CHAPTER 6

6.1 CONCEPT QUESTIONS, page 320

1. a. A set is a well-defined collection of objects. As an example, consider the set of all freshmen students in a college.
   b. Two sets A and B are equal if they have exactly the same elements.
   c. The empty set is the set that contains no elements.
2. a. If $A \cup B \subseteq A$, then $B \subseteq A$.
   b. If $A \cup B = \varnothing$, then both A and B are empty.
   c. If $A \cap B = B$, then $A = B$.
   d. If $A \cap B = \varnothing$, then A and B are disjoint.
3. a. If $A \subset B$, then $B^c \subset A^c$.
   b. If $A^c = \varnothing$, then $A = U$ where U is the universal set.

EXERCISES 6.1, page 320

1. $\{x \mid x$ is a gold medalist in the 2006 Winter Olympic Games$\}$

2. $\{x \mid x$ is a football team in the NFL$\}$

3. $\{x \mid x$ is an integer greater than 2 and less than 8$\}$

4. $\{x \mid x = 2n - 1;\ n$, an integer between 1 and 20 inclusive$\}$

5. $\{2,3,4,5,6\}$

6. $\{A, H, I, M, O, P, S, T, U\}$

7. $\{-2\}$

8. $\{\frac{-2}{1}\}$. Note that the answer is not unique.

9. a. True--the order in which the elements are listed is not important.
   b. False-- A is a set, not an element.

10. a. False--the symbol $\varnothing$ refers to the empty set. $\varnothing$ is not an element of any set.
    b. False. A set cannot be a proper subset of itself.

11. a. False. The empty set has no elements. b. False. 0 is an element and $\varnothing$ is a set.

12. a. False. $\{\varnothing\}$ contains an element but $\varnothing$ has no elements.
b. False. $\{a, b\}$ is a set and not an element of another set.

13. True.

14. False. There are silver medalists in the 2006 Winter Olympic Games.

15. a. True. 2 belongs to A. b. False. For example, 5 belongs to A but $5 \notin \{2,4,6\}$.

16. a. False. A does not contain the number 0. b. False. It is a subset, not an element.

17. a. and b. 18. a. b. and c.

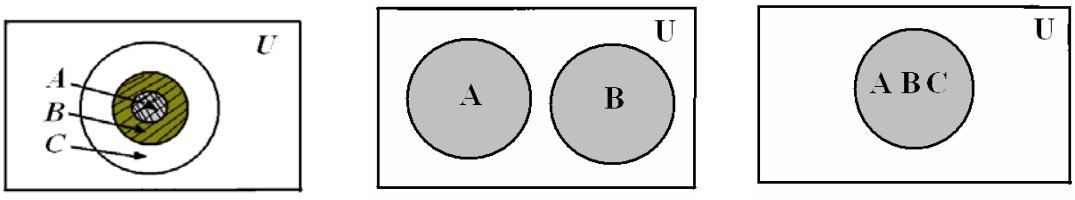
19. a. $\varnothing$, {1}, {2}, {1,2}
b. $\varnothing$, {1}, {2}, {3}, {1,2}, {1,3}, {2,3}, {1,2,3}
c. $\varnothing$, {1}, {2}, {3}, {4}, {1,2}, {1,3}, {1,4}, {2,3}, {2,4}, {3,4}, {1,2,3}, {1,2,4}, {2,3,4}, {1,3,4}, {1,2,3,4}

20. $\varnothing$, {IBM}, {U.S.Steel}, {Union Carbide}, {Boeing},{IBM,U.S.Steel}, {IBM, Union Carbide}, {IBM, Boeing}, {U.S. Steel, Union Carbide}, U.S.Steel, Boeing}, {Union Carbide, Boeing}, {IBM, U.S. Steel, Union Carbide}, {IBM, Union Carbide, Boeing}, {IBM, U.S. Steel, Boeing}, {U.S.Steel,Union Carbide, Boeing}, {IBM, U.S. Steel, Union Carbide, Boeing}. All except the last subset listed above are proper subsets.

21. {1, 2, 3, 4, 6, 8, 10} 22. {1, 2, 4, a, b} 23. {Jill, John, Jack, Susan, Sharon}

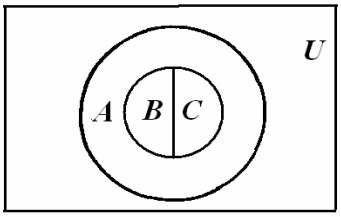
24. {GM, Ford, Chrysler, Daimler-Benz, Volkswagen, Toyota, Nissan}.

