CHAPTER 4

4.1 CONCEPT QUESTIONS, page 217

1. a. The objective function is to be maximized.
   b. All the variables involved in the problem are nonnegative.
   c. Each linear constraint may be written so that the expression involving the variables is less than or equal to a nonnegative constant.
2. a. It is transformed into a system of linear equations by the use of slack variables, if required. The objective function is rewritten in the form
   $-c_1x_2 - c_2x_2 - \cdots - c_nx_n + P = 0$.
   b. The optimal solution has been reached if all the entries in the last row to the left of the vertical line are nonnegative.
3. To find the *pivot column* locate the most negative entry to the left of the vertical line in the last row. The column containing this entry is the pivot column. To find the *pivot row*, divide each positive entry in the pivot column into its corresponding entry in the column of constants. The pivot row is the row corresponding to the smallest ratio thus obtained. The *pivot element* is the element common to both the pivot column and the pivot row.

EXERCISES 4.1, page 217

1. All entries in the last row of the simplex tableau are nonnegative and an optimal solution has been reached. We find

$$x = 30/7,\ y = 20/7,\ u = 0,\ v = 0,\ \text{ and }\ P = 220/7.$$

2. All entries in the last row of the simplex tableau are nonnegative and an optimal solution has been reached. We find

$$x = 0,\ y = 6,\ u = 0,\ v = 2,\ \text{and}\ P = 30.$$

3. The simplex tableau is not in final form because there is an entry in the last row that is negative. The entry in the first row, second column, is the next pivot element and has a value of 1/2.

4. All entries in the last row of the simplex tableau are nonnegative and an optimal solution has been reached. We find

$$x = 0,\ y = 16,\ z = 0,\ u = 28,\ v = 0,\ \text{and}\ P = 48.$$

5. The simplex tableau is in final form. We find

$$x = 1/3,\ y = 0,\ z = 13/3,\ u = 0,\ v = 6,\ w = 0 \text{ and } P = 17.$$

6. The simplex tableau is not in final form because there is an entry in the last row that is negative. The entry in the third row, first column, is the pivot element and has a value of 2.

7. The simplex tableau is not in final form because there are two entries in the last row that are negative. The entry in the third row, second column, is the pivot element and has a value of 1.

8. The simplex tableau is not in final form because there is an entry in the last row that is negative. The entry in the second row, sixth column, is the pivot element and has a value of 6/5.

9. The simplex tableau is in final form. The solutions are

$$x = 30,\ y = 10,\ z = 0,\ u = 0,\ v = 0, \text{ and } P = 60$$

and

$$x = 30,\ y = 0,\ z = 0,\ u = 10,\ v = 0, \text{ and } P = 60.$$

(There are infinitely many solutions.)

10. The simplex tableau is not in final form because there is an entry in the last row that is negative. The entry in the third row, first column is the pivot element and has a value of 2.

11. We obtain the following sequence of tableaus:

	x	y	u	v	P	Const	Ratio
$p.r. \rightarrow$	1	[1]	1	0	0	4	4
	2	1	0	1	0	5	5
	–3	–4	0	0	1	0	
		↑ $p.c.$					

$\xrightarrow[R_3+4R_1]{R_2-R_1}$

x	y	u	v	P	Const.
1	1	1	0	0	4
1	0	–1	1	0	1
1	0	4	0	1	16

The last tableau is in final form and we conclude that $x = 0$, $y = 4$, $u = 0$, $v = 1$, and $P = 16$.

12. The initial tableau is

x	y	u	v	P	Const.
1	1	1	0	0	80
3	0	0	1	0	90
–5	–3	0	0	1	0

Using the sequence of operations
1. $\frac{1}{3}R_2$ 2. $R_1 - R_2$; $R_3 + 5R_2$ 3. $R_3 + 3R_1$
we obtain the final tableau

x	y	u	v	P	Const
0	1	1	$-\frac{1}{3}$	0	50
1	0	0	$\frac{1}{3}$	0	30
0	0	3	$\frac{2}{3}$	1	300

from which we conclude that $x = 30$, $y = 50$, $u = 0$, $v = 0$, and $P = 300$.

13.

	x	y	u	v	P	Const	Ratio
p.r. →	1	[2]	1	0	0	12	12 / 2 = 6
	3	2	0	1	0	24	24 / 2 = 12
	–10	–12	0	0	1	0	
		↑ p.c.					

$\xrightarrow{\frac{1}{2}R_1}$

x	y	u	v	P	Const
$\frac{1}{2}$	1	$\frac{1}{2}$	0	0	6
3	2	0	1	0	24
–10	–12	0	0	1	0

$\xrightarrow[R_3+12R_1]{R_2-2R_1}$

	x	y	u	v	P	Const
	$\frac{1}{2}$	1	$\frac{1}{2}$	0	0	6
p.r. →	[2]	0	–1	1	0	12
	–4	0	6	0	1	72
	↑ p.c.					

Ratio
$6/(1/2)=12$
$12/2=6$

$\xrightarrow{\frac{1}{2}R_2}$

x	y	u	v	P	Const
$\frac{1}{2}$	1	$\frac{1}{2}$	0	0	6
1	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	6
–4	0	6	0	1	72

$\xrightarrow[R_3+4R_2]{R_1-\frac{1}{2}R_2}$

x	y	u	v	P	Const
0	1	$\frac{3}{4}$	$-\frac{1}{4}$	0	3
1	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	6
0	0	4	2	1	96

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The last tableau is in final form. We find that $x = 6$, $y = 3$, $u = 0$, $v = 0$, and $P = 96$.

14. Starting with the initial tableau

	x	y	u	v	P	Const
	3	5	1	0	0	78
p.r. →	[4]	1	0	1	0	36
	–5	–4	0	0	1	0
	↑ p.c.					

Ratio
$78/3=26$
$36/4=9$

we use the sequence of row operations

1. $\frac{1}{4}R_2$ 2. R_1 - $3R_2$; $R_3 + 5R_2$ 3. $\frac{4}{17}R_1$

4. $R_2 - \frac{1}{4}R_1$; $R_3 + \frac{11}{4}R_1$

to obtain the final tableau

x	y	u	v	P	Const.
0	1	$\frac{4}{17}$	$-\frac{3}{17}$	0	12
1	0	$-\frac{1}{17}$	$\frac{5}{17}$	0	6
0	0	$\frac{11}{17}$	$\frac{13}{17}$	1	78

and conclude that $x = 6$, $y = 12$, $u = 0$, $v = 0$, and $P = 78$.

15. We obtain the following sequence of tableaus:

	x	y	u	v	w	P	Const.	Ratio
	3	1	1	0	0	0	24	24
	2	1	0	1	0	0	18	18
p.r. →	1	[3]	0	0	1	0	24	8
	–4	–6	0	0	0	1	0	
		↑ p.c.						

$\xrightarrow{\frac{1}{3}R_3}$

x	y	u	v	w	P	Const.
3	1	1	0	0	0	24
2	1	0	1	0	0	18
$\frac{1}{3}$	1	0	0	$\frac{1}{3}$	0	8
–4	–6	0	0	0	1	0

$\xrightarrow[R_4+6R_3]{R_1-R_3 \quad R_2-R_3}$

	x	y	u	v	w	P	Const.	Ratio
p.r. →	[$\frac{8}{3}$]	0	1	0	$-\frac{1}{3}$	0	16	6
	$\frac{5}{3}$	0	0	1	$-\frac{1}{3}$	0	10	6
	$\frac{1}{3}$	1	0	0	$\frac{1}{3}$	0	8	24
	–2	0	0	0	2	1	48	
	↑ p.c.							

(Observe that we have a choice here.)

