CHAPTER 5

5.1 CONCEPT QUESTIONS, page 269

1. In simple interest, the interest is based on the original principal. In compound interest, interest earned is periodically added to the principal and thereafter earns interest at the same rate.

2. The accumulated amount of an investment is the sum of the principal and interest after a certain time period, and the present value of an investment is the amount of money that needs to be invested now so that a certain sum can be realized at some future date.

3. The effective rate of interest is the simple interest that would produce the same amount in 1 year as the nominal rate compounded m times a year.

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1. The interest is given by $I = (500)(2)(0.08) = 80$, or \$80.
   The accumulated amount is 500 + 80, or \$580.

2. The interest is given by $I = (1000)(3)(0.05) = 150$, or \$150.
   The accumulated amount is 1000 + 150, or \$1150.

3. The interest is given by $I = (800)(0.06)(0.75) = 36$, or \$36.
   The accumulated amount is 800 + 36, or \$836.

4. The interest is given by
   $$I = (1200)(\frac{7}{100})(\frac{2}{3}) = 56, \quad \text{or } \$56.$$
   The accumulated amount is 1200 + 56, or \$1256.

5. We are given that $A = 1160$, $t = 2$, and $r = 0.08$, and we are asked to find P. Since
   $$A = P(1 + rt)$$
   we see that
   $$P = \frac{A}{1+rt} = \frac{1160}{1+(0.08)(2)} = 1000, \text{ or } \$1000.$$

6. We are given that $A = 3100$, $t = 10/12$, and $r = 0.05$, and we are asked to find P. Since

$$A = P(1 + rt)$$

we see that $$P = \frac{A}{1+rt} = \frac{3100}{1+(0.05)(\frac{5}{6})} = 2976, \text{ or } \$2976.$$

7. We use the formula $I = Prt$ and solve for t when $I = 20$, $P = 1000$, and $r = 0.05$. Thus,

$$20 = 1000(0.05)(\frac{t}{365}), \text{ and } \quad t = \frac{365(20)}{50} = 146, \quad \text{or 146 days.}$$

8. We use the formula $I = Prt$ and solve for t when $I = 25$, $P = 1500$, and $r = 0.05$.

Thus, $$25 = 1500(0.05)(\frac{t}{365})$$

and $$t = \frac{365(25)}{75} \approx 122, \quad \text{or 122 days.}$$

9. We use the formula $A = P(1 + rt)$ with $A = 1075$, $P = 1000$, $t = 0.75$, and solve for r. Thus,

$$1075 = 1000(1 + 0.75r)$$
$$75 = 750r$$

or $$r = 0.10.$$

Therefore, the interest rate is 10 percent per year.

10. We use the formula $A = P(1 + rt)$ with $A = 1250$, $P = 1200$, $t = 2/3$, and solve for r.

Thus $$1250 = 1200(1 + \frac{2}{3}r)$$
$$50 = 800r$$

or $$r = 0.0625.$$

Therefore, the interest rate is 6.25 percent per year.

11. $A = 1000(1 + 0.07)^8 \approx 1718.19$, or \$1718.19.

12. $A = 1000(1 + 0.085)^6 \approx 1631.47$, or \$1631.47.

13. $$A = 2500\left(1+\frac{0.07}{2}\right)^{20} \approx 4974.47, \quad \text{or } \$4974.47.$$

14. $$A = 2500\left(1+\frac{0.09}{2}\right)^{21} \approx 6300.60, \quad \text{or } \$6300.60.$$

15. $A = 12,000\left(1+\frac{0.08}{4}\right)^{42} \approx 27,566.93$, or \$27,566.93.

16. $A = 42,000\left(1+\frac{0.0775}{4}\right)^{32} \approx 77,613.38$, or \$77,613.38.

17. $A = 150,000\left(1+\frac{0.14}{12}\right)^{48} \approx 261,751.04$, or \$261,751.04.

