$(\{1, 3, 5, 7, 9\} \cap \{2, 4, 6, 8\})$
$= \{1, 7\} \cup \varnothing = \{1, 7\}$.

b. $(A \cup B)^C = (\{1, 3, 5, 7, 9\} \cup \{1, 2, 4, 7, 8\})^C = \{1, 2, 3, 4, 5, 7, 8, 9\}^C$
$= \{6, 10\}$.
$A^C \cap B^C = \{1, 3, 5, 7, 9\}^C \cap \{1, 2, 4, 7, 8\}^C$
$= \{2, 4, 6, 8, 10\} \cap \{3, 5, 6, 9, 10\}$
$= \{6, 10\}$.

61. a. *r, u, v, w, x y* b. *v, r* 62. a. *r, u, v* b. *t, x, u, z, v, y ,s*

63. a. *t, y, s* b. *t ,s, w, x, z* 64. a. *v* b. *t, s*

65. $A \subset C$

66. If $A = B$, then $A \subseteq B$ and $B \subseteq A$. Next, if $A \subseteq B$ and $B \subseteq A$, then A and B contain exactly the same elements and so $A = B$.

67. False. Since every element in a set A belongs to A, A is a subset of itself.

68. True. Let $A = \{1, 2, 3\}$ and $B = \{1\}$. Then B is a proper subset of A, that is, of course, a subset of A. But A is not a proper subset of B.

69. True. If at least one of the sets A or B is nonempty, then $A \cup B \neq \varnothing$.

70. False. Take $A = \{1, 2\}$ and $B = \{3, 4\}$. 71. True. $(A \cup A^c)^c = U^c = \varnothing$.

72. True. Since $A \subseteq B$, every element in A is in B and so $A \cap B = A$.

6.2 CONCEPT QUESTIONS, page 326

1. a. If A and B are sets with $A \cap B = \varnothing$, then $n(A) + n(B) = n(A \cup B)$.
   b. If $n(A \cup B) \neq n(A) + n(B)$, then $A \cap B \neq \varnothing$.

2. If $n(A \cup B) = \varnothing$, then $n(B^c) = n(U) - n(B)$, and $n(A^c) = n(U) - n(A)$, and it follows

   that

$$n(B^c) - n(A^c) = n(U) - n(B) - n(U) + n(A)$$
$$= n(A) - n(B)$$

EXERCISES 6.2, page 326

1. $A \cup B = \{a,e,g,h,i,k,l,m,o,u\}$, and so $n(A \cup B) = 10$. Next, $n(A) + n(B) = 5 + 5 = 10$.

2. $A \cup B = \{0, 1, 2, 3, 4\} \cup \{-3, -2, -1\} = \{-3, -2, -1, 0, 1, 2, 3, 4\}$ and so $n(A \cup B) = 8$. Next, $n(A) + n(B) = 5 + 3 = 8$.

3. a. $A = \{2, 4, 6, 8\}$ and $n(A) = 4$. b. $B = \{6, 7, 8, 9, 10\}$ and $n(B) = 5$
   c. $A \cup B = \{2, 4, 6, 7, 8, 9, 10\}$ and $n(A \cup B) = 7$.
   d. $A \cap B = \{6, 8\}$ and $n(A \cap B) = 2$.

4. a. $A^c = \{3, 4, 5, 6, 7, b, c, d\}$ and so $n(A^c) = 8$
   b. $A \cap B^c = \{1, 2, a, e\} \cap \{5, 6, 7, d, e\} = \{e\}$ and so $n(A \cap B^c) = 1$
   c. $A \cup B^c = \{1, 2, a, e\} \cup \{5, 6, 7, d, e\} = \{1, 2, 5, 6, 7, a, d, e\}$ and so $n(A \cup B^c) = 8$.
   d. $A^c \cap B^c = \{3, 4, 5, 6, 7, b, c, d\} \cap \{5, 6, 7, d, e\} = \{5, 6, 7, d\}$ and so $n(A^c \cap B^c) = 4$.