$\xrightarrow{\frac{3}{8}R_1}$

x	y	u	v	w	P	Const.
1	0	$\frac{3}{8}$	0	$-\frac{1}{8}$	0	6
$\frac{5}{3}$	0	0	1	$-\frac{1}{3}$	0	10
$\frac{1}{3}$	1	0	0	$\frac{1}{3}$	0	8
–2	0	0	0	2	1	48

$\xrightarrow[R_4+2R_1]{R_2-\frac{5}{3}R_1 \quad R_3-\frac{1}{3}R_1}$

x	y	u	v	w	P	Const.
1	0	$\frac{3}{8}$	0	$-\frac{1}{8}$	0	6
0	0	$-\frac{5}{8}$	1	$-\frac{1}{8}$	0	0
0	1	$-\frac{1}{8}$	0	$\frac{3}{8}$	0	6
0	0	$\frac{3}{4}$	0	$\frac{7}{4}$	1	60

We deduce that $x = 6$, $y = 6$, $u = 0$, $v = 0$, $w = 0$, and $P = 60$.

16. The initial simplex tableau is

x	y	u	v	w	P	Const.
1	1	1	0	0	0	12
3	1	0	1	0	0	30
10	7	0	0	1	0	70
−15	−12	0	0	0	1	0

Using the sequence of row operations

1. $\frac{1}{10}R_3$ 2. R_1 - R_3; R_2 - $3R_3$; $R_4 + 15R_3$ 3. $\frac{10}{7}R_3$

4. $R_1 - \frac{3}{10}R_3$; $R_2 + \frac{11}{10}R_3$; $R_4 + \frac{3}{2}R_3$

we obtain the final simplex tableau

x	y	u	v	w	P	Const.
0	0	1	0	$-\frac{1}{7}$	0	2
$\frac{11}{7}$	0	0	1	$-\frac{1}{7}$	0	20
$\frac{10}{7}$	1	0	0	$\frac{1}{7}$	0	10
$\frac{15}{7}$	0	0	0	$\frac{12}{7}$	1	120

from which we deduce that $x = 0$, $y = 10$, $u = 2$, $v = 20$, and $P = 120$.

17. We obtain the following sequence of tableaus:

	x	y	z	u	v	P	Const.	Ratio
	1	1	1	1	0	0	8	$8/1 = 8$
p.r. →	3	2	[4]	0	1	0	24	$24/4 = 6$
	−3	−4	−5	0	0	1	0	
			↑ p.c.					

$\xrightarrow{\frac{1}{4}R_2}$

x	y	z	u	v	P	Const.
1	1	1	1	0	0	8
$\frac{3}{4}$	$\frac{1}{2}$	1	0	$\frac{1}{4}$	0	6
–3	–4	–5	0	0	1	0

$\xrightarrow[R_3+5R_2]{R_1-R_2}$

	x	y	z	u	v	P	Const.
p.r. →	$\frac{1}{4}$	$\boxed{\frac{1}{2}}$	0	1	$-\frac{1}{4}$	0	2
	$\frac{3}{4}$	$\frac{1}{2}$	1	0	$\frac{1}{4}$	0	6
	$\frac{3}{4}$	$-\frac{3}{2}$	0	0	$\frac{5}{4}$	1	30
		↑ p.c.					

Ratio

$2/(1/2) = 4$

$6/(1/2) = 12$

$\xrightarrow{2R_1}$

x	y	z	u	v	P	Const.
$\frac{1}{2}$	1	0	2	$-\frac{1}{2}$	0	4
$\frac{3}{4}$	$\frac{1}{2}$	1	0	$\frac{1}{4}$	0	6
$\frac{3}{4}$	$-\frac{3}{2}$	0	0	$\frac{5}{4}$	1	30

$\xrightarrow[R_3+\frac{3}{2}R_1]{R_2-\frac{1}{2}R_1}$

x	y	z	u	v	P	Const.
$\frac{1}{2}$	1	0	2	$-\frac{1}{2}$	0	4
$\frac{1}{2}$	0	1	–1	$\frac{1}{2}$	0	4
$\frac{3}{2}$	0	0	3	$\frac{1}{2}$	1	36

This last tableau is in final form. We find that $x = 0$, $y = 4$, $z = 4$, $u = 0$, $v = 0$, and $P = 36$.

18. The initial tableau is

x	y	z	u	v	P	Const.
1	1	3	1	0	0	15
4	4	3	0	1	0	65
–3	–3	–4	0	0	1	0

Using the following sequence of row operations

1. $\frac{1}{3}R_1$ 2. $R_2 - 3R_1; R_3 + 4R_1$ 3. $3R_1$ 4. $R_2 - 3R_1; R_3 + \frac{5}{3}R_1$

we obtain the final tableau

x	y	z	u	v	P	Const.
1	1	3	1	0	0	15
0	0	–9	–4	1	0	5
0	0	5	3	0	1	45

We find that the solutions are $x = 15$, $y = 0$, $z = 0$, $u = 0$, $v = 5$, and $P = 45$; and $x = 0$, $y = 15$, $z = 0$, $u = 0$, $v = 5$, and $P = 45$.

19.

	x	y	z	u	v	w	P	Const.
	3	10	5	1	0	0	0	120
p.r. →	5	[2]	8	0	1	0	0	6
	8	10	3	0	0	1	0	105
	–3	–4	–1	0	0	0	1	0
		↑ p.c.						

Ratio

$120/10 = 12$

$6/2 = 3$

$105/10 = 21/2$

$\xrightarrow{\frac{1}{2}R_2}$

x	y	z	u	v	w	P	Const.
3	10	5	1	0	0	0	120
$\frac{5}{2}$	1	4	0	$\frac{1}{2}$	0	0	3
8	10	3	0	0	1	0	105
–3	–4	–1	0	0	0	1	0

$\xrightarrow{\substack{R_1-10R_2\\R_3-10R_2\\R_4+4R_2}}$

x	y	z	u	v	w	P	Const.
–22	0	–35	1	–5	0	0	90
$\frac{5}{2}$	1	4	0	$\frac{1}{2}$	0	0	3
–17	0	–37	0	–5	1	0	75
7	0	15	0	2	0	1	12

The last tableau is in final form. We find that $x = 0$, $y = 3$, $z = 0$, $u = 90$, $v = 0$, $w = 75$, and $P = 12$.

20. From the initial tableau

x	y	z	u	v	w	P	Const.
2	1	1	1	0	0	0	14
4	2	3	0	1	0	0	28
2	5	5	0	0	1	0	30
–1	–2	1	0	0	0	1	0

we use the following row operations

1. $\frac{1}{5}R_3$ 2. $R_1 - R_3$; $R_2 - 2R_3$; $R_4 + 2R_3$ 3. $\frac{5}{8}R_1$

4. $R_2 - \frac{16}{5}R_1$; $R_3 - \frac{2}{5}R_1$; $R_4 + \frac{1}{5}R_1$

to obtain the final tableau

x	y	z	u	v	w	P	Const.
1	0	0	$\frac{5}{8}$	0	$-\frac{1}{8}$	0	5
0	0	1	–2	1	0	0	0
0	1	1	$-\frac{1}{4}$	0	$\frac{1}{4}$	0	4
0	0	3	$\frac{1}{8}$	0	$\frac{3}{8}$	1	13

and conclude that P attains an optimal value of 13 when $x = 5$, $y = 4$, $z = 0$, $u = 0$, $v = 0$, and $w = 0$.

21. We obtain the following sequence of tableaus:

	x	y	z	u	v	w	P	Const.	Ratio
	1	1	1	1	0	0	0	20	20
p.r. →	2	[4]	3	0	1	0	0	42	$10\frac{1}{2}$
	2	0	3	0	0	1	0	30	– – –
	–4	–6	–5	0	0	0	1	0	
		↑ p.c.							