18. $A = 180,000\left(1+\frac{0.09}{12}\right)^{75} \approx 315,247.47$, or \$315,247.47.

19. $A = 150,000\left(1+\frac{0.12}{365}\right)^{1095} = 214,986.69$, or \$214,986.69.

20. $A = 200,000\left(1+\frac{0.08}{365}\right)^{4(365)} = 275,416$, or \$275,416.

21. Using the formula

$$r_{eff} = \left(1+\frac{r}{m}\right)^{m} - 1$$

with r = 0.10 and m = 2, we have

$$r_{eff} = \left(1+\frac{0.10}{2}\right)^{2} - 1 = 0.1025, \quad \text{or 10.25 percent.}$$

22. Using the formula $r_{eff} = \left(1+\frac{r}{m}\right)^{m} - 1$

with r = 0.09 and m = 4, we have

$$r_{eff} = \left(1+\frac{0.09}{4}\right)^{4} - 1 \approx 0.09308, \text{ or 9.308 percent.}$$

23. Using the formula $r_{eff} = \left(1+\frac{r}{m}\right)^{m} - 1$

with r = 0.08 and m = 12, we have

$$r_{eff} = \left(1+\frac{0.08}{12}\right)^{12} - 1 \approx 0.08300, \quad \text{or 8.3 percent per year.}$$

24. The effective rate is given by $R = \left(1+\frac{0.08}{365}\right)^{365} - 1 = 0.08328$, or 8.328 percent per year.

25. The present value is given by $P = 40,000\left(1+\frac{0.08}{2}\right)^{-8} \approx 29,227.61$, or \$29,227.61.

26. The present value is given by $P = 40,000\left(1+\frac{0.08}{4}\right)^{-16} \approx 29,137.83$ or \$29,137.83.

27. The present value is given by
$$P = 40,000\left(1+\frac{0.07}{12}\right)^{-48} \approx 30,255.95, \quad \text{or \$30,255.95.}$$

28. The present value is given by $P = 40,000\left(1+\frac{0.09}{365}\right)^{-(365)(4)} \approx 27,908.29$, or \$27,908.29.

29. $A = 5000e^{0.08(4)} \approx 6885.64$, or \$6885.64.

30. $A = 6500e^{.07(6)} \approx 9892.75$, or approximately \$9892.75

31. Think of \$300 as the principal and \$306 as the accumulated amount at the end of 30 days. If r denotes the simple interest rate per annum, then we have $P = 300$, $A = 306$, $t = 1/12$, and we are required to find r. Using (1b) we have
$$306 = 300\left(1+\frac{r}{12}\right) = 300 + r\left(\frac{300}{12}\right)$$
and $\quad r = \left(\frac{12}{300}\right)6 = 0.24$, or 24 percent per year.

32. She would receive $A = P(1+rt) = 150,000[1+(0.12)(5)] = 240,000$, or \$240,000.

33. The Abdullahs will owe $A = P(1+rt) = 120,000[1+(0.12)(\frac{3}{12})] = 123,600$, or

$123,600.

34. Every six months, David will receive $Prt = 20,000(0.07)(\frac{1}{2}) = 700$, or \$700. Over the term, David will receive 20(700) or 14,000, or \$14,000.

35. Here P = 10,000, I = 3500, and t = 7, and so from Formula (1a), we have

$$3500 = 10000(r)7 \quad \text{and so} \quad r = \frac{3500}{70000} = 0.05.$$

So the bond pays simple interest at the rate of 5% per year.

36. Equation (1b), $A = P(1 + rt)$, may be rewritten in the form

$$A = (Pr)t + P \qquad \text{(Think of } A \text{ as a function of } t.)$$

This equation is in the slope-intercept form. The slope of the equation is equal to Pr and the A-intercept is equal to P.

