6.4 CONCEPT QUESTIONS, page 344

1. a. Given a set of distinct objects, a permutation of the set is an arrangement of these objects in a *definite order*.

   b. $P(n,r)=\dfrac{n!}{(n-r)!}$ so $P(5,3)=\dfrac{5!}{(5-3)!}=\dfrac{5\cdot 4}{2\cdot 1}=10$

2. $\dfrac{10!}{3!3!4!}=4200$

3. a. $C(n,r)=\dfrac{n!}{r!(n-r)!}$

   b. $C(6,3)=\dfrac{6!}{3!(6-3)!}=\dfrac{6\cdot 5\cdot 4}{3\cdot 2}=20$

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1. 3(5!) = 3(5)(4)(3)(2)(1) = 360.

2. 2(7!) = 2(7)(6)(5)(4)(3)(2)(1) = 10,080.

3. $\dfrac{5!}{2!3!}=5(2)=10.$

4. $\dfrac{6!}{4!2!}=3(5)=15.$

5. $P(5,5)=\dfrac{5!}{(5\text{-}5)!}=\dfrac{5!}{0!}=120$

6. $P(6,6)=\dfrac{6!}{0!}=720$

7. $P(5,2)=\dfrac{5!}{(5-2)!}=\dfrac{5!}{3!}=(5)(4)=20$

8. $P(5,3)=\dfrac{5!}{(5-3)!}=\dfrac{5!}{2!}=(5)(4)(3)=60$

9. $P(n,1)=\dfrac{n!}{(n-1)!}=n$

10. $P(k,2)=\dfrac{k!}{(k-2)!}=k(k-1)$

11. $C(6,6)=\dfrac{6!}{6!0!}=1$

12. $C(8,8)=\dfrac{8!}{8!0!}=1$

13. $C(7,4)=\dfrac{7!}{4!3!}=\dfrac{7\cdot 6\cdot 5}{3\cdot 2}=35$

14. $C(9,3)=\dfrac{9!}{6!3!}=\dfrac{9\cdot 8\cdot 7}{3\cdot 2}=84$

15. $C(5,0)=\frac{5!}{5!0!}=1$

16. $C(6,5)=\frac{6!}{5!1!}=6$

17. $C(9,6)=\frac{9!}{3!6!}=\frac{9\cdot 8\cdot 7}{3\cdot 2}=84$

18. $C(10,3)=\frac{10!}{3!7!}=\frac{10\cdot 9\cdot 8}{3\cdot 2}=120$

19. $C(n,2)=\frac{n!}{(n-2)!2!}=\frac{n(n-1)}{2}$

20. $C(7,r)=\frac{7!}{r!(7-r)!}$

21. $P(n,n-2)=\frac{n!}{(n-(n-2))!}=\frac{n!}{(n-n+2)!}=\frac{n!}{2}$

22. $C(n,n-2)=\frac{n!}{[n-(n-2)]!(n-2)!}=\frac{n!}{2!(n-2)!}=\frac{n(n-1)}{2}$

23. Order is important here since the word "*glacier*" is different from "*reicalg*", so this is a permutation.

24. Order is not important here since the order in which the members of the board of directors are chosen is not important.

25. Order is not important here. Therefore, we are dealing with a combination. If we consider a sample of three cellphones of which one is defective, it does not matter whether the defective cellphone is the first member of our sample, the second member of our sample, or the third member of our sample. The net result is a sample of three cellphones of which one is defective.

26. Order is important here since 327 is different from 732, so we are dealing with a permutation.

27. The order is important here. Therefore, we are dealing with a permutation. Consider, for example, 9 books on a library shelf. Each of the 9 books would have a call number, and the books would be placed in order of their call numbers; that is, a call number of 902 would come before a call number of 910.

28. The order is not important here as the selection *AB* would be considered the same as the selection *BA*. Therefore, we are dealing with a combination.

29. The order is not important here, and consequently we are dealing with a combination. It would not matter if the hand *Q Q Q 5 5* were dealt or the hand *5 5 Q Q Q*. In each case the hand would consist of three queens and a pair.