$\xrightarrow{\frac{1}{4}R_2}$

x	y	z	u	v	w	P	Const.
1	1	1	1	0	0	0	20
$\frac{1}{2}$	1	$\frac{3}{4}$	0	$\frac{1}{4}$	0	0	$\frac{21}{2}$
2	0	3	0	0	1	0	30
–4	–6	–5	0	0	0	1	0

$\xrightarrow[R_4+6R_2]{R_1-R_2}$

	x	y	z	u	v	w	P	Const.	Ratio
	$\frac{1}{2}$	0	$\frac{1}{4}$	1	$-\frac{1}{4}$	0	0	$\frac{19}{2}$	19
	$\frac{1}{2}$	1	$\frac{3}{4}$	0	$\frac{1}{4}$	0	0	$\frac{21}{2}$	21
p.r. →	$\boxed{2}$	0	3	0	0	1	0	30	15
	–1	0	$-\frac{1}{2}$	0	$\frac{3}{2}$	0	1	63	
	↑ p.c.								

$\xrightarrow{\frac{1}{2}R_3}$

x	y	z	u	v	w	P	Const.
$\frac{1}{2}$	0	$\frac{1}{4}$	1	$-\frac{1}{4}$	0	0	$\frac{19}{2}$
$\frac{1}{2}$	1	$\frac{3}{4}$	0	$\frac{1}{4}$	0	0	$\frac{21}{2}$
1	0	$\frac{3}{2}$	0	0	$\frac{1}{2}$	0	15
–1	0	$-\frac{1}{2}$	0	$\frac{3}{2}$	0	1	63

$R_1-\frac{1}{2}R_3$, $R_2-\frac{1}{2}R_3$, R_4+R_3 →

x	y	z	u	v	w	P	Const.
0	0	$-\frac{1}{2}$	1	$-\frac{1}{4}$	$-\frac{1}{4}$	0	2
0	1	0	0	$\frac{1}{4}$	$-\frac{1}{4}$	0	3
1	0	$\frac{3}{2}$	0	0	$\frac{1}{2}$	0	15
0	0	1	0	$\frac{3}{2}$	$\frac{1}{2}$	1	78

So the solution is $x = 15$, $y = 3$, $z = 0$, $u = 2$, $v = 0$, $w = 0$, and $P = 78$.

22. From the initial tableau

x	y	z	u	v	w	P	Const.
3	1	–1	1	0	0	0	80
2	1	–1	0	1	0	0	40
–1	1	1	0	0	1	0	80
–1	–4	2	0	0	0	1	0

we use the following row operations
1. $R_1 - R_2;\ R_3 - R_2;\ R_4 + 4R_2$ 2. $\frac{1}{2}R_3$ 3. $R_2 + R_3;\ R_4 + 2R_3$
to obtain the final simplex tableau

x	y	z	u	v	w	P	Const.
1	0	0	1	-1	0	0	40
$\frac{1}{2}$	1	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0	60
$-\frac{3}{2}$	0	1	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	20
4	0	0	0	3	1	1	200

and conclude that the solution is $x = 0$, $y = 60$, $z = 20$, $u = 40$, $v = 0$, $w = 0$, and $P = 200$.

23. We obtain the following sequence of tableaus:

	x	y	z	u	v	w	P	Const.	Ratio
p.r. →	[2]	1	1	1	0	0	0	10	$10/2 = 5$
	3	5	1	0	1	0	0	45	$45/3 = 15$
	2	5	1	0	0	1	0	40	$40/2 = 20$
	-12	-10	-5	0	0	0	1	0	
	↑ p.c.								

$\xrightarrow{\frac{1}{2}R_1}$

x	y	z	u	v	w	P	Const.
1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	5
3	5	1	0	1	0	0	45
2	5	1	0	0	1	0	40
-12	-10	-5	0	0	0	1	0

$\xrightarrow[R_4+12R_1]{R_2-3R_1,\ R_3-2R_1}$

	x	y	z	u	v	w	P	Const.	Ratio
	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	5	$5/(1/2) = 10$
	0	$\frac{7}{2}$	$-\frac{1}{2}$	$-\frac{3}{2}$	1	0	0	30	$30/(7/2) = 60/7$
p.r. →	0	[4]	0	-1	0	1	0	30	$30/4 = 15/2$
	0	-4	1	6	0	0	1	60	
		↑ .p.c							

$\xrightarrow{\frac{1}{4}R_3}$

x	y	z	u	v	w	P	Const.
1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	5
0	$\frac{7}{2}$	$-\frac{1}{2}$	$-\frac{3}{2}$	1	0	0	30
0	1	0	$-\frac{1}{4}$	0	$\frac{1}{4}$	0	$\frac{15}{2}$
0	–4	1	6	0	0	1	60

$\xrightarrow[R_4+4R_3]{R_1-\frac{1}{2}R_3,\ R_2-\frac{7}{2}R_3}$

x	y	z	u	v	w	P	Const.
1	0	$\frac{1}{2}$	$\frac{5}{8}$	0	$-\frac{1}{8}$	0	$\frac{5}{4}$
0	0	$-\frac{1}{2}$	$-\frac{5}{8}$	1	$-\frac{7}{8}$	0	$\frac{15}{4}$
0	1	0	$-\frac{1}{4}$	0	$\frac{1}{4}$	0	$\frac{15}{2}$
0	0	1	5	0	1	1	90

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This last tableau is in final form, and we conclude that $x = 5/4$, $y = 15/2$, $z = 0$, $u = 0$, $v = 15/4$, $w = 0$, and $P = 90$.

24. We obtain the following sequence of tableaus, where t, u, v and w are slack variables.

x	y	z	t	u	v	w	P	Const.	Ratio
2	[1]	3	1	0	0	0	0	10	10
4	1	2	0	1	0	0	0	56	56
6	4	3	0	0	1	0	0	126	$31\frac{1}{2}$
2	1	1	0	0	0	1	0	32	32
–2	–6	–6	0	0	0	0	1	0	

$\xrightarrow{R_2-R_1,\ R_3-4R_1,\ R_4-R_1,\ R_5+6R_1}$

x	y	z	t	u	v	w	P	Const.
2	1	3	1	0	0	0	0	10
2	0	–1	–1	1	0	0	0	46
–2	0	–9	–4	0	1	0	0	86
0	0	–2	–1	0	0	1	0	22
10	0	12	6	0	0	0	1	60

The last simplex tableau is in final form, and we have the solution $x = 0$, $y = 10$, $z = 0$, $t = 0$, $u = 46$, $v = 86$, $w = 22$, and $P = 60$.

25. We obtain the following sequence of tableaus, where u, v and w are slack variables.

	x	y	z	u	v	w	P	Const.	Ratio
p.r. →	[2]	1	2	1	0	0	0	7	$\frac{7}{2}$
	2	3	1	0	1	0	0	8	4
	1	2	3	0	0	1	0	7	7
	–24	–16	–23	0	0	0	1	0	
	↑ p.c.								

$\xrightarrow{\frac{1}{2}R_1}$

x	y	z	u	v	w	P	Const.
[1]	$\frac{1}{2}$	1	$\frac{1}{2}$	0	0	0	$\frac{7}{2}$
2	3	1	0	1	0	0	8
1	2	3	0	0	1	0	7
–24	–16	–23	0	0	0	1	0

$\xrightarrow[R_4+24R_1]{R_2-2R_1,\ R_3-R_1}$

	x	y	z	u	v	w	P	Const.	Ratio
	1	$\frac{1}{2}$	1	$\frac{1}{2}$	0	0	0	$\frac{7}{2}$	7
p.r. →	0	[2]	–1	–1	1	0	0	1	$\frac{1}{2}$
	0	$\frac{3}{2}$	2	$-\frac{1}{2}$	0	1	0	$\frac{7}{2}$	$\frac{7}{3}$
	0	–4	1	12	0	0	1	84	
		↑ p.c.							

