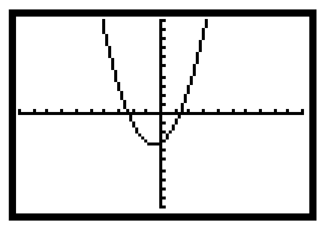
$xy + y = 2x$

$y(x+1) = 2x$

$y = \dfrac{2x}{x+1}$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \dfrac{2x}{x+1}$

7. a) The graph of $f(x) = x^2+x-3$ is shown below. It is not one-to-one since there are many horizontal lines that cross the graph more than once. The function does not have an inverse that is a function.



8. We find $(f^{-1} \circ f)(x)$ and $(f \circ f^{-1})(x)$ and check to see that each is x.

$(f^{-1} \circ f)(x) = f^{-1}(f(x)) = f^{-1}(-4x+3) =$

$\dfrac{3-(-4x+3)}{4} = \dfrac{3+4x-3}{4} = \dfrac{4x}{4} = x$

$(f \circ f^{-1})(x) = f(f^{-1}(x)) = f\left(\dfrac{3-x}{4}\right) =$

$-4\left(\dfrac{3-x}{4}\right) + 3 = -3 + x + 3 = x$

9. Replace $f(x)$ with y: $y = \dfrac{1}{x-4}$

Interchange x and y: $x = \dfrac{1}{y-4}$

Solve for y: $x(y-4) = 1$

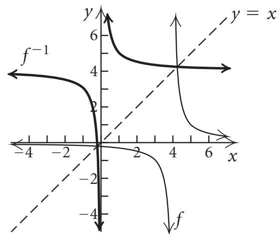
$xy - 4x = 1$

$xy = 4x + 1$

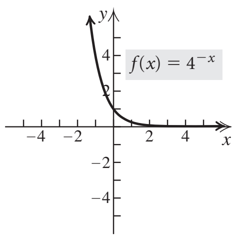
$y = \dfrac{4x+1}{x}$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \dfrac{4x+1}{x}$

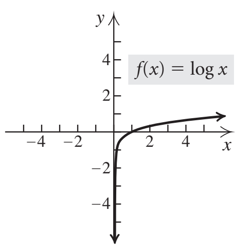
The domain of $f(x)$ is $(-\infty, 4) \cup (4, \infty)$ and the range of $f(x)$ is $(-\infty, 0) \cup (0, \infty)$. Thus, the domain of f^{-1} is $(-\infty, 0) \cup (0, \infty)$ and the range of f^{-1} is $(-\infty, 4) \cup (4, \infty)$.



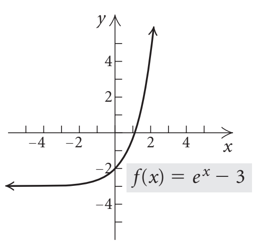
10.



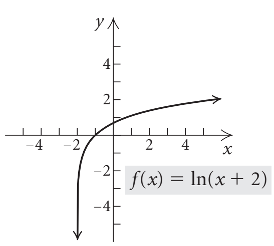
11.



12.



13.



14. $\log 0.00001 = -5$ because the exponent to which we raise 10 to get 0.00001 is -5.

15. $\ln e = 1$ because the exponent to which we raise e to get e is 1.

16. $\ln 1 = 0$ because the exponent to which we raise e to get 1 is 0.

17. $\log_4 \sqrt[5]{4} = \log_4 4^{1/5} = \dfrac{1}{5}$ because the exponent to which we raise 4 to get $4^{1/5}$ is $\dfrac{1}{5}$.