25. a. b. c.



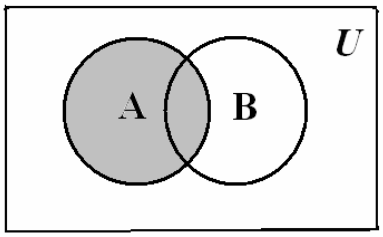
26.

a.

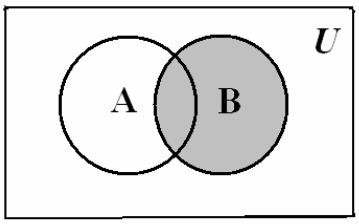


b. *i.* False *ii.* True *iii.* False

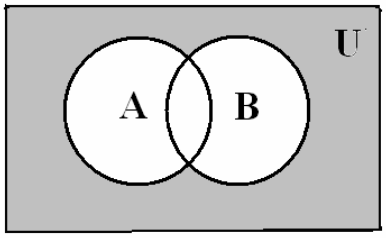
27. a.



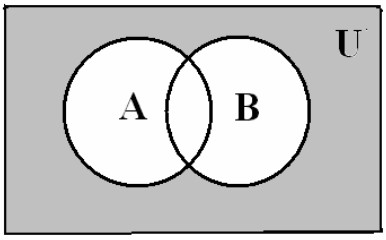
b.



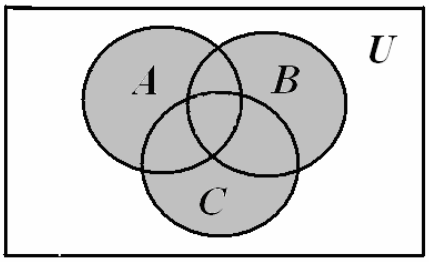
28. a.



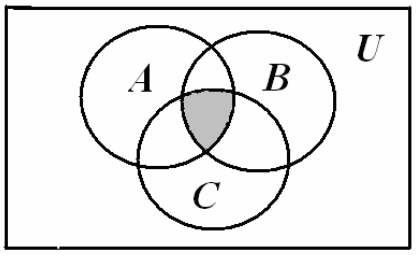
b.



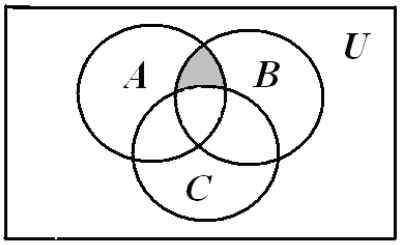
29. a.



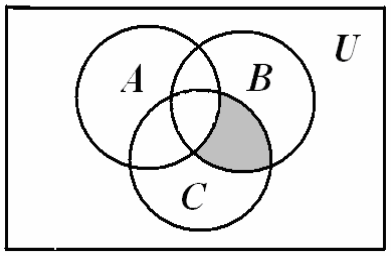
b.



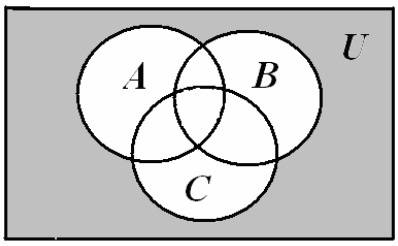
30. a.



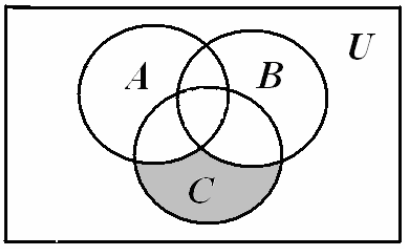
b.



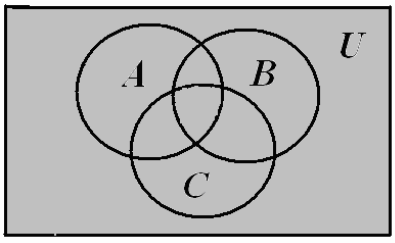
31. a.



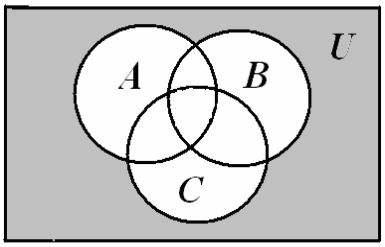
b.