$\xrightarrow{\frac{1}{2}R_2}$

x	y	z	u	v	w	P	Const.
1	$\frac{1}{2}$	1	$\frac{1}{2}$	0	0	0	$\frac{7}{2}$
0	1	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{2}$
0	$\frac{3}{2}$	2	$-\frac{1}{2}$	0	1	0	$\frac{7}{2}$
0	–4	1	12	0	0	1	84

$\xrightarrow[R_4+4R_2]{R_1-\frac{1}{2}R_2,\ R_3-\frac{3}{2}R_2}$

	x	y	z	u	v	w	P	Const.	Ratio
	1	0	$\frac{5}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	0	0	$\frac{13}{4}$	$\frac{13}{5}$
	0	1	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{2}$	--
p.r. →	0	0	$\boxed{\frac{11}{4}}$	$\frac{1}{4}$	$-\frac{3}{4}$	1	0	$\frac{11}{4}$	1
	0	0	−1	10	2	0	1	86	
			↑ p.c.						

$\xrightarrow{\frac{4}{11}R_3}$

x	y	z	u	v	w	P	Const.
1	0	$\frac{5}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	0	0	$\frac{13}{4}$
0	1	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{2}$
0	0	1	$\frac{1}{11}$	$-\frac{3}{11}$	$\frac{4}{11}$	0	1
0	0	−1	10	2	0	1	86

$\xrightarrow[R_4+R_3]{R_1-\frac{5}{4}R_3,\ R_2+\frac{1}{2}R_3}$

x	y	z	u	v	w	P	Const.
1	0	0	$\frac{7}{11}$	$\frac{13}{22}$	$-\frac{5}{11}$	0	2
0	1	0	$-\frac{5}{11}$	$\frac{4}{11}$	$\frac{2}{11}$	0	1
0	0	1	$\frac{1}{11}$	$-\frac{3}{11}$	$\frac{4}{11}$	0	1
0	0	0	$\frac{111}{11}$	$\frac{19}{11}$	$\frac{4}{11}$	1	87

This last tableau is in final form and we conclude that P attains a maximum value of 87 when $x = 2$, $y = 1$, and $z = 1$.

26. From the initial tableau

x	y	z	u	v	w	P	Const.
2	1	2	1	0	0	0	14
2	4	1	0	1	0	0	26
1	2	3	0	0	1	0	28
−2	−2	−1	0	0	0	1	0

we use the following row operations

1. $\frac{1}{4}R_2$ 2. $R_1 - R_2;\ R_3 - 2R_2;\ R_4 + 2R_2$ 3. $\frac{2}{3}R_1$

4. $R_2 - \frac{1}{2}R_1;\ R_3 - \frac{1}{2}R_1;\ R_4 + R_1$

to obtain the final tableau

x	y	z	u	v	w	P	Const.
1	0	$\frac{7}{6}$	$\frac{2}{3}$	$-\frac{1}{6}$	0	0	5
0	1	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	0	0	4
0	0	$\frac{5}{2}$	0	$-\frac{1}{2}$	1	0	15
0	0	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	0	1	18

The solution is $x = 5$, $y = 4$, $z = 0$, $u = 0$, $v = 0$, $w = 15$, and $P = 18$.

27. Pivoting about the x-column in the initial simplex tableau, we have

	x	y	z	u	v	P	Const.	Ratio
	3	3	–2	1	0	0	100	100 / 3
p.r. →	[5]	5	3	0	1	0	150	150 / 5
	–2	–2	4	0	0	1	0	
	↑ p.c.							

$\xrightarrow{\frac{1}{5}R_2}$

x	y	z	u	v	P	Const.
3	3	–2	1	0	0	100
1	1	$\frac{3}{5}$	0	$\frac{1}{5}$	0	30
–2	–2	4	0	0	1	0

$\xrightarrow[R_3+2R_2]{R_1-3R_2}$

x	y	z	u	v	P	Const.
0	0	$-\frac{19}{5}$	1	$-\frac{3}{5}$	0	10
1	1	$\frac{3}{5}$	0	$\frac{1}{5}$	0	30
0	0	$\frac{26}{5}$	0	$\frac{2}{5}$	1	60

and we see that one optimal solution occurs when $x = 30$, $y = 0$, $z = 0$, and $P = 60$. Similarly, pivoting about the y-column, we obtain another optimal solution: $x = 0$, $y = 30$, $z = 0$, and $P = 60$.

28. We tabulate the given information

	Product A	*Product B*	*Time available (min)*
Machine I	6	9	300
Machine II	5	4	180
Profit per unit	3	4	

Let *x* and *y* denote the number of units of Product *A* and Product *B* to be produced. Then the required linear programming problem is:

Maximize the objective function $P = 3x + 4y$ subject to the constraints

$$6x + 9y \leq 300$$

$$5x + 4y \leq 180$$

$$x \geq 0,\ y \geq 0$$

The initial simplex tableau is

x	y	u	v	P	*Const*
6	9	1	0	0	300
5	4	0	1	0	180
–3	–4	0	0	1	0

Using the following sequence of row operations

1. $\frac{1}{9}R_1$ 2. $R_2 - 4R_1$; $R_3 + 4R_1$ 3. $\frac{3}{7}R_2$ 4. $R_1 - \frac{2}{3}R_2$; $R_3 + \frac{1}{3}R_2$

we obtain the final tableau

x	y	u	v	P	*Const*
0	1	$\frac{15}{63}$	$-\frac{2}{7}$	0	20
1	0	$-\frac{4}{21}$	$\frac{3}{7}$	0	20
0	0	$\frac{8}{21}$	$\frac{1}{7}$	1	140

and conclude that $x = 20$, $y = 20$, $u = 0$, $v = 0$, and $P = 140$. They should make 20 units of each product. The maximum profit is $140. There is no time left unused because $u = v = 0$.

29. Let the number of model *A* and model *B* fax machines made each month be *x* and *y*, respectively. Then we have the following linear programming problem:

Maximize $P = 30x + 40y$ subject to

$$100x + 150y \leq 600,000$$
$$x + \quad y \leq \quad 2,500$$
$$x \geq 0, \ y \geq 0$$

Using the simplex method, we obtain the following sequenceof tableaus:

x	y	u	v	P	*Const*	*Ratio*
100	150	1	0	0	600,000	4000
1	$\boxed{1}$	0	1	0	2,500	2500
–30	–40	0	0	1	0	

$\xrightarrow[R_3 + 40R_2]{R_1 - 150R_2}$

x	y	u	v	P	*Const*
–50	0	1	–150	0	225000
1	1	0	1	0	2500
10	0	0	40	1	100000

We conclude that the maximum monthly profit is $100,000, and this occurs when 0 model *A* and 2500 model *B* fax machines are produced.