30. This is a permutation since the order in which the letters are chosen is important.

31. The number of 4-letter permutations is $P(4,4)=\frac{4!}{0!}=4\cdot 3\cdot 2\cdot 1=24$.

32. The number of 3-letter permutations is $P(5,3)=\frac{5!}{2!}=5\cdot 4\cdot 3=60$.

33. The number of seating arrangements is $P(4,4)=\frac{4!}{0!}=24$.

34. The number of different ways they can line up is $P(5,5)=\frac{5!}{0!}=120$.

35. The number of different batting orders is $P(9,9)=\frac{9!}{0!}=362{,}880$.

36. The number of different voting lists is $P(6,6)=\frac{6!}{0!}=720$.

37. The number of different ways the 3 candidates can be selected is

$$C(12,3)=\frac{12!}{9!3!}=\frac{12\cdot 11\cdot 10}{3\cdot 2\cdot 1}=220.$$

38. The number of different ways the investor can select the four mutual funds is

$$C(8,4)=\frac{8!}{4!4!}=\frac{8\cdot 7\cdot 6\cdot 5}{4\cdot 3\cdot 2\cdot 1}=70.$$

39. There are 10 letters in the word *ANTARCTICA*, 3*A*s, 1*N*, 2*T*s, 1*R*, 2*C*s, and 1*I*. Therefore, we use the formula for the permutation of n objects, not all distinct:

$$\frac{n!}{n_1!n_2!\cdots n_r!}=\frac{10!}{3!2!2!}=151{,}200.$$

40. There are 11 letters in the word *PHILIPPINES*, 3*P*s, 1*H*, 3*I*s, 1*L*s, 1*N*, 1*E*, and 1*S*.

Therefore, we use the formula for the permutation of n objects, not all distinct:

$$\frac{n!}{n_1!n_2!\cdots n_r!}=\frac{11!}{3!3!}=1{,}108{,}800\,.$$

41. The vowels cannot be permuted among themselves and may be considered as identical. So we can view the problem as that of finding the number of permutations of 7 letters, taken all together, where 2 of the letters are identical. Thus, the result is

$$\frac{7!}{2!(1!)^5}=(7)(6)(5)(4)(3)=2520$$

42. In this case order is important so the problem involves a permutation. The number of ways is given by $P(8,5)=\frac{8!}{3!}=(8)(7)(6)(5)(4)=6720\,.$

43. Here we use Formula (7). The number of distinct numbers is given by

$$\frac{5!}{3!1!1!}=20$$

44. Here we use Formula (7). The number of different signals that can be made is given by $P(7,7)=\frac{9!}{2!4!3!}=\frac{(9)(8)(7)(6)(5)}{(3)(2)(2)}=1260\,.$

45. The number of ways the 3 sites can be selected is

$$C(12,3)=\frac{12!}{9!3!}=\frac{12\cdot 11\cdot 10}{3\cdot 2\cdot 1}=220\,.$$

46. The number of ways the student can select the two books is

$$C(10,2)=\frac{10!}{8!2!}=\frac{10\cdot 9}{2\cdot 1}=45\,.$$

47. The number of ways in which the sample of 3 microprocessors can be selected is

$$C(100,3) = \frac{100!}{97!3!} = \frac{100 \cdot 99 \cdot 98}{3 \cdot 2 \cdot 1} = 161{,}700.$$

48. The five courses can be assigned to the members of the group in

$$P(5,5) = \frac{5!}{0!} = 5!, \quad \text{or } 120 \text{ ways.}$$

49. In this case order is important, as it makes a difference whether a commercial is shown first, last, or in between. The number of ways that the director can schedule the commercials is given by $P(6,6) = 6! = 720$.