18. $\ln x = 4 \Rightarrow x = e^4$

19. $3^x = 5.4 \Rightarrow x = \log_3 5.4$

20. $\ln 16 \approx 2.7726$

21. $\log 0.293 \approx -0.5331$

22. $\log_6 10 = \dfrac{\log 10}{\log 6} \approx 1.2851$

23.
$$\begin{aligned} &2\log_a x - \log_a y + \frac{1}{2}\log_a z \\ &= \log_a x^2 - \log_a y + \log_a z^{1/2} \\ &= \log_a \frac{x^2 z^{1/2}}{y}, \text{ or } \log_a \frac{x^2\sqrt{z}}{y} \end{aligned}$$

24.
$$\begin{aligned} \ln \sqrt[5]{x^2 y} &= \ln(x^2 y)^{1/5} \\ &= \frac{1}{5}\ln x^2 y \\ &= \frac{1}{5}(\ln x^2 + \ln y) \\ &= \frac{1}{5}(2\ln x + \ln y) \\ &= \frac{2}{5}\ln x + \frac{1}{5}\ln y \end{aligned}$$

25.
$$\begin{aligned} \log_a 4 &= \log_a \left(\frac{8}{2}\right) \\ &= \log_a 8 - \log_a 2 \\ &\approx 0.984 - 0.328 \\ &\approx 0.656 \end{aligned}$$

26. $\ln e^{-4t} = -4t \quad (\log_a a^x = x)$

27.
$$\begin{aligned} \log_{25} 5 &= x \\ 25^x &= 5 \\ (5^2)^x &= 5^1 \\ 5^{2x} &= 5^1 \\ 2x &= 1 \\ x &= \frac{1}{2} \end{aligned}$$

The solution is $\dfrac{1}{2}$.

28.
$$\begin{aligned} \log_3 x + \log_3(x+8) &= 2 \\ \log_3 x(x+8) &= 2 \\ x(x+8) &= 3^2 \\ x^2 + 8x &= 9 \\ x^2 + 8x - 9 &= 0 \\ (x+9)(x-1) &= 0 \end{aligned}$$

$x = -9$ *or* $x = 1$

The number 1 checks, but -9 does not. The solution is 1.

29.
$$\begin{aligned}3^{4-x} &= 27^x\\ 3^{4-x} &= (3^3)^x\\ 3^{4-x} &= 3^{3x}\\ 4-x &= 3x\\ 4 &= 4x\\ x &= 1\end{aligned}$$

The solution is 1.

30.
$$\begin{aligned}e^x &= 65\\ \ln e^x &= \ln 65\\ x &= \ln 65\\ x &\approx 4.174\end{aligned}$$

The solution is 4.174.

31. $R = \log \dfrac{10^{6.6} \cdot I_0}{I_0} = \log\ 10^{6.6} = 6.6$

32. $k = \dfrac{\ln 2}{45} \approx 0.0154 \approx 1.54\%$

33. a)
$$\begin{aligned}1144.54 &= 1000e^{3k}\\ 1.14454 &= e^{3k}\\ \ln 1.14454 &= \ln e^{3k}\\ \ln 1.14454 &= 3k\\ \frac{\ln 1.14454}{3} &= k\\ 0.045 &\approx k\end{aligned}$$

The interest rate is about 4.5%.

b) $P(t) = 1000e^{0.045t}$

c) $P(8) = 1000e^{0.045 \cdot 8} \approx \$1433.33$

d) $T = \dfrac{\ln 2}{0.045} \approx 15.4$ yr

34. The graph of $f(x) = 2^{x-1} + 1$ is the graph of $g(x) = 2^x$ shifted right 1 unit and up 1 unit. Thus C is the correct graph.

35.
$$\begin{aligned}4^{\sqrt[3]{x}} &= 8\\ (2^2)^{\sqrt[3]{x}} &= 2^3\\ 2^{2\sqrt[3]{x}} &= 2^3\\ 2\sqrt[3]{x} &= 3\\ \sqrt[3]{x} &= \frac{3}{2}\\ x &= \left(\frac{3}{2}\right)^3\\ x &= \frac{27}{8}\end{aligned}$$

The solution is $\dfrac{27}{8}$.