30. Let x denote the number of model *A* hibachis and y the number of model *B* hibachis to be manufactured. Then the problem is equivalent to the following linear programming problem:

Maximize $P = 2x + 1.5y$ subject to

$$3x + 4y \leq 1000$$
$$6x + 3y \leq 1200$$
$$x \geq 0, \ y \geq 0$$

The initial simplex tableau is

x	y	u	v	P	*Const*
3	4	1	0	0	1000
6	3	0	1	0	1200
–2	$-\frac{3}{2}$	0	0	1	0

Using the following sequence of row operations,

1. $\frac{1}{6}R_2$ 2. $R_1 - 3R_2$; $R_3 + 2R_2$ 3. $\frac{2}{5}R_1$ 4. $R_2 - \frac{1}{2}R_1$; $R_3 + \frac{1}{2}R_1$

we obtain the final tableau

x	y	u	v	P	Const
0	1	$\frac{2}{5}$	$-\frac{1}{5}$	0	160
1	0	$-\frac{1}{5}$	$\frac{4}{15}$	0	120
0	0	$\frac{1}{5}$	$\frac{7}{30}$	1	480

We conclude that the company should produce 120 model A and 160 model B hibachis to realize a profit of \$480. There are no raw materials left because $u = v = 0$.

31. Suppose the farmer plants x acres of Crop A and y acres of Crop B. Then the problem is

Maximize $P = 150x + 200y$ subject to

$$x + y \le 150$$
$$40x + 60y \le 7400$$
$$20x + 25y \le 3300$$
$$x \ge 0,\ y \ge 0$$

Using the simplex method, we obtain the following sequence of tableaus:

x	y	u	v	w	P	Const
1	1	1	0	0	0	150
40	60	0	1	0	0	7400
20	25	0	0	1	0	3300
–150	–200	0	0	0	1	0

$\xrightarrow[R_4+150R_1]{R_2-40R_1,\ R_3-20R_1}$

	x	y	u	v	w	P	Const	Ratio
	1	1	1	0	0	0	150	150
	0	20	–40	1	0	0	1400	700
p.r. →	0	[5]	–20	0	1	0	300	60
	0	–50	150	0	0	1	22500	
		↑ p.c.						

$\xrightarrow{\frac{1}{5}R_3}$

x	y	u	v	w	P	*Const*
1	1	1	0	0	0	150
0	20	–40	1	0	0	1400
0	1	–4	0	$\frac{1}{5}$	0	60
0	–50	150	0	0	1	22500

$\xrightarrow[R_4+50R_3]{R_1-R_3,\ R_2-20R_3}$

	x	y	u	v	w	P	*Const*	*Ratio*
	1	0	5	0	$-\frac{1}{5}$	0	90	18
p.r. →	0	0	$\boxed{40}$	1	–4	0	200	5
	0	1	–4	0	$\frac{1}{5}$	0	60	– –
	0	0	–50	0	10	1	25500	
			↑ p.c.					

$\xrightarrow{\frac{1}{40}R_2}$

x	y	u	v	w	P	*Const*
1	0	5	0	$-\frac{1}{5}$	0	90
0	0	1	$\frac{1}{40}$	$-\frac{1}{10}$	0	5
0	1	–4	0	$\frac{1}{5}$	0	60
0	0	–50	0	10	1	25500

$\xrightarrow[R_4+50R_2]{R_1-5R_2,\ R_3+4R_2}$

x	y	u	v	w	P	*Const*
1	0	0	$-\frac{1}{8}$	$\frac{3}{10}$	0	65
0	0	1	$\frac{1}{40}$	$-\frac{1}{10}$	0	5
0	1	0	$\frac{1}{10}$	$-\frac{1}{5}$	0	80
0	0	0	$\frac{5}{4}$	5	1	25750

The last tableau is in final form. We find $x = 65$, $y = 80$, and $P = 25{,}750$. So the maximum profit of \$25,750 is realized by planting 65 acres of Crop *A* and 80 acres of Crop *B*. Since $u = 5$, we see that there are 5 acres of land left unused.

32. Let x (in thousands) denote the amount invested in project A and y (in thousands) the amount invested in project B. Then we have the following linear programming

problem: Maximize $P = 0.1x + 0.15y$

$$x + y \le 500$$
$$y \le 0.4(x + y)$$
$$x \ge 0, y \ge 0$$

which can be rewritten as

Maximize $P = \frac{1}{10}x + \frac{3}{20}y$ subject to

$$x + \ \ y \le 500$$
$$-2x + 3y \le 0$$
$$x \ge 0, y \ge 0$$

The initial simplex tableau is

x	y	u	v	P	Const
1	1	1	0	0	500
-2	$\boxed{3}$	0	1	0	0
$-\frac{1}{10}$	$-\frac{3}{20}$	0	0	1	0

Using the following sequence of row operations

1. $\frac{1}{3}R_2$ 2. $R_1 - R_2$; $R_3 + \frac{3}{20}R_2$ 3. $\frac{3}{5}R_1$ 4. $R_2 + \frac{2}{3}R_1$; $R_3 + \frac{1}{5}R_1$

we obtain the final simplex tableau

x	y	u	v	P	Const
1	0	$\frac{3}{5}$	$-\frac{1}{5}$	0	300
0	1	$\frac{2}{5}$	$\frac{1}{5}$	0	200
0	0	$\frac{3}{25}$	$\frac{1}{100}$	1	60

The last tableau is in final form, and we see that the optimal solution is $x = 300$, $y = 200$, and $P = 60$. So the financier should invest \$300,000 in project A and \$200,000 in project B. The maximum return is \$60,000.

33. Suppose Ashley invests x, y, and z dollars in the money market fund, the international equity fund, and the growth-and-income fund, respectively. Then the objective function is $P = 0.06x + 0.1y + 0.15z$. The constraints are

$$x + y + z \le 250{,}000; \quad z \le 0.25(x + y + z); \text{ and } \quad y \le 0.5(x + y + z).$$

The last two inequalities simplify to

$$-\frac{1}{4}x - \frac{1}{4}y + \frac{3}{4}z \le 0 \quad \text{ or } \quad -x - y + 3z \le 0$$

and $\quad -\frac{1}{2}x+\frac{1}{2}y-\frac{1}{2}z\le 0,\quad$ or $\quad -x+y-z\le 0.$

So the required linear programming problem is

Maximize $P = 0.06x + 0.1y + 0.15z = \frac{3}{50}x+\frac{1}{10}y+\frac{3}{20}z$ subject to

$$\begin{aligned} x+y+\ z &\le 250000 \\ -x-y+3z &\le 0 \\ -x+y-\ z &\le 0 \\ x\ge 0, y\ge 0, z &\ge 0 \end{aligned}$$

Let u, v, and w be slack variables. We obtain the following tableaus:

	x	y	z	u	v	w	P	Const.	Ratio
	1	1	1	1	0	0	0	250000	250000
p.r. →	−1	−1	[3]	0	1	0	0	0	0
	−1	1	−1	0	0	1	0	0	−−
	$-\frac{3}{50}$	$-\frac{1}{10}$	$-\frac{3}{20}$	0	0	0	1	0	
			↑ p.c						

$\xrightarrow{\frac{1}{3}R_2}$

x	y	z	u	v	w	P	Const.
1	1	1	1	0	0	0	250000
$-\frac{1}{3}$	$-\frac{1}{3}$	[1]	0	$\frac{1}{3}$	0	0	0
−1	1	−1	0	0	1	0	0
$-\frac{3}{50}$	$-\frac{1}{10}$	$-\frac{3}{20}$	0	0	0	1	0

$\xrightarrow[\substack{R_3+R_2 \\ R_4+\frac{3}{20}R_2}]{R_1-R_2}$

x	y	z	u	v	w	P	Const.	Ratio
$\frac{4}{3}$	$\frac{4}{3}$	0	1	$-\frac{1}{3}$	0	0	250000	$\frac{250000}{4/3}=187500$
$-\frac{1}{3}$	$-\frac{1}{3}$	1	0	$\frac{1}{3}$	0	0	0	−−
$-\frac{4}{3}$	[$\frac{2}{3}$]	0	0	$\frac{1}{3}$	1	0	0	0
$-\frac{11}{100}$	$-\frac{3}{20}$	0	0	$\frac{1}{20}$	0	1	0	
	↑ p.c.							