50. The number of ways they can line up to purchase their ticket is

$$P(7,7) = 7! = 5040.$$

51. The inquiries can be directed in

$$P(12,6) = \frac{12!}{6!} = 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 = 665{,}280, \quad \text{or } 665{,}280 \text{ ways.}$$

52. The driver can be selected in $C(4,1) = 4$ ways. Next, the 5 remaining passengers can then be arranged in $P(5,5) = 120$ ways. Then by the multiplication principle, the number of possible seating arrangements is given by

$$C(4,1) \times P(5,5) = 4 \cdot 120 = 480.$$

53. a. The ten books can be arranged in

$$P(10,10) = 10! = 3{,}628{,}800 \text{ ways.}$$

b. If books on the same subject are placed together, then they can be arranged on the shelf

$$P(3,3) \times P(4,4) \times P(3,3) \times P(3,3) = 5184 \text{ ways.}$$

Here we have computed the number of ways the mathematics books can be arranged times the number of ways the social science books can be arranged times the number of ways the biology books can be arranged times the number of ways the 3 sets of books can be arranged.

54. a. The number of ways is $P(8,8) = \frac{8!}{0!} = 8!$, or 40,320 .

b. The four married couples can be seated in $P(4,4) \cdot 2^4 = \frac{4!}{0!}(16)$, or 384, ways.

c. The number of ways is $P(4,4) \cdot P(4,4) \cdot 2$, or 1152 .

55. Notice that order is certainly important here.

a. The number of ways that the 20 featured items can be arranged is given by

$$P(20,20) = 20! = 2.43 \times 10^{18}.$$

b. If items from the same department must appear in the same row, then the number of ways they can be arranged on the page is

Number of ways of arranging the rows × Number of ways of arranging the items in each of the 5 rows

$$P(5,5) \bullet P(4,4)\times P(4,4)\times P(4,4)\times P(4,4)\times P(4,4)$$

$$= 5! \times (4!)^5 = 955{,}514{,}880.$$

56. Using the formula for the permutation of n objects, not all distinct, with $n = 12$, and $n_1 = n_2 = n_3 = n_4 = 3$, we see that the number of ways the inquiries can be directed to the agents is

$$\frac{12!}{3!3!3!3!}, \quad \text{or } 369{,}600.$$

57. a. $P(12,9) = \dfrac{12!}{3!} = 79{,}833{,}600$

b. $C(12,9) = \dfrac{12!}{3!9!} = 220$

c. $C(12,9)\cdot C(3,2) = 220\cdot 3 = 660$

58. The number of ways is given by

$$2\{C(3,3) + [C(4,3) - C(3,3)] + [C(5,3) - C(4,3)]\} = 20$$

(number of players)[(number of ways to win in exactly 3 sets)
+ (number of ways to win in exactly 4 sets)
+ (number of ways to win in exactly 5 sets)]

59. The number of ways is given by

$$2\,\{C(2,2) + [C(3,2) - C(2,2)]\} = 2[1 + (3-1)] = 2\times 3 = 6$$

(number of players)[number of ways to win in exactly 2 sets + number of ways to win in exactly 3 sets]

60. The number of ways of selecting a panel of 12 jurors and 2 alternate jurors is given

by $C(30,12)\cdot C(18,2)=\dfrac{30!}{12!16!2!}=1.32\times 10^{10}$.

61. The number of ways the measure can be passed is

$$C(3,3) \times [C(8,6) + C(8,7) + C(8,8)] = 37.$$

Here three of the three permanent members must vote for passage of the bill and this can be done in $C(3,3) = 1$ way. Of the 8 nonpermanent members who are voting 6 can vote for passage of the bill, or 7 can vote for passage, or 8 can vote for passage. Therefore, there are

$$C(8,6) + C(8,7) + C(8,8) = 37 \text{ ways}$$

that the nonpermanent members can vote to ensure passage of the measure. This gives 1 × 37 = 37 ways that the members can vote so that the bill is passed.

62. a. The number of ways that the 10 questions can be selected from the 15 questions is

$$C(15,10)=\frac{15!}{10!5!}=3003.$$

b. The number of ways the 10 questions can be selected if exactly 2 of the first 3 questions must be answered is $C(3,2)\cdot C(12,8)=\dfrac{3!}{1!2!}\cdot\dfrac{12!}{4!8!}=1485$.