37. The rate that you would expect to pay is

$$A = 580(1 + 0.08)^5 \approx 852.21, \text{ or \$852.21 per day.}$$

38. The amount that the typical family of four would expect to pay for food six years from now is

$$A = 600(1 + 0.03)^6 \approx 716.43, \text{ or \$716.43.}$$

39. The amount that they can expect to pay is given by

$$A = 210{,}000(1 + 0.05)^4 \approx 255{,}256, \text{ or approximately \$255,256.}$$

40. The generating capacity at the end of the decade will have to be

$$A = P(1.08)^{10} = 2.1589P,$$

or 215.89 percent of the current generating capacity. Therefore, the utility company will have to increase its generating capacity by an amount of 115.89 percent of its current generating capacity.

41. The investment will be worth $A = 1.5\left(1 + \frac{0.055}{2}\right)^{20} = 2.58064$, or approximately \$2.58 million.

42. We use Formula (3) with P = 10,000, r = 0.1082, m = 4 and $t = \frac{11}{2}$ giving the worth of Chris' account as

$$A = 10{,}000\left(1 + \frac{0.1082}{4}\right)^{(4)(\frac{11}{2})} \approx 17{,}989.327 \text{ or approximately \$17,989.33}$$

43. We use Formula (3) with P = 15,000, r = 0.098, m = 12, and t = 4 giving the worth of Jodie's account as

$$A = 15,000\left(1+\frac{0.098}{12}\right)^{(12)(4)} \approx 22,163.753, \text{ or approximately } \$22,163.75.$$

44. If the money has earned interest at the rate of 8 percent compounded annually, he will receive $A = (1.08)^{21}(10,000) \approx 50,338.34$, or \$50,338.34.
If the money has earned interest at the rate of 8 percent compounded quarterly, he will receive $A = \left(1+\frac{0.08}{4}\right)^{4(21)}(10,000) \approx 52,773.32$, or \$52,773.32.
If the money has earned interest at the rate of 8 percent compounded monthly, he will receive $A = \left(1+\frac{0.08}{12}\right)^{12(21)}(10,000) \approx 53,357.25$, or \$53,357.25.

45. Using the formula $P = A\left(1+\frac{r}{m}\right)^{-mt}$, we have

$$P = 40,000\left(1+\frac{0.085}{4}\right)^{-20} \approx 26,267.49, \text{ or } \$26,267.49.$$

46. Using the formula $P = A\left(1+\frac{r}{m}\right)^{-mt}$, we see that she purchased the note for

$$P = 10,000\left(1+\frac{0.085}{2}\right)^{-8} \approx 7167.89, \text{ or } \$7167.89.$$

47. a. They should set aside

$$P = 100,000(1 + 0.085)^{-13} \approx 34,626.88, \text{ or } \$34,626.88.$$

b. They should set aside

$$P = 100,000\left(1+\frac{0.085}{2}\right)^{-26} \approx 33,886.16, \text{ or } \$33,886.16.$$

c. They should set aside

$$P = 100,000\left(1+\frac{0.085}{4}\right)^{-52} \approx 33,506.76, \text{ or } \$33,506.76.$$

48. Anthony originally invested

$$P = 22,289.22\left(1+\frac{0.08}{4}\right)^{-20} \approx 15,000, \text{ or } \$15,000.$$

49. The effective rate of interest for the Bendix Mutual Fund is

$$r_{eff} = \left(1+\frac{0.104}{4}\right)^{4} - 1 \approx 0.1081 \quad \text{or} \quad 10.81\%/\text{yr}$$

whereas the effective rate of interest for the Acme Mutual fund is

$$r_{eff} = \left(1+\frac{0.106}{2}\right)^{4} - 1 \approx 0.1088 \quad \text{or} \quad 10.88\%/\text{yr}.$$

We conclude that the Acme Mutual Fund has a better rate of return.