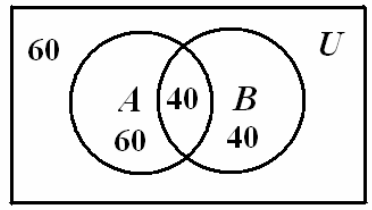
5. Using the results of Exercise 3, we see that $n(A \cup B) = 7$ and $n(A) + n(B) - n(A \cap B) = 4 + 5 - 2 = 7$.

6. $A \cup B = \{a, e, i, o, u\} \cup \{b, d, e, o, u\} = \{a, b, d, e, i, o, u\}$ and $n(A \cup B) = 7$
   $A = \{a, e, i, o, u\}$ so $n(A) = 5$, $B = \{b, d, e, o, u\}$ and $n(B) = 5$, and
   $A \cap B = \{a, e, i, o, u\} \cap \{b, d, e, o, u\} = \{e, o, u\}$ so that $n(A \cap B) = 3$.
   Therefore, $n(A \cup B) = n(A) + n(B) - n(A \cap B) = 5 + 5 - 3 = 7$.

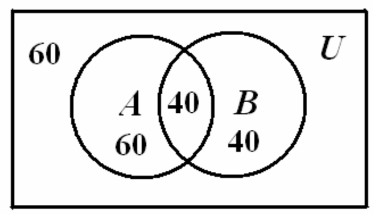
7. Since $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
   $n(B) = n(A \cup B) + n(A \cap B) - n(A) = 30 + 5 - 15 = 20$.

8. $n(A \cap B) = n(A) + n(B) - n(A \cup B) = 10 + 8 - 15 = 3$.

9. Refer to the Venn diagram at the right.
   a. $n(A \cup B) = 60 + 40 + 40 = 140$.
   b. $n(A^C) = 40 + 60 = 100$.
   c. $n(A \cap B^C) = 60$.



10. Refer to the Venn diagram at the right.
   a. $n(A^C \cap B) = 40$.
   b. $n(B^C) = 60 + 60 = 120$.
   c. $n(A^C \cap B^C) = n(A \cup B)^C = 60$.



11. $n(A \cup B) = n(A) + n(B) - n(A \cap B) = 6 + 10 - 3 = 13$.

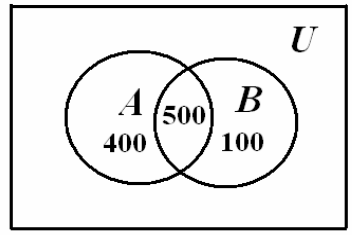
12. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ so
$n(A) = n(A \cup B) + n(A \cap B) - n(B) = 14 + 3 - 6 = 11$.

13. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ so
$n(A \cap B) = n(A) + n(B) - n(A \cup B) = 4 + 5 - 9 = 0$.

14. $n(A \cap B \cap C)$
$= n(A \cup B \cup C) - n(A) - n(B) - n(C) + n(A \cap B) + n(A \cap C) + n(B \cap C)$
$= 31 - 16 - 16 - 14 + 6 + 5 + 6 = 2$.

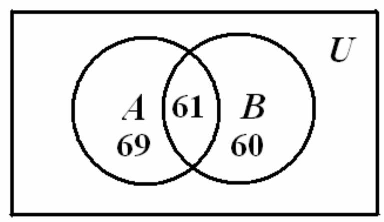
15. $n(A \cap B \cap C)$
$= n(A \cup B \cup C) - n(A) - n(B) - n(C) + n(A \cap B) + n(A \cap C) + n(B \cap C)$
so $n(C) = n(A \cup B \cup C) - n(A \cap B \cap C) - n(A) - n(B)$
$+ n(A \cap B) + n(A \cap C) + n(B \cap C$
$= 25 - 2 - 12 - 12 + 5 + 5 + 4 = 13$.

16. Let
A = {x | x is a subscriber to the daily morning edition}
and
B = {x | x is a subscriber to the Sunday *L.A. Times*}.



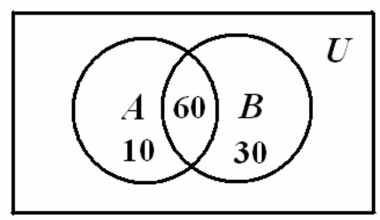
Then, we are given that $n(A)$ = 900, $n(A \cap B)$ = 500, and $n(A \cup B)$ = 1000. Refer to the Venn diagram at the right. Since $n(A \cup B) = n(A) + n(B) - n(A \cap B),$ we see that
$n(B) = n(A \cup B) + n(A \cap B) - n(A) = 1000 + 500 - 900 = 600$
Next, $(B \cap A^C) = n(B) - n(A \cap B)$ = 600 - 500 = 100.