32. a.



b.



33. a. $A^C = \{2, 4, 6, 8, 10\}$
b. $B \cup C = \{2, 4, 6, 8, 10\} \cup \{1, 2, 4, 5, 8, 9\} = \{1, 2, 4, 5, 6, 8, 9, 10\}$
c. $C \cup C^C = U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

34. a. $C \cap C^C = \varnothing$
b. $(A \cap C)^C = \{1, 5, 9\}^C = \{2, 3, 4, 6, 7, 8, 10\}$
c. $A \cup (B \cap C) = \{1, 3, 5, 7, 9\} \cup \{2, 4, 8\} = \{1, 2, 3, 4, 5, 7, 8, 9\}$

35. a. $(A \cap B) \cup C = C = \{1, 2, 4, 5, 8, 9\}$

b. $(A \cup B \cup C)^C = \varnothing$

c. $(A \cap B \cap C)^C = U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

36. a. $A^c = \{2, 4, 6, 8, 10\}$ and $C^C = \{3, 6, 7, 10\}$. Therefore, $B \cap C^C = \{6, 10\}$ and so $A^C \cap (B \cap C^C) = \{6, 10\}$.

b. $A \cup B^C = \{1, 3, 5, 7, 9\}$, and $B \cap C^C = \{6, 10\}$, and $(A \cup B^C) \cup (B \cap C^C) = \{1, 3, 5, 6, 7, 9, 10\}$.

c. $(A \cup B) = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} = U$. Therefore, $(A \cup B)^C = \varnothing$ and so $(A \cup B)^C \cap C^C = \varnothing \cap C^C = \varnothing$

37. a. The sets are not disjoint. 4 is an element of both sets.
b. The sets are disjoint as they have no common elements.

38. a. The sets are disjoint as they do not contain any common elements.
b. The sets are not disjoint as 0 is an element of both sets.

39. a. The set of all employees at the Universal Life Insurance Company who do not drink tea.
b. The set of all employees at the Universal Life Insurance Company who do not drink coffee.

40. a. The set of all employees at the Universal Life Insurance Company who drink tea and/or coffee.
b. The set of all employees at the Universal Life Insurance Company who drink both tea and coffee.

41. a. The set of all employees at the Universal Life Insurance Company who drink tea but not coffee.
b. The set of all employees at the Universal Life Insurance Company who drink coffee but not tea.

42. a. The set of all employees at the Universal Life Insurance Company who drink neither tea nor coffee.
b. The set of all employees at the Universal Life Insurance Company who drink neither tea nor coffee.

43. a. The set of all employees at the hospital who are not doctors.
b. The set of all employees at the hospital who are not nurses.

44. a. The set of all employees at the hospital who are nurses and/or doctors.
b. The set of all employees at the hospital who are male nurses.

45. a. The set of all employees at the hospital who are female doctors.
b. The set of all employees at the hospital who are both doctors and administrators.

46. a. The set of all employees at the hospital who are female nurses.
b. The set of all employees at the hospital who are neither doctors nor nurses.

47. a. $D \cap F$ b. $R \cap F^C \cap L^C$ 48. a. $D \cap (F \cup L)$ b. $D^C \cup L$

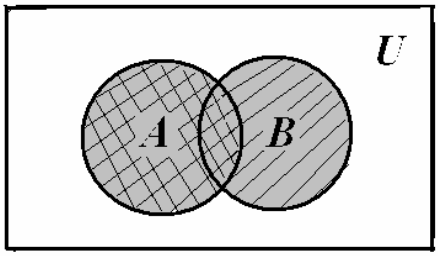
49. a. B^C b. $A \cap B$ c. $A \cap B \cap C^C$

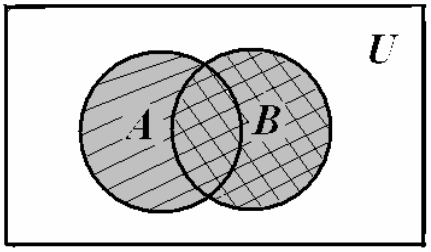
50. a. $A^C \cap B \cap C^C$ b. $A \cup B \cup C$ c. $A \cap B \cap C$

51. a. Region 1: $A \cap B \cap C$ is the set of tourists who used all three modes of transportation over a 1-week period in London.
b. Regions 1 and 4: $A \cap C$ is the set of tourists who have taken the underground and a bus over a 1-week period in London.
c. Regions 4, 5, 7, and 8: B^c is the set of tourists who have not taken a cab over a 1-week period in London.