63. a. If no preference is given to any student, then the number of ways of awarding the 3 teaching assistantships is $C(12,3)=\dfrac{12!}{3!9!}=220$.

b. If it is stipulated that one particular student receive one of the assistantships, then the remaining two assistantships must be awarded to two of the remaining 11 students. Thus, the number of ways is $C(11,2)=\dfrac{11!}{2!9!}=55$.

c. If at least one woman is to be awarded one of the assistantships, and the group of students consists of seven men and five women, then the number of ways the assistantships can be awarded is given by

$$C(5,1)\times C(7,2) + C(5,2)\times C(7,1) + C(5,3)$$

$$=\frac{5!}{4!1!}\cdot\frac{7!}{5!2!}+\frac{5!}{3!2!}\cdot\frac{7!}{6!1!}+\frac{5!}{3!2!}=105+70+10=185.$$

64. a. The number of ways the subcommittee can be chosen is

$$C(9,4)=\frac{9!}{5!4!}=126.$$

b. The number of ways the subcommittee can be chosen if it must include 2

Republicans and 2 Democrats is $C(5,2)\cdot C(4,2)=\frac{5!}{3!2!}\cdot\frac{4!}{2!2!}=60.$

65. The number of ways of awarding 3 contracts to 7 different firms is given by

$$P(7,3)=\frac{7!}{4!}=210\,.$$

The number of ways of awarding the 3 contracts to 2 different firms (one firm gets 2 contracts) from a choice of 7 different firms is

$C(7,2)\times P(3,2) = 126.$ (First pick the two firms, and then award the 3 contracts.)

Therefore, the number of ways the contracts can be awarded if no firm is to receive more than 2 contracts is given by 210 + 126 = 336.

66. a. The number of ways of choosing the 5 executive trainees is

$$C(20,5)=\frac{20!}{15!5!}=15{,}504\,.$$

b. The number of ways of choosing 2 male and 3 female trainees is

$$C(10,2)\cdot C(10,3)=\frac{10!}{8!2!}\cdot\frac{10!}{7!3!}=5400\,.$$

67. The number of different curricula that are available for the student's consideration is given by

$$C(5,1)\times C(3,1)\times C(6,2)\times C(4,1) + C(5,1)\times C(3,1)\times C(6,2)\times C(3,1)$$

$$=\frac{5!}{4!1!}\cdot\frac{3!}{2!1!}\cdot\frac{6!}{4!2!}\cdot\frac{4!}{3!1!}+\frac{5!}{4!1!}\cdot\frac{3!}{2!1!}\cdot\frac{6!}{4!2!}\cdot\frac{3!}{2!1!}$$

$$= (5)(3)(15)(4) + (5)(3)(15)(3) = 900 + 675 = 1575.$$

68. The number of ways is given by $C(10,2) + C(10,1) + C(10,0) = 45 + 10 + 1 = 56$
or $C(10,8) + C(10,9) + C(10,10) = 56.$

69. The number of ways of dealing a straight flush (5 cards in sequence in the same suit is given by

the number of ways of selecting 5 cards in sequence in the same suit	×	the number of ways of selecting a suit	
10	•	$C(4,1) =$	40.

70. The number of ways of dealing a straight (but not a straight flush) is

the number of ways a card of each suit can be selected × the number of ways of selecting 5 cards in sequence - the number of ways of selecting a straight flush

$$[C(4,1)C(4,1)C(4,1)C(4,1)C(4,1)](10) - 40$$

$$= 4^5(10) - 40 = 10{,}200$$

71. The number of ways of dealing a flush (5 cards in one suit that are not all in sequence) is given by

the number of ways of selecting 5 cards in one suit - the number of ways of selecting 5 cards in one suit in sequence

$$4C(13,5) - 4(10)$$

$$= 5148 - 40 = 5108.$$

72. The number of ways of dealing 4 of a kind is

the number of different cards in one suit × the number of ways of selecting four cards from four cards × the number of ways of picking the remaining card

$$13 \bullet C(4,4) \bullet C(48,1)$$

$$= (13)(1)(48) = 624.$$

73. The number of ways of dealing a full house (3 of a kind and a pair) is given by

the number of ways of picking 3 of a kind from a given rank × the number of ways of picking a pair from the 12 remaining ranks

