5.2 CONCEPT QUESTIONS, page 282

1. The term is fixed. The periodic payments are of the same size. The payments are made at the end of the payment period. The payments coincide with the interest conversion periods.

2. In an ordinary annuity the payments are made at the end of each payment period, whereas, in an annuity due, the payments are made at the beginning of each period.

3. The future value S of an annuity of n payments of R dollars each, paid at the end of each investment period into an account that earns interest at the rate of i per period, is

$$S = R\left[\frac{(1+i)^n - 1}{i}\right].$$

An example is given by a retirement fund in which an employee makes a monthly deposit of a fixed amount for a certain period of time.

4. The present value P of an annuity of n payments of R dollars each, paid at the end of each investment period into an account that earns interest at the rate of i per period is

$$P = R\left[\frac{1-(1+i)^{-n}}{i}\right].$$

An example is given by the present value of a car loan at the time it is taken out.

EXERCISES 5.2, page 283

1. $S = 1000\left[\frac{(1+0.1)^{10} - 1}{0.1}\right] = 15{,}937.42$, or \$15,937.42.

2. $S = 1500\left[\frac{\left(1+\frac{0.09}{2}\right)^{16} - 1}{\frac{0.09}{2}}\right] \approx 34{,}079.01$, or \$34,079.01.

3. $S = 1800\left[\frac{\left(1+\frac{0.08}{4}\right)^{24} - 1}{\frac{0.08}{4}}\right] \approx 54{,}759.35$, or \$54,759.35.

4. $S = 500\left[\dfrac{\left(1+\dfrac{0.11}{2}\right)^{24}-1}{\dfrac{0.11}{2}}\right] \approx 23{,}769.00$, or \$23,769.00.

5. $S = 600\left[\dfrac{\left(1+\dfrac{0.12}{4}\right)^{36}-1}{\dfrac{0.12}{4}}\right] \approx 37{,}965.57$, or \$37,965.57.

6. $S = 150\left[\dfrac{\left(1+\dfrac{0.10}{12}\right)^{180}-1}{\dfrac{0.10}{12}}\right] \approx 62{,}170.55$, or \$62,170.55.

7. $S = 200\left[\dfrac{\left(1+\dfrac{0.09}{12}\right)^{243}-1}{\dfrac{0.09}{12}}\right] \approx 137{,}209.97$, or \$137,209.97.

8. $S = 100\left[\dfrac{\left(1+\dfrac{0.075}{52}\right)^{(\frac{15}{2}\cdot 52)}-1}{\dfrac{0.075}{52}}\right] \approx 52{,}301.15$, or \$52,301.15.

9. $P = 5000\left[\dfrac{1-(1+0.08)^{-8}}{0.08}\right] \approx 28{,}733.19$, or \$28,733.19.

10. $P = 1200\left[\dfrac{1-\left(1+\dfrac{0.10}{2}\right)^{-12}}{\dfrac{0.10}{2}}\right] \approx 10{,}635.90$, or \$10,635.90.

11. $P = 4000\left[\dfrac{1-(1+0.09)^{-5}}{0.09}\right] \approx$ 15,558.61, or $15,558.61.

12. $P = 3000\left[\dfrac{1-\left(1+\dfrac{0.11}{2}\right)^{-12}}{\dfrac{0.11}{2}}\right] \approx$ 25,855.55, or $25,855.55.

13. $P = 800\left[\dfrac{1-\left(1+\dfrac{0.12}{4}\right)^{-28}}{\dfrac{0.12}{4}}\right] \approx$ 15,011.29, or $15,011.29.

14. $P = 150\left[\dfrac{1-\left(1+\dfrac{0.08}{12}\right)^{-120}}{\dfrac{0.08}{12}}\right] \approx$ 12,363.22, or $12,363.22.

15. She will have $S = 1500\left[\dfrac{(1+0.08)^{25}-1}{0.08}\right] \approx$ 109,658.91, or $109,658.91.

16. Jackson will have $S = 100\left[\dfrac{\left(1+\dfrac{0.08}{12}\right)^{72}-1}{\dfrac{0.08}{12}}\right] \approx$ 9,202.53, or $9,202.53.