$\xrightarrow{\frac{3}{2}R_3}$

x	y	z	u	v	w	P	$Const.$
$\frac{4}{3}$	$\frac{4}{3}$	0	1	$-\frac{1}{3}$	0	0	250000
$-\frac{1}{3}$	$-\frac{1}{3}$	1	0	$\frac{1}{3}$	0	0	0
-2	$\boxed{1}$	0	0	$\frac{1}{2}$	$\frac{3}{2}$	0	0
$-\frac{11}{100}$	$-\frac{3}{20}$	0	0	$\frac{1}{20}$	0	1	0

$\xrightarrow[\substack{R_2+\frac{1}{3}R_3 \\ R_4+\frac{3}{20}R_2}]{R_1-\frac{4}{3}R_3}$

x	y	z	u	v	w	P	$Const.$	$Ratio$
$\boxed{4}$	0	0	1	-1	-2	0	250000	62500
-1	0	1	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$--$
-2	1	0	0	$\frac{1}{2}$	$\frac{3}{2}$	0	0	0
$-\frac{41}{100}$	0	0	0	$\frac{3}{20}$	$\frac{9}{40}$	1	0	

↑
p.c.

$\xrightarrow{\frac{1}{4}R_1}$

x	y	z	u	v	w	P	$Const.$
$\boxed{1}$	0	0	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$	0	62500
-1	0	1	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0
-2	1	0	0	$\frac{1}{2}$	$\frac{3}{2}$	0	0
$-\frac{41}{100}$	0	0	0	$\frac{3}{20}$	$\frac{9}{40}$	1	0

$\xrightarrow[\substack{R_3+2R_2 \\ R_4+\frac{41}{100}R_1}]{R_2+R_1}$

x	y	z	u	v	w	P	$Const.$
1	0	0	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$	0	62500
0	0	1	$\frac{1}{4}$	$\frac{1}{4}$	0	0	62500
0	1	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	125000
0	0	0	$\frac{41}{400}$	$\frac{19}{400}$	$\frac{1}{50}$	1	25625

The last tableau is in final form, and we see that $x = 62{,}500$, $y = 125{,}000$, $z = 62{,}500$, and $P = 25{,}625$. So, Ashley should invest $62,500 in the money market fund, $125,000 in the international equity fund, and $62,500 in the growth-and-income fund. Her maximum return will be $25,625.

34. Refer to Exercise 33, page 253 of this manual. The linear programming problem is

Maximize $P = 45x + 20y$ subject to

$$40x + 16y \le 3200$$
$$3x + 4y \le 520$$
$$x \ge 0,\ y \ge 0$$

The initial simplex tableau is

x	y	u	v	P	*Const*
[40]	16	1	0	0	3200
3	4	0	1	0	520
−45	−20	0	0	1	0

Using the following sequence of row operations

1. $\frac{1}{40}R_1$ 2. $R_2 - 3R_1$; $R_3 + 45R_1$ 3. $\frac{5}{14}R_2$ 4. $R_1 - \frac{2}{5}R_2$; $R_3 + 2R_2$

we obtain the final tableau

x	y	u	v	P	*Const*
1	0	$\frac{31}{280}$	$-\frac{1}{7}$	0	40
0	1	$-\frac{45}{112}$	$-\frac{1}{7}$	0	100
0	0	$\frac{347}{112}$	$\frac{5}{7}$	1	3800

We conclude that Winston should manufacture 40 tables and 100 chairs for a maximum profit of \$3800.

35. We wish to maximize $P = 18x + 12y + 15z$ subject to

$$2x + y + 2z \le 900$$
$$3x + y + 2z \le 1080$$
$$2x + 2y + z \le 840$$
$$x \ge 0,\ y \ge 0,\ z \ge 0$$

Let u, v, and w be slack variables. We obtain the following tableaus:

	x	y	z	u	v	w	P	Const.	Ratio
	2	1	2	1	0	0	0	900	450
p.r. →	[3]	1	2	0	1	0	0	1080	360
	2	2	1	0	0	1	0	840	420
	−18	−12	−15	0	0	0	1	0	

↑ p.c. (column x)

$\xrightarrow{\frac{1}{3}R_2}$

x	y	z	u	v	w	P	Const.
2	1	2	1	0	0	0	900
1	$\frac{1}{3}$	$\frac{2}{3}$	0	$\frac{1}{3}$	0	0	360
2	2	1	0	0	1	0	840
−18	−12	−15	0	0	0	1	0

$\xrightarrow[R_3-2R_2,\ R_4+18R_2]{R_1-2R_2}$

x	y	z	u	v	w	P	Const.
0	$\frac{1}{3}$	$\frac{2}{3}$	1	$-\frac{2}{3}$	0	0	180
1	$\frac{1}{3}$	$\frac{2}{3}$	0	$\frac{1}{3}$	0	0	360
0	[$\frac{4}{3}$]	$-\frac{1}{3}$	0	$-\frac{2}{3}$	1	0	120
0	−6	−3	0	6	0	1	6480

$\xrightarrow{\frac{3}{4}R_3}$

x	y	z	u	v	w	P	Const.
0	$\frac{1}{3}$	$\frac{2}{3}$	1	$-\frac{2}{3}$	0	0	180
1	$\frac{1}{3}$	$\frac{2}{3}$	0	$\frac{1}{3}$	0	0	360
0	1	$-\frac{1}{4}$	0	$-\frac{1}{2}$	$\frac{3}{4}$	0	90
0	−6	−3	0	6	0	1	6480

$\xrightarrow[R_2-\frac{1}{3}R_3,\ R_4+6R_3]{R_1-\frac{1}{3}R_3}$

x	y	z	u	v	w	P	Const.
0	0	$\boxed{\frac{3}{4}}$	1	$-\frac{1}{2}$	$-\frac{1}{4}$	0	150
1	0	$\frac{3}{4}$	0	$\frac{1}{2}$	$-\frac{1}{4}$	0	330
0	1	$-\frac{1}{4}$	0	$-\frac{1}{2}$	$\frac{3}{4}$	0	90
0	0	$-\frac{9}{2}$	0	6	$\frac{9}{2}$	1	7020

$\xrightarrow{\frac{4}{3}R_1}$

x	y	z	u	v	w	P	Const.
0	0	1	$\frac{4}{3}$	$-\frac{2}{3}$	$-\frac{1}{3}$	0	200
1	0	$\frac{3}{4}$	0	$\frac{1}{2}$	$-\frac{1}{4}$	0	330
0	1	$-\frac{1}{4}$	0	$-\frac{1}{2}$	$\frac{3}{4}$	0	90
0	0	$-\frac{9}{2}$	0	6	$\frac{9}{2}$	1	7020

$\xrightarrow[\substack{R_3+\frac{1}{4}R_1 \\ R_4+\frac{9}{2}R_1}]{R_2-\frac{3}{4}R_1}$

x	y	z	u	v	w	P	Const.
0	0	1	$\frac{4}{3}$	$-\frac{2}{3}$	$-\frac{1}{3}$	0	200
1	0	0	-1	1	0	0	180
0	1	0	$\frac{1}{3}$	$-\frac{2}{3}$	$\frac{2}{3}$	0	140
0	0	0	6	3	3	1	7920

The last tableau is in final form and we conclude that the company will realize a maximum profit of \$7920 if they produce 180 units of product *A*, 140 units of product *B*, and 200 units of product *C*. Since $u = v = w = 0$, there are no resources left over.