50. The effective rate of interest for the Fleet Street Savings Bank is

$$r_{eff} = \left(1+\frac{0.0425}{52}\right)^{52} - 1 \approx 0.0434 \quad \text{or} \quad 4.34\%/\text{yr}$$

whereas the effective rate of interest for the Washington Bank is

$$r_{eff} = \left(1+\frac{0.04125}{365}\right)^{365} - 1 \approx 0.0421 \quad \text{or} \quad 4.21\%/\text{yr}.$$

Thus, the Fleet Savings Bank offers a better rate of interest.

51. The present value of the \$8000 loan due in 3 years is given by

$$P = 8000\left(1+\frac{0.10}{2}\right)^{-6} = 5969.72, \text{ or } \$5969.72.$$

The present value of the \$15,000 loan due in 6 years is given by

$$P = 15,000\left(1+\frac{0.10}{2}\right)^{-12} = 8352.56, \text{ or } \$8352.56.$$

Therefore, the amount the proprietors of the inn will be required to pay at the end of 5 years is given by

$$A = 14,322.28\left(1+\frac{0.10}{2}\right)^{10} = 23,329.48, \quad \text{or } \$23,329.48.$$

52. To find the effective rate of interest corresponding to a nominal rate of 9%/yr compounded annually, we use the formula $r_{eff} = \left(1+\frac{r}{m}\right)^{m} - 1$, with $r = 0.09$ and

$m = 1$. We have

$$r_{eff} = (1 + 0.09)^1 - 1 = 0.09, \text{ or 9 percent per year.}$$

To find the effective rate of interest corresponding to a nominal rate of 9%/yr compounded semiannually, we use the formula $r_{eff} = \left(1+\frac{r}{m}\right)^m - 1$ with $r = 0.09$ and $m = 2$. We have

$$r_{eff} = \left(1+\frac{0.09}{2}\right)^2 - 1 \approx 0.092025, \text{ or 9.2025 percent per annum.}$$

To find the effective rate of interest corresponding to a nominal rate of 9%/yr compounded quarterly, we compute $r_{eff} = \left(1+\frac{0.09}{4}\right)^4 - 1 \approx 0.093083$, or 9.3083 percent per annum. To find the effective rate of interest corresponding to a nominal rate of 9%/yr compounded monthly, we compute

$$r_{eff} = \left(1+\frac{0.09}{12}\right)^{12} - 1 \approx 0.093807, \text{ or 9.3807 percent per annum.}$$

53. Let $A = 10{,}000$, $r = 0.0525$, and $t = 10$. Using Formula (7), we have

$$P = 10000(1+0.0525)^{-10} \approx 5994.86$$

Thus, Juan should pay $5994.86 for the bond.

54. His revenue in 2006 was (1.2)(240,000); his revenue in 2007 was (1.2)(1.3)(240,000). So, his revenue in 2010 will be at least $(1.2)(1.3)(1.25)^3(240{,}000) = 731{,}250$ or $731,250.

55. The projected online retail sales for 2008 are

$$(1.332)(1.278)(1.305)(1.199)(1.243)(1.14)(1.176)(1.105)(23.5)$$
$$\approx 115.26, \text{ or approximately \$115.3 billion.}$$

56. We want the value of a 2006 dollar in the year 2003. Denoting this value by x, we have

$$(1.016)(1.023)(1.027)(1.034)x = 1$$

or $x \approx 0.9060$. Thus, the purchasing power is approximately 91 cents.

57. Suppose \$1 is invested in each investment.

Investment A: Accumulated amount is $\left(1+\frac{0.1}{2}\right)^8 \approx 1.47746$.

Investment B: Accumulated amount is $e^{0.0975(4)} \approx 1.47698$.

So Investment A has a higher rate of return.

58. He needs $45,000e^{0.03(10)} \approx 60,743.65$ or approximately \$60,744 per year.

59. If they invest the money at 10.5 percent compounded quarterly, they should set aside $P = 70,000\left(1+\frac{0.105}{4}\right)^{-28} \approx 33,885.14$, or \$33,885.14.

If they invest the money at 10.5 percent compounded continuously, they should set aside $P = 70{,}000e^{-0.735} = 33{,}565.38$, or \$33,565.38.