17. Let A denote the set of prisoners in the Wilton County Jail who were accused of a felony and B the set of prisoners in that jail who were accused of a misdemeanor. Then we are given that
$n(A \cup B) = 190$



Refer to the diagram at the right.
Then the number of prisoners who were accused of both a felony and a misdemeanor is given by $(A \cap B) = n(A) + n(B) - n(A \cup B)$
= 130 + 121 - 190 = 61.

18. Let A = {x | x has *FM* circuitry},
B = { x | x has *AM* circuitry}.

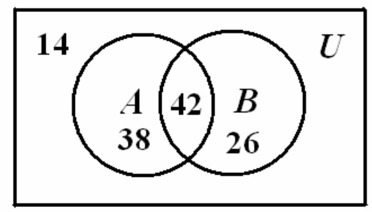


Then $n(A)$ = 70, $n(B)$ = 90, and $n(A \cup B)$ = 100. Refer to the Venn diagram at the right.The number of radios with both *FM* and *AM* circuitry is given by
$n(A \cap B) = n(A) + n(B) - n(A \cup B)$
= 70 + 90 - 100 = 60.
The number of radios that could receive FM transmission only is given by
$n(A \cap B^C) = n(A) - n(A \cap B)$ = 70 - 60 = 10.
The number of radios that could receive *AM* transmission only is given by
$n(A^C \cap B) = n(B) - n(A \cap B)$ = 90 - 60 = 30.

19. Let U denote the set of all customers surveyed, and let
A = { $x \in U \mid x$ buys brand A}
B = { $x \in U \mid x$ buys brand B}.

Then $n(U) = 120$, $n(A) = 80$,
$n(B) = 68$, and $n(A \cap B) = 42$.

Refer to the diagram at the right.



a. The number of customers who buy at least one of these brands is

$$n(A \cup B) = 80 + 68 - 42 = 106.$$

b. The number who buy exactly one of these brands is

$$n(A \cap B^C) + n(A^C \cap B) = 38 + 26 = 64$$

c. The number who buy only brand A is $n(A \cap B^C) = 38$.

d. The number who buy none of these brands is $n[(A \cup B)^C] = 120 - 106 = 14$.

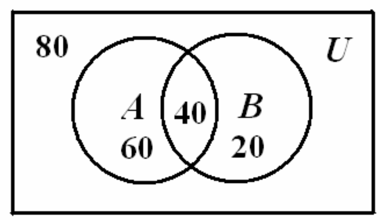
20. Let U denote the set of all members of the sports club who were surveyed, and let

$A = \{ x \in U \mid x$ plans to attend the Summer Olympic Games$\}$

$B = \{ x \in U \mid x$ plans to attend the Winter Olympic Games$\}$.

Then $n(U) = 200$, $n(A) = 100$, $n(B) = 60$, and $n(A \cap B) = 40$.

Refer to the diagram at the right.



a. The number of members who plan to attend at least one of the two games is

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$= 100 + 60 - 40 = 120.$$

b. The number of members who plan to attend exactly one of the games is

$$n(A \cap B^C) + n(A^C \cap B) = 60 + 20 = 80.$$

c. The number of members who plan to attend the Summer Olympic Games only is $n(A \cap B^C) = 60$.

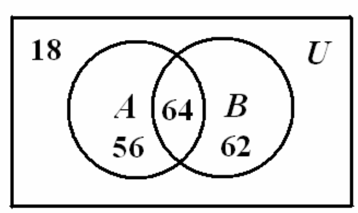
d. The number of members who do not plan to attend either of the games is $U - n(A \cup B) = 200 - 120 = 80$.

21. Let U denote the set of 200 investors and let

$A = \{ x \in U \mid x$ uses a discount broker$\}$

$B = \{ x \in U \mid x$ uses a full-service broker$\}$.