36. Refer to Exercise 14, page 228 of this manual. The linear programming problem is

$$\text{Maximize } P = 10x + 15y \text{ subject to}$$
$$\begin{aligned} 1.5x + \ \ 2y &\le \ \ 3{,}600 \\ 30x + 35y &\le 66{,}000 \\ 5x + \ \ 8y &\le 13{,}600 \\ x \ge 0, y &\ge 0 \end{aligned}$$

The initial simplex tableau is given by

x	y	u	v	w	P	$Const$
$\frac{3}{2}$	2	1	0	0	0	3600
6	7	0	1	0	0	66000
5	$\boxed{8}$	0	0	1	0	13600
-10	-15	0	0	0	1	0

Using the following sequence of row operations

1. $\frac{1}{8}R_3$ 2. $R_1-2R_2;\ R_2-7R_3;\ R_4+15R_3$ 3. $4R_1$
4. $R_2-\frac{13}{8}R_1;\quad R_3-\frac{5}{8}R_1;\quad R_4+\frac{5}{8}R_1$

We obtain the final tableau

x	y	u	v	w	P	$Const$
1	0	4	0	-1	0	800
0	0	$-\frac{13}{2}$	1	$\frac{5}{8}$	0	52800
0	1	$-\frac{5}{2}$	0	$\frac{3}{4}$	0	1200
0	0	$\frac{5}{2}$	0	$\frac{5}{4}$	1	26000

Thus, $x = 800$, $y = 1200$, and P = 26,000. We conclude that 800 pandas and 1200 Saint Bernards should be produced for a maximum profit of $26,000.

37. Suppose the Excelsior Company buys x, y, and z minutes of morning, afternoon, and evening commercials, respectively. Then we wish to maximize

$$P = 200{,}000x + 100{,}000y + 600{,}000z \text{ subject to}$$
$$\begin{aligned} 3000x+1000y+12{,}000z &\le 102{,}000 \\ z &\le 6 \\ x+\quad y+\quad z &\le 25 \\ x\ge 0, y\ge 0, z &\ge 0 \end{aligned}$$

Using the simplex method, we obtain the following sequence of tableaus.

x	y	z	u	v	w	P	Const.	Ratio
[3000]	1000	12,000	1	0	0	0	102,000	17/2
0	0	1	0	1	0	0	6	6
1	1	1	0	0	1	0	25	25
−200,000	−100,000	−600,000	0	0	0	1	0	

$\xrightarrow[R_4+600{,}000R_2]{R_1-12{,}000R_2 \quad R_3-R_2}$

x	y	z	u	v	w	P	Const.	Ratio
3000	1000	0	1	−12,000	0	0	30,000	10
0	0	1	0	1	0	0	6	– –
1	1	0	0	−1	1	0	19	19
−200,000	−100,000	0	0	600,000	0	1	3,600,000	

$\xrightarrow{\frac{1}{3000}R_1}$

x	y	z	u	v	w	P	Const.
1	$\frac{1}{3}$	0	$\frac{1}{3000}$	−4	0	0	10
0	0	1	0	1	0	0	6
1	1	0	0	−1	1	0	19
−200,000	−100,000	0	0	600,000	0	1	3,600,000

$\xrightarrow[R_4+200{,}000R_1]{R_3-R_1}$

x	y	z	u	v	w	P	Const.
1	$\frac{1}{3}$	0	$\frac{1}{3000}$	−4	0	0	10
0	0	1	0	1	0	0	6
0	$\frac{2}{3}$	0	$-\frac{1}{3000}$	[3]	1	0	9
0	$-\frac{100,000}{3}$	0	$\frac{200}{3}$	−200,000	0	1	5,600,000

$\xrightarrow{\frac{1}{3}R_3}$

x	y	z	u	v	w	P	Const.
1	$\frac{1}{3}$	0	$\frac{1}{3000}$	−4	0	0	10
0	0	1	0	1	0	0	6
0	$\frac{2}{9}$	0	$-\frac{1}{9000}$	1	$\frac{1}{3}$	0	3
0	$-\frac{100,000}{3}$	0	$\frac{200}{3}$	−200,000	0	1	5,600,000

$\xrightarrow[R_4+200{,}000R_3]{R_1+4R_3 \quad R_2-R_3}$

x	y	z	u	v	w	P	Const.
1	$\frac{11}{9}$	0	$-\frac{1}{9000}$	0	$\frac{4}{3}$	0	22
0	$-\frac{2}{9}$	1	$\frac{1}{9000}$	0	$-\frac{1}{3}$	0	3
0	$\frac{2}{9}$	0	$-\frac{1}{9000}$	1	$\frac{1}{3}$	0	3
0	$\frac{100{,}000}{9}$	0	$\frac{400}{9}$	0	$\frac{200{,}000}{3}$	1	6,200,000

We conclude that $x = 22$, $y = 0$, $z = 3$, $u = 0$, $v = 3$, and $P = 6{,}200{,}000$. Therefore, the company should buy 22 minutes of morning and 3 minutes of evening advertising time, thereby maximizing their exposure to 6,200,000 viewers.

38. The problem is to maximize $P = 0.12x + 0.10y + 0.06z$ subject to

$$\begin{aligned} x + y - z &\le 0 \\ x - 3y + z &\le 0 \\ 5x + y - 3z &\le 0 \\ x + y + z &\le 200{,}000 \end{aligned}$$

The initial simplex tableau is

x	y	z	t	u	v	w	P	Const.
1	1	–1	1	0	0	0	0	0
1	–3	1	0	1	0	0	0	0
5	1	–3	0	0	1	0	0	0
1	1	1	0	0	0	1	0	200,000
–0.12	–0.10	–0.06	0	0	0	0	1	0

Using the following sequence of row operations,
1. $R_2 - R_1$; $R_3 - 5R_1$. $R_4 - R_1$; $R_5 + 0.12R_1$ 2. $\frac{1}{2}R_2$
3. $R_1 + R_2$; $R_3 - 2R_2$; $R_4 - 2R_2$; $R_5 + 0.18R_2$
4. $\frac{1}{4}R_4$ 6. $R_1 + R_4$; $R_2 + 2R_4$; $R_5 + 0.34R_4$
we obtain the final tableau

x	y	z	t	u	v	w	P	Const.
1	0	0	0.5	0.25	0	0.25	0	50,000
0	0	1	–0.5	0	0	0.5	0	100,000
0	0	0	–4	–1	1	0	0	0
0	1	0	0	–0.25	0	0.25	0	50,000
0	0	0	0.03	0.005	0	0.085	1	17,000

Therefore, the maximum return on the investment each year is \$17,000 when Sharon invests \$50,000 in growth funds, \$50,000 in balanced funds, and \$100,000 in income funds.

39. We first tabulate the given information:

Department	*Model A*	*Model B*	*Model C*	*Time available*
Fabrication	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{3}{2}$	310
Assembly	1	1	$\frac{3}{4}$	205
Finishing	1	1	$\frac{1}{2}$	190
Profit	26	28	24	

Let x, y, and z denote the number of units of model A, model B, and model C to be produced, respectively. Then the required linear programming problem is

$$
\begin{aligned}
&\text{Maximize } P = 26x + 28y + 24z \text{ subject to} \\
&\tfrac{5}{4}x + \tfrac{3}{2}y + \tfrac{3}{2}z \le 310 \\
&x + y + \tfrac{3}{4}z \le 205 \\
&x + y + \tfrac{1}{2}z \le 190 \\
&x \ge 0, y \ge 0, z \ge 0
\end{aligned}
$$

Using the simplex method, we obtain the following tableaus:

	x	y	z	u	v	w	P	Const.	Ratio
	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{3}{2}$	1	0	0	0	310	$206\frac{2}{3}$
	1	1	$\frac{3}{4}$	0	1	0	0	205	205
p.r. →	1	$\boxed{1}$	$\frac{1}{2}$	0	0	1	0	190	190
	–26	–28	–24	0	0	0	1	0	
		↑ p.c.							