60. a. If inflation over the next 15 years is 6 percent, then Maria's first year's pension will be worth $P = 40{,}000e^{-0.9} = 16{,}262.79$, or \$16,262.79.

b. If inflation over the next 15 years is 8 percent, then Maria's first year's pension will be worth $P = 40{,}000e^{-1.2} = 12{,}047.77$, or \$12,047.77.

c. If inflation over the next 15 years is 12 percent, then Maria's first year's pension will be worth $P = 40{,}000e^{-1.8} = 6611.96$, or \$6,611.96.

61. $P(t) = V(t)e^{-rt} = 80{,}000e^{\sqrt{t}/2}e^{-rt} = 80{,}000e^{(\sqrt{t}/2-0.09t)}$

$P(4) = 80{,}000e^{1-0.09(4)} \approx 151{,}718.47$, or approximately \$151,718.

62. a. We obtain a family of straight lines with varying slope and P-intercept as P increases. For a fixed rate of interest, the accumulated amount A grows at the rate of Pr units per year starting initially with an amount of \$$P$.

b. We obtain a family of straight lines emanating from the point (0, P) and with varying slope as r increases. For a fixed principal, the accumulated amount A grows at the rate Pr units per year starting initially with an amount of \$$P$.

63. By definition, $A = P(1+r_{\text{eff}})^t$. So

$$(1+r_{\text{eff}})^t = \frac{A}{P}, \quad 1+r_{\text{eff}} = \left(\frac{A}{P}\right)^{1/t} \text{ and } \quad r_{eff} = \left(\frac{A}{P}\right)^{1/t} - 1.$$

64. According to the result of Exercise 63, $r_{\text{eff}} = (A/P)^{1/t} - 1$. Here $P = 40,000$, $A = 70,000$, and $t = 5$. So, the required effective rate is

$$r_{\text{eff}} = \left(\frac{70,000}{40,000}\right)^{1/5} - 1 \approx 11.84$$

or 11.84%.

65. Using the formula $r_{eff} = \left(\frac{A}{P}\right)^{1/t} - 1$ with A = 256,000, P = 200,000, and t = 6, we have

$$r_{eff} = \left(\frac{250,000}{200,000}\right)^{1/6} - 1 \approx 0.042$$

or 4.2%.

66. Using the formula $r_{eff} = \left(\frac{A}{P}\right)^{1/t} - 1$ with A = 32,100, P = 25,250, and t = 2, we have

$$r_{eff} = \left(\frac{32,100}{25,250}\right)^{1/2} - 1 \approx 0.1275$$

or 12.75%.

67. Using the formula $r_{eff} = \left(\frac{A}{P}\right)^{1/t} - 1$ with A = 10,000, P = 6,724.53, and t = 7, we have

$$r_{eff} = \left(\frac{10,000}{6,724.53}\right)^{1/7} - 1 \approx 0.0583$$

or 5.83%.

68. Using the formula $r_{eff} = \left(\frac{A}{P}\right)^{1/t} - 1$ with A = 5170.42, P = 5000, and t = 245/365, we have

$$r_{eff} = \left(\frac{5170.42}{5000}\right)^{1/(245/365)} - 1 = \left(\frac{5170.42}{5000}\right)^{(365/245)} - 1 \approx 0.0512$$

or 5.12%.

69. True. $A = P(1+rt) = Prt$ is a linear function of t.

70. False. Under compound interest $A = P(1+r)^t$ (m = 1), whereas under simple

interest, $A = P(1 + rt)$.

71. True. With $m = 1$, the effective rate is $r_{eff} = \left(1+\frac{r}{1}\right)^1 - 1 = r$.

72. False. If Susan had gotten annual increases of 5 percent over 5 years, her salary would have been $A = 50,000(1+0.05)^5 = 63,814.08$, or approximately \$63,814 after 5 years and not \$60,000.