$$13C(4,3) \bullet 12C(4,2)$$

$$= 13(4) \bullet (12)(6) = 3744.$$

74. The number of ways of dealing 2 pairs is

the number of ways of picking 2 of a kind from a given rank × the number of ways of picking the first pair r × the number of ways of picking the second pair × the number of ways of picking the remaining card

$$C(13,2) \bullet C(4,2) \bullet C(4,2) \bullet C(44,1)$$

$$= (78)(6)(6)(44) = 123{,}552.$$

75. The bus will travel a total of 6 blocks. Each route must include 2 blocks running north and south and 4 blocks running east and west. To compute the total number of possible routes, it suffices to compute the number of ways the 2 blocks running north and south can be selected from the six blocks. Thus,

$$C(6,2) = \frac{6!}{2!4!} = 15.$$

76. The number of ways the series can be completed is

$$C(4,0) \times 2 + C(4,1) \times 2 + C(5,2) \times 2 + C(6,3) \times 2 = 70.$$

In each case we are computing the number of ways a team can lose--
0, 1, 2, and 3 games to win the series.

77. The number of ways that the quorum can be formed is given by

$$C(12,6)+C(12,7)+C(12,8)+C(12,9)+C(12,10)+C(12,11)+C(12,12)$$

$$=\frac{12!}{6!6!}+\frac{12!}{7!5!}+\frac{12!}{8!4!}+\frac{12!}{9!3!}+\frac{12!}{10!2!}+\frac{12!}{11!1!}+\frac{12!}{12!0!}$$

$$=924+792+495+220+66+12+1=2510.$$

78. In the case of the circular permutation of 5 objects *A*, *B*, *C*, *D*, and *E*, observe that the permutations *ABCDE*, *BCDEA*, *CDEAB*, *DEABC*, and *EABCD* are not distinguishable. Therefore, if there are N different arrangements of the five objects on a circle, there are $5N$ permutations of these objects on the line. But there are $n!$ ways of permuting 5 objects on the line. So $5N = 5!$, or $N = 4!$. Generalizing this result, we see that there are n (beginning with any of the n objects) permutations on a circle that are not distinguishable. Therefore, if there are N different arrangements of the n objects on a circle, there are Nn permutations of these objects on the line. So, $Nn = n!$, or $N = \frac{n!}{n} = (n - 1)!$.

79. Using the formula given in Exercise 78, we see that the number of ways of seating the 5 commentators at a round table is

$$(5 - 1)! = 4! = 24.$$

80. The number of ways of seating the guests is $\frac{4!4!}{2!2!}=144$.

81. The number of possible corner points is $C(8,3)=\frac{8!}{5!3!}=56.$

82. The number of possible corner points is $C(15,5)=\frac{15!}{10!5!}=3003.$

83. True.

84. True. $P(n,r)=\frac{n!}{(n-r)!}$ and $C(n,r)=\frac{n!}{(n-r)!r!}$ and so $P(n,r)=r!\,C(n,r)$.

85. True. $C(n,r)=\dfrac{n!}{(n-r)!r!}$ and $C(n,n-r)=\dfrac{n!}{[n-(n-r)]!(n-r)!}=\dfrac{n!}{r!(n-r)!}$.

So, $C(n,r)=C(n,n-r)$.

86. False

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1. 1.307674368×10^{12}
2. $2.43290200818\times10^{18}$
3. $2.56094948229\times10^{16}$
4. $4.14303931642\times10^{16}$
5. 674,274,182,400
6. 29,654,109,720
7. 133,784,560
8. 1,562,275
9. 4,656,960
10. 10,939,622,400
11. Using the multiplication principle, the number of 10-question exams she can set is given by $C(25,3)\times C(40,5)\times C(30,2)=658{,}337{,}004{,}000$.
12. The number of inquiries that can be handled is $\dfrac{100!}{20!20!20!20!20!}$ or, $1.09491541553\times10^{66}$, ways.