Refer to the diagram at the right.



a. The number of investors who use at least one kind of broker is
$n(A \cup B) = n(A) + n(B) - n(A \cap B) = 120 + 126 - 64 = 182.$

b. The number of investors who use exactly one kind of broker is
$n(A \cap B^C) + n(A^C \cap B) = 56 + 62 = 118.$

c. The number of investors who use only discount brokers is $n(A \cap B^C) = 56$.

d. The number of investors who don't use a broker is
$n(A \cup B)^C = n(U) - n(A \cup B) = 200 - 182 = 18.$

22. Let U denote the set of 50 employees at the downtown store, and let

$A = \{ x \in U \mid x \text{ takes the subway to work}\}$

$B = \{ x \in U \mid x \text{ takes the bus to work}\}.$

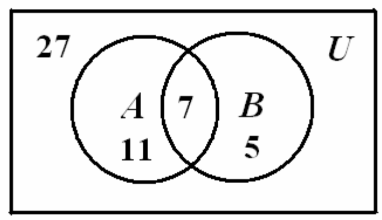
Then we are given that $n(A) = 18$, $n(B) = 12$,
and $n(A \cap B) = 7$. Refer to the diagram at the right.

a. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$= 18 + 12 - 7 = 23.$

b. $n(B \cap A^C) = 5$

c. $n[(B \cap A^C) \cup (A \cap B^C)] = 11 + 5 = 16.$

d. $n(U) - n(A \cup B) = 50 - 23 = 27$

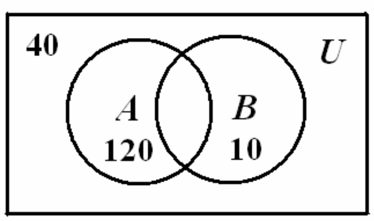


23. Let U denote the set of 200 households in the survey and let

$A = \{x \in U \mid x \text{ owns a desktop computer}\}$

$B = \{x \in U \mid x \text{ owns a laptop computer}\}$

Referring to the figure that follows, we see that the number of households that own both desktop and laptop computers is $n(A \cap B) = 200 - 120 - 10 - 40 = 30.$



24. Let U denote the set of 400 households in the survey, and let

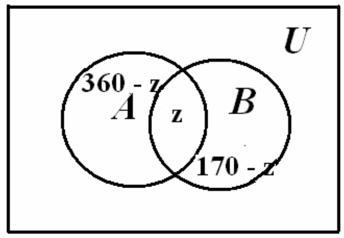
$A = \{x \in U \mid x \text{ owns one or more VCRs}\}$

$B = \{x \in U \mid x \text{ owns one or more DVD players}\}$

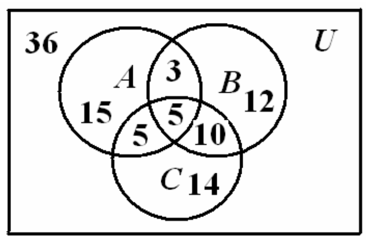
Referring to the diagram that follows, where z denotes $n(A \cap B)$, we have $(360 - z) + z + (170 - z) + 19 = 400$, or $z = 149$.

Therefore, the number of households owning one or more DVD players only is

$n(A^c \cap B) = 170 - z = 170 - 149 = 21$.



In Exercises 25 - 28, refer to the figure that follows.



25. a. $n(A \cup B \cup C) = 64$ b. $n(A^C \cap B \cap C) = 10$

26. a. $n[A \cap (B \cup C)] = 13$ b. $n[A \cap (B \cup C)^C] = 15$

27. a. $n(A^C \cap B^C \cap C^C) = n[(A \cup B \cup C)^C] = 36$ b. $n[A^C \cap (B \cup C)] = 36$

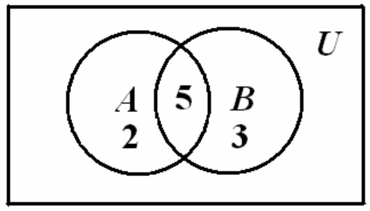
28. a. $n[A \cup (B \cap C)] = 38$ b. $n(A^C \cap B^C \cap C^C)^C = n[(A \cup B \cup C)] = 64$.