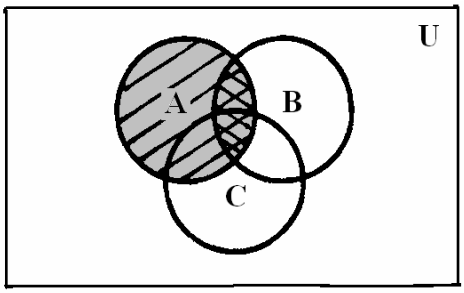
52. a. Region 3: $A \cap B \cap C^C$ is the set of tourists who have taken the underground and a
cab but not a bus over a 1-week period in London.
b. Regions 4 and 6: $(A \cap B^C \cap C) \cup (A^C \cap B \cap C^C)$ is the set of tourists who have taken the underground and a bus but have not taken a cab or those tourists who have taken a cab but have not taken the underground or a bus.
c. Regions 5, 6, and 7:
$$(C \cap A^C \cap B^C) \cup (B \cap C^C \cap A^C) \cup (A \cap B^C \cap C^C)$$
The set of tourists who have taken a bus only, a cab only, or the underground only.

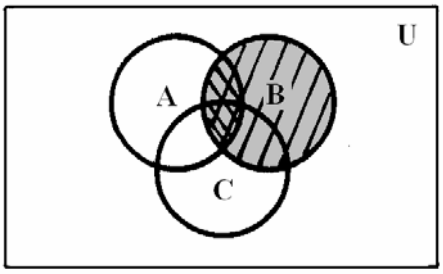
53. $A \subset A \cup B$ $B \subset A \cup B$



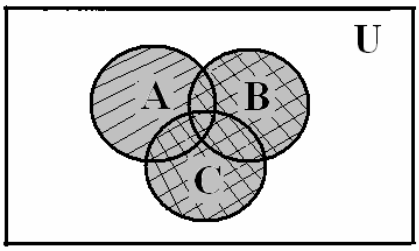


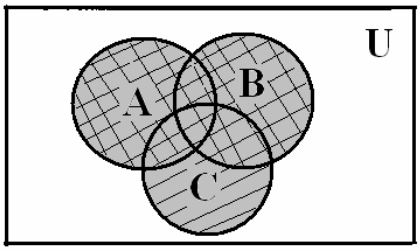
54. $A \cap B \subset A$ $A \cap B \subset B$



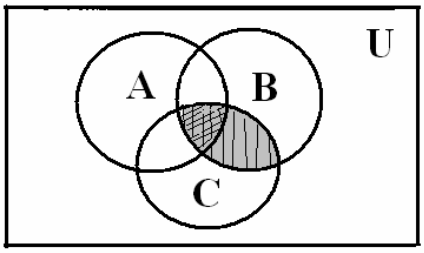


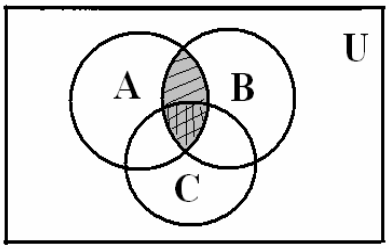
55. $A \cup (B \cup C) = (A \cup B) \cup C$



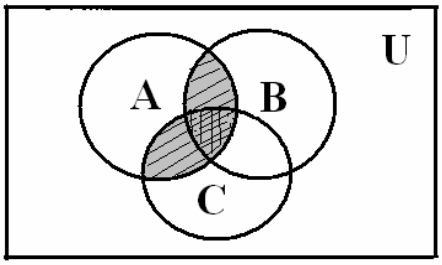


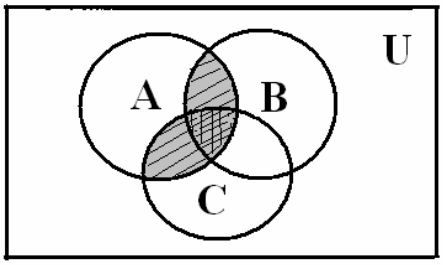
56. $A \cap (B \cap C) = (A \cap B) \cap C$