17. On October 31, Linda's account will be worth

$$S = 40\left[\dfrac{\left(1+\dfrac{0.07}{12}\right)^{11}-1}{\dfrac{0.07}{12}}\right] \approx 453.06, \text{ or } \$453.06.$$

One month later, this account will be worth $A = (453.06)\left(1+\frac{0.07}{12}\right) = 455.70$, or $455.70.

18. He will have an amount given by

$$S = 5000\left[\frac{(1+0.085)^{25}-1}{0.085}\right] \approx 393{,}338.96, \text{ or } \$393{,}338.96.$$

19. The amount in Colin's employee retirement account is given by

$$S = 100\left[\frac{\left(1+\frac{0.07}{12}\right)^{144}-1}{\frac{0.07}{12}}\right] \approx 22{,}469.50, \text{ or } \$22{,}469.50.$$

The amount in Colin's IRA is given by

$$S = 2000\left[\frac{(1+0.09)^{8}-1}{0.09}\right] \approx 22{,}056.95, \text{ or } \$22{,}056.95.$$

Therefore, the total amount in his retirement fund is given by
22,469.50 + 22,056.95 = 44,526.45, or $44,526.45.

20. They will have

$$S = 150\left[\frac{\left(1+\frac{0.08}{12}\right)^{36}-1}{\frac{0.08}{12}}\right] \approx 6{,}080.33, \text{ or } \$6080.33.$$

21. To find how much Karen has at age 65, we use formula (10) with $R = 150$, $i = \frac{r}{m} = \frac{0.05}{12}$, and $n = mt = (12)(40) = 480$, giving

$$S = 150\left[\frac{\left(1+\frac{0.05}{12}\right)^{480}-1}{\frac{0.05}{12}}\right] \approx 228{,}903.0235$$

or $228,903.02. To find how much Matt will have upon attaining the age of 65, use

Formula (10) with $R = 250$, $i = \frac{r}{m} = \frac{0.05}{12}$, and $n = mt = (12)(30) = 360$ giving

$$S = 250\left[\frac{\left(1+\frac{0.05}{12}\right)^{360} - 1}{\frac{0.05}{12}}\right] \approx 208{,}064.6588\text{, or \$208,064.66.}$$

So Karen will have the bigger nest egg.

22. The sum of \$150,000 will grow into $S_1 = 150{,}000\left(1+\frac{0.08}{4}\right)^{(4)(20)} \approx 731{,}315.873$.

The annuity will grow into

$$S_2 = R\left[\frac{(1+i)^{mt} - 1}{i}\right] = 3000\left[\frac{\left(1+\frac{0.08}{4}\right)^{80} - 1}{\frac{0.08}{4}}\right] \approx 581{,}315.873.$$

So Luis will have \$1,312,631.75 in his retirement account.

23. The equivalent cash payment is given by

$$P = 450\left[\frac{1-\left(1+\frac{0.09}{12}\right)^{-24}}{\frac{0.09}{12}}\right] \approx 9850.12\text{, or \$9850.12.}$$

24. We use the formula for the present value of an annuity obtaining

$$P = 420\left[\frac{1-\left(1+\frac{0.12}{12}\right)^{-36}}{\frac{0.12}{12}}\right] \approx 12{,}645.15\text{, or \$12,645.15.}$$

Since her down payment was \$4000, the cash price of the car was \$16,645.15.

25. We use the formula for the present value of an annuity obtaining

$$P = 22\left[\frac{1-\left(1+\frac{0.18}{12}\right)^{-36}}{\frac{0.18}{12}}\right] \approx 608.54, \text{ or } \$608.54.$$

26. We first find the present value of the 19 future payments. Thus,

$$P = 50{,}000\left[\frac{1-(1+0.08)^{-19}}{0.08}\right] \approx 480{,}179.96, \text{ or } \$480{,}179.96.$$

Therefore, the commission should have \$480,179.96 + \$50,000, or \$530,179.96 in the bank initially.