29. Let U denote the set of all economists surveyed, and let

$A = \{ x \in U \mid x \text{ had lowered his estimate of the consumer inflation rate}\}$

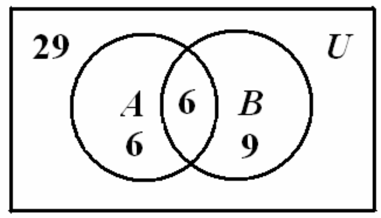
$B = \{ x \in U \mid x \text{ had raised his estimate of the } GDP \text{ growth rate}\}$.

Refer to the diagram that follows.



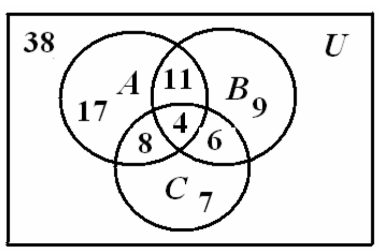
Then $n(U) = 10$, $n(A) = 7$, $n(B) = 8$, and $n(A \cap B^C) = 2$. Then the number of economists who had both lowered their estimate of the consumer inflation rate and raised their estimate of the *GDP* rate is given by $n(A \cap B) = 5$.

30. Let U denote the set of 50 states and let
$A = \{x | x$ had an increase in the dropout rate during the past 2 years$\}$
and
$B = \{x | x$ had a dropout rate of at least 30 percent during the past 2 years$\}$.
Refer to the diagram at the right.



a. The number of states that had both a dropout rate of at least 30 percent and an increase in the dropout rate over the two-year period is $n(A \cap B) = 6$.
b. The number of states that had a dropout rate that was less than 30 percent but that had increased over the two-year period is $n(B^C \cap A) = 6$.

31. Let U denote the set of 100 college students who were surveyed and let
$A = \{x \in U | x$ is a student who reads *Time* magazine$\}$
$B = \{x \in U | x$ is a student who reads *Newsweek* magazine$\}$
and $C = \{x \in U | x$ is a student who reads *U.S. News and World Report* magazine$\}$
Refer to the diagram that follows.



Then $n(A) = 40$, $n(B) = 30$, $n(C) = 25$, $n(A \cap B) = 15$,

$n(A \cap C) = 12$, $n(B \cap C) = 10$, and $n(A \cap B \cap C) = 4$.

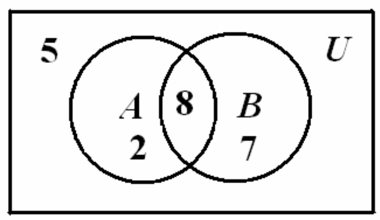
a. The number of students surveyed who read at least one magazine is
$n(A \cup B \cup C) = 17 + 11 + 4 + 8 + 6 + 7 + 9 = 62$

b. The number of students surveyed who read exactly one magazine is
$$n(A \cap B^C \cap C^C) + n(A^C \cap B \cap C^C) + n(A^C \cap B^C \cap C)$$
$$= 17 + 9 + 7 = 33.$$

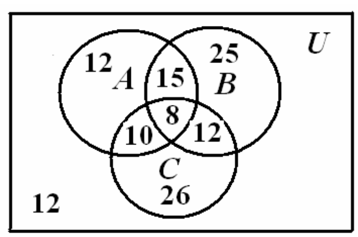
c. The number of students surveyed who read exactly two magazines is
$$n(A \cap B \cap C^C) + n(A^C \cap B \cap C) + n(A \cap B^C \cap C)$$
$$= 11 + 6 + 8 = 25.$$

d. The number of students surveyed who did not read any of these magazines is
$$n(A \cup B \cup C)^C = 100 - 62 = 38.$$

32. Let U denote the set of states surveyed, and let
$A = \{ x \in U \mid x$ is a state that had an average composite test score of at least 1000$\}$
$B = \{ x \in U \mid x$ is a state that had an increaseof at least 10 points in the average composite score$\}$
Then $n(U) = 22$, $n(A) = 10$, $n(B) = 15$, and $n(A \cap B) = 8$.
Refer to the diagram at the right.
a. $n(B \cap A^C) = 7$ b. $n(A \cap B^C) = 2$.



