CHAPTER 7

7.1 CONCEPT QUESTIONS, page 359

1. An experiment is an activity with observable results. Examples vary.

2. Two events E and F are mutually exclusive if they cannot occur at the same time. Examples vary. E and F are mutually exclusive if $E \cap F = \varnothing$.

EXERCISES 7.1, page 359

1. $E \cup F = \{a, b, d, f\}$; $E \cap F = \{a\}$.

2. $F \cup G = \{a, b, c, d, e, f\}$; $F \cap G = \varnothing$.

3. $F^C = \{b, c, e\}$; $E \cap G^C = \{a, b\} \cap \{a, d, f\} = \{a\}$.

4. $E^C = \{c, d, e, f\}$; $F^C \cap G = \{b, c, e\} \cap \{b, c, e\} = \{b, c, e\}$.

5. Since $E \cap F = \{a\}$ is not a null set, we conclude that E and F are not mutually exclusive.

6. $E \cup F = \{a, b, d, f\}$ and $E \cap F^C = \{a, b\} \cap \{b, c, e\} = \{b\}$. Since $\{b\}$ is an element of both sets, they are not mutually exclusive.

7. $E \cup F \cup G = \{2, 4, 6\} \cup \{1, 3, 5\} \cup \{5, 6\} = \{1, 2, 3, 4, 5, 6\}$.

8. $E \cap F \cap G = \{2, 4, 6\} \cap \{1, 3, 5\} \cap \{5, 6\} = \varnothing$.

9. $(E \cup F \cup G)^C = \{1, 2, 3, 4, 5, 6\}^C = \varnothing$.

10. $(E \cap F \cap G)^C = \{1, 2, 3, 4, 5, 6\}$.

11. Yes, $E \cap F = \varnothing$; that is, E and F do not contain any common elements.

12. No. 5 is an element of both sets.

13. $E^c = \{2, 4, 6\}^c = \{1, 3, 5\} = F$ and so E and F are complementary.

14. $F^c = \{1, 3, 5\}^c = \{2, 4, 6\} \neq G$ and so F and G are not complementary.

15. $E \cup F$

16. $E \cap F$

17. G^C

18. $(E \cap F^C)$

19. $(E \cup F \cup G)^C$

20. $(E \cap F^C \cap G^C)$

21. a. Refer to Example 4, page 356.
$E = \{(2, 1), (3, 1), (4, 1), (5, 1), (6, 1), (3, 2), (4, 2), (5, 2), (6, 2),$
$(4, 3), (5, 3), (6, 3), (5, 4), (6, 4), (6, 5)\}$
b. $E = \{(1, 2), (2, 4), (3, 6)\}$

22. a. Refer to Example 4, page 356.
$E = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5),$
$(3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (6, 1)\}$
b. $E = \{(4, 5), (4, 6), (5, 4), (6, 4)\}$

23. $\varnothing$, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$, $\{b, c\}$, $\{a, b, c\}$.

24. a. $\varnothing$, $\{1\}$, $\{2\}$, $\{3\}$, $\{1, 2\}$, $\{1, 3\}$, $\{2, 3\}$, $\{1, 2, 3\}$. b. 4 c. 4

25. a. $S = \{R, B\}$ b. $\varnothing$, $\{B\}$, $\{R\}$, $\{B, R\}$

26. a. $\{A, C, E, H, M, S, T, U\}$ b. $\{A, E, U\}$

27. a. $S = \{(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2),$
$(T, 3), (T, 4), (T, 5), (T, 6)\}$
b. $E = \{(H, 2), (H, 4), (H, 6)\}$

28. a. $S = \{1, 2, 3, 4, 5\}$ b. $E = \{2\}$ c. $F = \{1, 3, 5\}$

29. a. Here $S = \{1, 2, 3, 4, 5, 6\}$, $E = \{2\}$, and $F = \{2, 4, 6\}$. Since $E \cap F = \{2\} \neq \varnothing$, we conclude that E and F are not mutually exclusive.
b. $E^c = \{1, 3, 4, 5, 6\} \neq F$ and so E and F are not complementary.