27. With an \$2400 monthly payment, the present value of their loan would be

$$P = 2400\left[\frac{1-\left(1+\frac{0.075}{12}\right)^{-360}}{\frac{0.075}{12}}\right] \approx 343{,}242.31, \text{ or } \$343{,}242.31.$$

With a \$3000 monthly payment, the present value of their loan would be

$$P = 3000\left[\frac{1-\left(1+\frac{0.075}{12}\right)^{-360}}{\frac{0.075}{12}}\right] \approx 429{,}052.88 \text{ or } \$429{,}052.88.$$

Since they intend to make a \$40,000 down payment, the range of homes they should consider is \$383,242 to \$469,053.

28. With a \$2400 monthly payment, the present value of their loan would be

$$P = 2400\left[\frac{1-\left(1+\frac{0.08}{12}\right)^{-360}}{\frac{0.08}{12}}\right] \approx 327{,}080.39, \text{ or } \$327{,}080.39.$$

With a \$3000 monthly payment, the present value of their loan would be

$$P = 3000\left[\frac{1-\left(1+\frac{0.08}{12}\right)^{-360}}{\frac{0.08}{12}}\right] \approx 408{,}850.48, \text{ or } \$408{,}850.48.$$

Since they intend to make a \$40,000 down payment, the range of homes they should consider is approximately \$367,080 to \$448,850.

29. The lower limit of their investment is

$$A = 2400\left[\frac{1-\left(1+\frac{0.07}{12}\right)^{-180}}{\frac{0.07}{12}}\right] + 40{,}000 \approx 307{,}014.30$$

or approximately \$307,104. The upper limit of their investment is

$$A = 3000\left[\frac{1-\left(1+\frac{0.07}{12}\right)^{-180}}{\frac{0.07}{12}}\right] + 40{,}000 \approx 373{,}767.87$$

or approximately \$373,768. Therefore, the price range of houses they should consider is \$307,014.30 to \$373,767.87.

30. The sum of \$5000 will grow into $5000\left(1+\frac{0.06}{12}\right)^{(12)(5)} \approx 6744.25$, or \$6744.25.

The annuity of \$200/month will grow into $200\left[\frac{\left(1+\frac{0.06}{12}\right)^{60}-1}{\frac{0.06}{12}}\right] \approx 13954.01$

or \$13,954.01. So Lauren's account will be worth 6744.25 + 13954.01, or approximately \$20,698.26.

31. The deposits of \$200/month into the bank account for a period of 2 years will grow into a sum of

$$A_1 = 200\frac{\left(1+\frac{0.06}{12}\right)^{24}-1}{\frac{0.06}{12}} \approx 5086.391, \quad \text{or } \$5086.39.$$

For the next 3 years, this amount will grow into a sum of

$$A_2 = A_1\left(1+\frac{0.06}{12}\right)^{36} \approx 6086.785, \text{ or } \$6086.79.$$

The deposits of $300/month will grow into a sum of

$$A_3 = 300\left[\frac{\left(1+\frac{0.06}{12}\right)^{36}-1}{\frac{0.06}{12}}\right] \approx 11800.831, \text{ or } \$11{,}800.83.$$

Therefore, at the end of 5 years. He will have

$$A_2 + A_3 = 6086.785 + 11800.831 \approx 17887.616, \quad \text{or approximately } \$17{,}887.62.$$

32. By age 40, Jessica will have

$$200\left[\frac{\left(1+\frac{0.05}{12}\right)^{(15)(12)}-1}{\frac{0.05}{12}}\right] = 53,457.78864,, \text{ or approximately } \$53{,}457.79.$$

So by age 65, Jessica will have $53,457.79\left(1+\frac{0.05}{12}\right)^{(25)(12)} \approx 186{,}102.09$

or approximately $186,102.09. By age 65, Alex will have

$$300\left[\frac{\left(1+\frac{0.05}{12}\right)^{(25)(12)}-1}{\frac{0.05}{12}}\right] \approx 178,652.91 \text{ or approximately } \$178{,}652.91.$$

So Jessica will end up with a bigger nest egg.

33. False. This statement would be true only if the interest rate is equal to zero.

34. True. See the formula in the text.

USING TECHNOLOGY EXERCISES 5.2, page 287

1. $59,622.15	2. $55,718.57	3. $8453.59	4. $20,460.98
5. $35,607.23	6. $45,983.53	7. $13,828.60	8. $18,344.08