33. Let U denote the set of all customers surveyed, and let
$A = \{ x \in U \mid x$ buys brand $A\}$
$B = \{ x \in U \mid x$ buys brand $B\}$.
$C = \{ x \in U \mid x$ buys brand $C\}$.
Refer to the figure at the right. Then
$n(U) = 120$, $n(A \cap B \cap C^C) = 15$,
$n(A^C \cap B \cap C^C) = 25$,
$n(A^C \cap B^C \cap C) = 26$,
$n(A \cap B \cap C^C) = 15$, $n(A \cap B^C \cap C) = 10$,
$n(A^C \cap B \cap C) = 12$, and $n(A \cap B \cap C) = 8$.



a. The number of customers who buy at least one of these brands is
$$n(A \cup B \cup C) = 12 + 15 + 25 + 12 + 8 + 10 + 26 = 108.$$

b. The number who buy labels A and B but not C is $n(A \cap B \cap C^C) = 15$

c. The number who buy brand A is $n(A) = 12 + 10 + 15 + 8 = 45$.

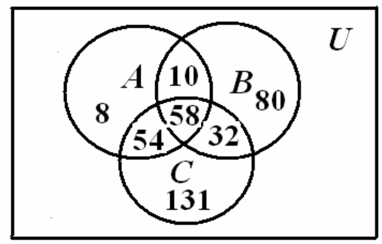
d. The number who buy none of these brands is

$$n[(A \cup B \cup C)^C] = 120 - 108 = 12.$$

34. Let $A = \{ x \in U \mid x \text{ ate breakfast}\}$

$B = \{ x \in U \mid x \text{ ate lunch}\}$

$C = \{ x \in U \mid x \text{ ate dinner}\}$



Refer to the diagram at the right.

Then

$n(A) = 130$, $n(B) = 180$, $n(C) = 275$,

$n(A \cap B) = 68$, $n(A \cap C) = 112$,

and $n(B \cap C) = 90$, and $n(A \cap B \cap C) = 58$.

a. $n(A \cup B \cup C) = 80 + 10 + 8 + 131 + 54 + 58 + 32 = 373$.

b. $n(A \cap B^C \cap C^C) + n(A^C \cap B^C \cap C) + n(A^C \cap B \cap C^C)$

$= 8 + 80 + 131 = 219$.

c. $n(A^C \cap B^C \cap C) = 131$.

d. $n(A \cap B \cap C^C) + n(A^C \cap B \cap C) + n(A \cap B^C \cap C) = 10 + 32 + 54 = 96$.

35. Let U denote the set of 200 employees surveyed, and let

$A = \{x \in U \mid x \text{ had investments in stock funds}\}$

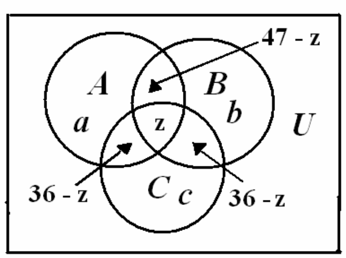
$B = \{x \in U \mid x \text{ had investments in bond funds}\}$

$C = \{x \in U \mid x \text{ had investments in money market funds}\}$

Then

$n(U) = 200$, $n(A) = 141$, $n(B) = 91$, $n(C) = 60$, $n(A \cap B) = 47$,

$n(A \cap C) = 36$, $n(B \cap C) = 36$, and $n(A^c \cap B^c \cap C^c) = n[(A \cup B \cup C)^c] = 5$



Letting $n(A \cap B \cap C) = z$ and using the fact that $n(A \cap B) = 47$, $n(A \cap C) = 36$, and

$n(B \cap C) = 36,$ leads to the Venn diagram shown. Next, using the fact that $n(A) = 141,\ n(B) = 91,$ and $n(C) = 60$ leads to

$$a + (36 - z) + (47 - z) + z = 141$$

$$b + (47 - z) + (36 - z) + z = 91$$

$$c + (36 - z) + (36 - z) + z = 60$$

$$a + b + c + (36 - z) + (47 - z) + (36 - z) + z + 5 = 200$$

which simplifies to $a - z = 58,\ b - z = 8,\ c - z = -12,\ a + b + c - 2z = 76.$