30. Here $S = \{1, 2, 3, 4, 5, 6\}$, $E = \{2, 4, 6\}$, and $F = \{1, 3, 5\}$. Since $E \cap F = \emptyset$, we conclude that E and F are mutually exclusive.
b. $E^c = \{1, 3, 5\} = F$ and so E and F are complementary events.

31. $S = \{ddd, ddn, dnd, ndd, dnn, ndn, nnd, nnn\}$

32. $S = \{A^+, A^-, B^+, B^-, AB^+, AB^-, O^+, O^-\}$

33. a. $\{ABC, ABD, ABE, ACD, ACE, ADE, BCD, BCE, BDE, CDE\}$;
b. 6 c. 3 d. 6

34. a. $S = \{t \mid t > 0\}$ b. $E = \{t \mid 2 \le t \le 3\}$

35. a. E^C b. $E^C \cap F^C$ c. $E \cup F$ d. $(E \cap F^C) \cup (E^C \cap F)$

36. a. $S = \{t \mid t > 0\}$ b. $E = \{t \mid 0 < t < 90\}$ c. $F = \{t \mid t > 365\}$

37. a. $S = \{t \mid t > 0\}$ b. $E = \{t \mid 0 < t \le 2\}$ c. $F = \{t \mid t > 2\}$

38. a. $S = \{(L, f), (L, o), (L, u), (M, f), (M, o), (M, u), (U, f), (U, o), (U, u)\}$
b. $E_1 = \{(L, f), (M, f), (U, f)\}$
c. $E_2 = \{(M, o), (U, o)\}$
d. $E_3 = \{(L, o),(L, u),(M, o),(M, u)\}$

39. a. $S = \{0, 1, 2, 3, ... ,10\}$ b. $E = \{0, 1, 2, 3\}$ c. $F = \{5, 6, 7, 8, 9, 10\}$

40. a. $S = \{(L, R), (L, D), (L, I), (M, R), (M, D), (M, I), (U, R), (U, D),(U, I)\}$
b. $E_1 = \{(L, D), (M, D), (U, D)\}$
c. $E_2 = \{(U, R)\}$ d. $E_3 = \{(M, R), (M, I)\}$

41. a. $S = \{0, 1,2, ..., 20\}$ b. $E = \{0, 1, 2, ..., 9\}$ c. $F = \{20\}$

42. $\{(A, E), (A, F), (A, G), (A, H), (B, E), (B, F), (B, G), (B, H),$
$(C, E), (C, F), (C, G), (C, H), (D, E), (D, F), (D, G), (D, H)\}$

43. Let S denote the sample space of the experiment that is the set of 52 cards. Then $E=\{x\in S \mid x \text{ is an ace}\}$ and $F=\{x\in S \mid x \text{ is a spade}\}$ and $E\cap F=\{x\in S \mid x \text{ is the ace of spades}\}$. Now $n(E) = 4$, $n(F) = 13$, and $n(E\cap F)=1$. Also, $E\cup F=\{x\in S \mid x \text{ is an ace or a spade}\}$ and $n(E\cup F)=16$, and

$$n(E)+n(F)-n(E\cap F)=4+13-1=16=n(E\cup F).$$

44. If E is an event of an experiment then E^c is the event containing the elements in S that are not in E. Therefore $E \cap E^C = \varnothing$ and the two sets are mutually exclusive.

45. $E^C \cap F^C = (E \cup F)^C$ by DeMorgan's Law. Since $(E \cup F) \cap (E \cup F)^C = \varnothing$, they are mutually exclusive.

46. The number of events of this experiment is 2^n.

47. False. Let $E=\{1, 2, 3\}$, $F=\{4, 5, 6\}$, and $G=\{4, 5\}$. Then $E\cap F=\varnothing$ and $E\cap G=\varnothing$, but $F \cap G=\{4,5\}\neq\varnothing$.

48. True. The sample space is $S=\{(1, 2), (1, 3), (2, 1), (2, 3), (3, 1), (3, 2)\}$.