73. We use formula (3) with $A = 6500$, $P = 5000$, $m = 12$, and $r = 0.12$. Thus

$$6500 = 5000\left(1+\tfrac{0.12}{12}\right)^{12t};\ \ (1.01)^{12t} = \tfrac{6500}{5000} = 1.3;\ \ 12t\ln(1.01) = \ln 1.3;$$

$$t = \frac{\ln 1.3}{12\ln 1.01} \approx 2.197$$

So, it will take approximately 2.2 years.

74. We use formula (3) with $A = 15,000$, $P = 12,000$, $m = 12$, and $r = 0.08$. Thus,

$$15,000 = 12,000\left(1+\tfrac{0.08}{12}\right)^{12t};\ (1+\tfrac{0.08}{12})^{12t} = \tfrac{15,000}{12,000} = 1.25;\ 12t\ln(1+\tfrac{0.08}{12}) = \ln 1.25;$$

$$t = \frac{\ln 1.25}{12\ln(1+\frac{0.08}{12})} \approx 2.799$$

So it will take approximately 2.8 years.

75. We use formula (3) with $A = 4000$, $P = 2000$, $m = 12$, and $r = 0.09$. Thus,

$$4000 = 2000\left(1+\tfrac{0.09}{12}\right)^{12t};\ \left(1+\tfrac{0.09}{12}\right)^{12t} = 2;\ 12\text{t}\ \ln\left(1+\tfrac{0.09}{12}\right) = \ln 2 \text{ and}$$

$$t = \frac{\ln 2}{12\ln\left(1+\frac{0.09}{12}\right)} \approx 7.73.$$

So it will take approximately 7.7 years.

76. We use formula (3) with $A = 15000$, $P = 5000$, $m = 365$, and $r = 0.08$. Thus

$$15000 = 5000\left(1+\tfrac{0.08}{365}\right)^{365t} \quad \text{to obtain} \quad t = \frac{\ln\left(\frac{15000}{5000}\right)}{365\ln\left(1+\frac{0.08}{365}\right)} \approx 13.73.$$

So, it will take approximately 13.7 years.

77. We use formula (5) with $A = 6000$, $P = 5000$, and $t = 3$. Thus,

$$6000 = 5000e^{3r}$$

$$e^{3r} = \frac{6000}{5000} = 1.2;$$

Next, taking the logarithm of each side of the equation, we have

$3r = \ln 1.2$ [The natural logarithm of e^{3r} is $3r$.]

$r = \frac{\ln 1.2}{3} \approx 0.6077$

So the interest rate is 6.08% per year.

78. We use formula (5) with $A = 8000$, $P = 4000$, and $t = 5$. Thus

$$8000 = 4000e^{5r}$$

obtaining $r = \frac{\ln\left(\frac{8000}{4000}\right)}{5} \approx 0.13863$. So the interest rate is 13.86% per year.

79. We use formula (5) with $A = 7000$, $P = 6000$, and $r = 0.075$. Thus

$$7000 = 6000e^{0.075t}$$

$$e^{0.075t} = \frac{7000}{6000} = \frac{7}{6}$$

Next, taking the logarithm of each side, we have

$0.075t \ln e = \ln \frac{7}{6}$ [The natural logarithm of $e^{0.075t}$ is $0.075t$.]

and $t = \frac{\ln \frac{7}{6}}{0.075} \approx 2.055$.

So, it will take 2.06 years.

80. We use formula (5) with $A = 16{,}000$, $P = 8000$, and $r = 0.08$. Thus

$$16{,}000 = 8000e^{0.08t}$$

obtaining $t = \frac{\ln 2}{0.08} \approx 8.664$. So, it will take 8.7 years.

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1. \$5872.78 2. \$3,712.07 3. \$475.49 4. \$567.35 5. 8.95%/yr

6. 11.158% 7. 10.20%/yr 8. 4.447% 9. \$29,743.30 10. \$94,038.74

11. \$53,303.25 12. \$62,244.96