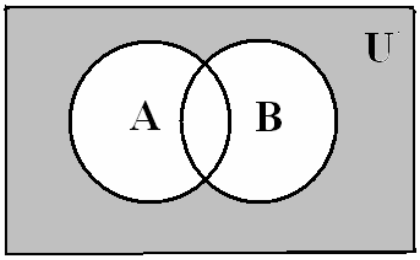


57. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$





58. $(A \cup B)^C = A^C \cap B^C$



59. a. $A \cup (B \cup C) = \{1, 3, 5, 7, 9\} \cup (\{1, 2, 4, 7, 8\} \cup \{2, 4, 6, 8\})$
$= \{1, 3, 5, 7, 9\} \cup \{1, 2, 4, 6, 7, 8\}$
$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
$(A \cup B) \cup C = (\{1, 3, 5, 7, 9\} \cup (\{1, 2, 4, 7, 8\}) \cup \{2, 4, 6, 8\})$
$= \{1, 2, 3, 4, 5, 7, 8, 9\} \cup \{2, 4, 6, 8\}$
$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

b. $A \cap (B \cap C) = \{1, 3, 5, 7, 9\} \cap (\{1, 2, 4, 7, 8\} \cap \{2, 4, 6, 8\})$
$= \{1, 3, 5, 7, 9\} \cap (\{2, 4, 8\}$
$= \varnothing$
$(A \cap B) \cap C = (\{1, 3, 5, 7, 9\} \cap \{1, 2, 4, 7, 8\}) \cap \{2, 4, 6, 8\}$
$= \{1, 7\} \cap \{2, 4, 6, 8\}$
$= \varnothing.$

60. a. $A \cap (B \cup C) = \{1, 3, 5, 7, 9\} \cap (\{1, 2, 4, 7, 8\} \cup \{2, 4, 6, 8\})$
$= \{1, 3, 5, 7, 9\} \cap \{1, 2, 4, 6, 7, 8\}$
$= \{1, 7\}.$

$(A \cap B) \cup (A \cap C) = (\{1, 3, 5, 7, 9\} \cap \{1, 2, 4, 7, 8\}) \cup$

$$(\{1, 3, 5, 7, 9\} \cap \{2, 4, 6, 8\})$$
$$= \{1, 7\} \cup \varnothing = \{1, 7\}.$$

b. $(A \cup B)^C = (\{1, 3, 5, 7, 9\} \cup \{1, 2, 4, 7, 8\})^C = \{1, 2, 3, 4, 5, 7, 8, 9\}^C$
$= \{6, 10\}.$
$A^C \cap B^C = \{1, 3, 5, 7, 9\}^C \cap \{1, 2, 4, 7, 8\}^C$
$= \{2, 4, 6, 8, 10\} \cap \{3, 5, 6, 9, 10\}$
$= \{6, 10\}.$

61. a. *r, u, v, w, x y* b. *v, r* 62. a. *r, u, v* b. *t, x, u, z, v, y ,s*

63. a. *t, y, s* b. *t ,s, w, x, z* 64. a. *v* b. *t, s*

65. $A \subset C$

66. If $A = B$, then $A \subseteq B$ and $B \subseteq A$. Next, if $A \subseteq B$ and $B \subseteq A$, then A and B contain exactly the same elements and so $A = B$.

67. False. Since every element in a set A belongs to A, A is a subset of itself.

68. True. Let $A = \{1, 2, 3\}$ and $B = \{1\}$. Then B is a proper subset of A, that is, of course, a subset of A. But A is not a proper subset of B.

69. True. If at least one of the sets A or B is nonempty, then $A \cup B \neq \varnothing$.

70. False. Take $A = \{1, 2\}$ and $B = \{3, 4\}$. 71. True. $(A \cup A^c)^c = U^c = \varnothing$.

72. True. Since $A \subseteq B$, every element in A is in B and so $A \cap B = A$.

6.2 CONCEPT QUESTIONS, page 326

1. a. If A and B are sets with $A \cap B = \varnothing$, then $n(A) + n(B) = n(A \cup B)$.
   b. If $n(A \cup B) \neq n(A) + n(B)$, then $A \cap B \neq \varnothing$.

2. If $n(A \cup B) = \varnothing$, then $n(B^c) = n(U) - n(B)$, and $n(A^c) = n(U) - n(A)$, and it follows that