Solving, we find $a = 80,\ b = 30,\ c = 10,$ and $z = 22.$ Therefore,

a. The number of employees surveyed who had invested in all three investments is $n(A \cap B \cap C) = z = 22.$

b. The number who had invested in stock funds only is given by $n(A \cap B^c \cap C^c) = a = 80.$

36. Let U denote the set of 300 individual investors surveyed, and let

$$A = \{x \in U \mid x \text{ subscribed to the NYT}\}$$

$$B = \{x \in U \mid x \text{ subscribed to the WSJ}\}$$

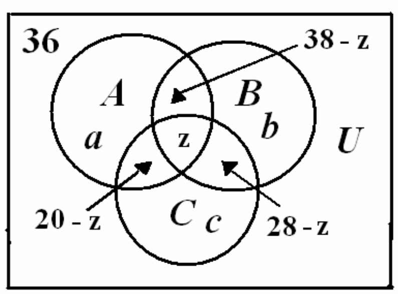
$$C = \{x \in U \mid x \text{ subscribed to the UST}\}$$

Then, $n(U) = 300,\ n(A) = 122,\ n(B) = 150,\ n(C) = 62,$

$n(B \cap C) = 28,\ n(A \cap C) = 20,$ and $n(A \cap B) = 38,$

$n(A^c \cap B^c \cap C^c) = n[(A \cup B \cup C)^c = 36.$

Letting $n(A \cap B \cap C) = z,$ and using the fact that $n(A \cap B) = 38,\ n(B \cap C) = 28,$ and $n(A \cap C) = 20$ leads to the Venn diagram shown.



Next, using the fact that $n(A) = 122,\ n(B) = 150,\ n(C) = 62,$ and $n(U) = 300$ leads to

$$a + (20 - z) + (38 - z) + z = 122$$

$$b + (38 - z) + (28 - z) + z = 150$$

$$c + (20 - z) + (28 - z) + z = 62$$

$$a+b+c+(20-z)+(38-z)+(28-z)+z+36=300$$

which simplifies to $a-z=64,\ b-z=84,\ c-z=14,\ a+b+c-2z=178.$ Solving, we find $a=80,\ b=100,\ c=30,$ and $z=16$. Therefore,

a. The number of individual investors surveyed who subscribe to all three newspapers is $n(A\cap B\cap C)=z=16.$

b. The number is $n(A\cap B^c\cap C^c)+n(A^c\cap B\cap C^c)+n(A^c\cap B^c\cap C)=a+b+c$
$=80+100+30=210.$

37. True. $n(A\cup B)=n(A)+n(B)-n(A\cap B)$.

38. True. If $A\subseteq B,$ then $B=A\cup(A^c\cap B)$ and $A\cap(A^c\cap B)=\varnothing.$ Therefore, $n(B)=n(A)+n(A^c\cap B).$

39. True. If $A\cap B\neq\varnothing$, then $n(A\cup B)=n(A)+n(B)-n(A\cap B)$.

40. Write Equation (4) as $n(D\cup E) = n(D) + n(E) - n(D\cap E)$. Then let $D = A\cup B$ and $E=C$ so that

$$\begin{aligned}
n(A\cup B\cup C) &= n(A\cup B) + n(C) - n[(A\cup B)\cap C]\\
&= n(A) + n(B) - n(A\cap B) + n(C) - n[(A\cup B)\cap C]\\
&= n(A) + n(B) - n(A\cap B) + n(C) - \{n[(A\cap C)\cup(B\cap C)]\}\\
&= n(A) + n(B) - n(A\cap B) + n(C)\\
&\quad - [n(A\cap C)+n(B\cap C)-n(A\cap C\cap B\cap C)]\\
&= n(A) + n(B)+ n(C) - n(A\cap B)\\
&\quad - n(A\cap C) - n(B\cap C)+n(A\cap B\cap C).
\end{aligned}$$

6.3 CONCEPT QUESTIONS, page 333

1. If a task T_1 can be performed in N_1 ways, a task T_2 can be performed in N_2 ways, …, and finally a task T_n can be performed in N_n ways, then the number of ways of performing the tasks $T_1, T_2, ..., T_n$ in succession is given by $N_1 N_2 \cdots N_n$.

2. *AI*, *AII*, *BI*, *BII*