$\xrightarrow{R_1-\frac{3}{2}R_3,\; R_2-R_3,\; R_4+28R_3}$

	x	y	z	u	v	w	P	Const.	Ratio
p.r. →	$-\frac{1}{4}$	0	$\boxed{\frac{3}{4}}$	1	0	$-\frac{3}{2}$	0	25	$33\frac{1}{3}$
	0	0	$\frac{1}{4}$	0	1	–1	0	15	60
	1	1	$\frac{1}{2}$	0	0	1	0	190	380
	2	0	–10	0	0	28	1	5320	
			↑ p.c.						

$\xrightarrow{\frac{4}{3}R_1}$

x	y	z	u	v	w	P	Const.
$-\frac{1}{3}$	0	1	$\frac{4}{3}$	0	–2	0	$\frac{100}{3}$
0	0	$\frac{1}{4}$	0	1	–1	0	15
1	1	$\frac{1}{2}$	0	0	1	0	190
2	0	–10	0	0	28	1	5320

$\xrightarrow{R_2-\frac{1}{4}R_1,\; R_3-\frac{1}{2}R_1,\; R_4+10R_1}$

	x	y	z	u	v	w	P	Const.	Ratio
	$-\frac{1}{3}$	0	1	$\frac{4}{3}$	0	–2	0	$\frac{100}{3}$	– –
p.r. →	$\boxed{\frac{1}{12}}$	0	0	$-\frac{1}{3}$	1	$-\frac{1}{2}$	0	$\frac{20}{3}$	80
	$\frac{7}{6}$	1	0	$-\frac{2}{3}$	0	2	0	$\frac{520}{3}$	$148\frac{4}{7}$
	$-\frac{4}{3}$	0	0	$\frac{40}{3}$	0	8	1	$\frac{16960}{3}$	
	↑ p.c.								

$\xrightarrow{12R_2}$

x	y	z	u	v	w	P	Const.
$-\frac{1}{3}$	0	1	$\frac{4}{3}$	0	-2	0	$\frac{100}{3}$
1	0	0	-4	12	-6	0	80
$\frac{7}{6}$	1	0	$-\frac{2}{3}$	0	2	0	$\frac{520}{3}$
$-\frac{4}{3}$	0	0	$\frac{40}{3}$	0	8	1	$\frac{16960}{3}$

$$\xrightarrow[R_4+\frac{4}{3}R_2]{R_1+\frac{1}{3}R_2,\ R_3-\frac{7}{6}R_2}$$

x	y	z	u	v	w	P	Const.
0	0	1	0	4	-4	0	60
1	0	0	-4	12	-6	0	80
0	1	0	4	-14	9	0	80
0	0	0	8	16	0	1	5760

The last tableau is in final form. We see that $x = 80$, $y = 80$, $z = 60$, $u = 0$, $v = 0$, $w = 0$, and $P = 5760$. So, by producing 80 units each of Models A and B, and 60 units of Model C, the company stands to make a profit of \$5760. Since $u = v = w = 0$, there are no resources left over.

40. The linear programming problem is

Maximize $P = 3400x + 4000y + 5000z$ subject to

$$\begin{aligned} 6x + 8y + 10z &\le 8200 \\ 24x + 22y + 20z &\le 21800 \\ 18x + 21y + 30z &\le 23700 \\ x \ge 0, y \ge 0, z &\ge 0 \end{aligned}$$

The initial simplex tableau is

x	y	z	u	v	w	P	Const.
6	8	10	1	0	0	0	8,200
24	22	20	0	1	0	0	21,800
18	21	30	0	0	1	0	23,700
-3400	-4000	-5000	0	0	0	1	0

Using the sequence of row operations

1. $\frac{1}{30}R_3$;   2. $R_1 - 10R_3$; $R_2 - 20R_3$; $R_4 + 5000R_3$
3. $R_2 - 8R_1$; $R_3 - \frac{7}{10}R_1$; $R_4 + 500R_1$ 4. $\frac{1}{12}R_2$ 5. $R_3 - \frac{3}{5}R_2$; $R_4 + 400R_2$

we obtain the final simplex tableau

x	y	z	u	v	w	P	Const.
0	1	0	1	0	$-\frac{1}{3}$	0	300
1	0	0	$-\frac{2}{3}$	$\frac{1}{12}$	$\frac{1}{6}$	0	300
0	0	1	$-\frac{3}{10}$	$-\frac{1}{20}$	$\frac{1}{6}$	0	400
0	0	0	$\frac{700}{3}$	$\frac{100}{3}$	$\frac{200}{3}$	1	4,220,000

from which we conclude that maximum profit of $4,220,000 is obtained if Boise produces 300 standard, 300 deluxe, and 400 luxury models. Since $u = \frac{25}{6}$, we see that there are $4\frac{1}{6}$ hours left over.

41. Let x, y, and z denote the number (in thousands) of bottles of formula *I*, formula *II*, and formula *III*, respectively, produced. The resulting linear programming problem is

Maximize $P = 180x + 200y + 300z$ subject to

$$\begin{aligned} \tfrac{5}{2}x + 3y + 4z &\le 70 \\ x &\le 9 \\ y &\le 12 \\ z &\le 6 \\ x \ge 0,\ y \ge 0,\ z &\ge 0 \end{aligned}$$

Using the simplex method, we have

	x	y	z	s	t	u	v	P	*Const.*	*Ratio*
	$\frac{5}{2}$	3	4	1	0	0	0	0	70	$17\frac{1}{2}$
	1	0	0	0	1	0	0	0	9	– –
	0	1	0	0	0	1	0	0	12	– –
$p.r. \rightarrow$	0	0	[1]	0	0	0	1	0	6	6
	–180	–200	–300	0	0	0	0	1	0	
			$\uparrow$ *p.c.*							

$\xrightarrow[R_5+300R_4]{R_1-4R_4}$

	x	y	z	s	t	u	v	P	*Const.*	*Ratio*
	$\frac{5}{2}$	3	0	1	0	0	–4	0	46	$15\frac{1}{3}$
	1	0	0	0	1	0	0	0	9	– –
$p.r. \rightarrow$	0	[1]	0	0	0	1	0	0	12	12
	0	0	1	0	0	0	1	0	6	– –
	–180	–200	0	0	0	0	300	1	1800	
		$\uparrow$ *p.c.*								

$\xrightarrow[R_5+200R_3]{R_1-3R_3}$

x	y	z	s	t	u	v	P	*Const.*	*Ratio*
$\frac{5}{2}$	0	0	1	0	–3	–4	0	10	4
1	0	0	0	1	0	0	0	9	9
0	1	0	0	0	1	0	0	12	– –
0	0	1	0	0	0	1	0	6	– –
–180	0	0	0	0	200	300	1	4200	

$\xrightarrow{\frac{2}{5}R_1}$

x	y	z	s	t	u	v	P	*Const.*
1	0	0	$\frac{2}{5}$	0	$-\frac{6}{5}$	$-\frac{8}{5}$	0	4
1	0	0	0	1	0	0	0	9
0	1	0	0	0	1	0	0	12
0	0	1	0	0	0	1	0	6
–180	0	0	0	0	200	300	1	4200

$\xrightarrow[R_5+180R_1]{R_2-R_1}$

x	y	z	s	t	u	v	P	Const.	Ratio
1	0	0	$\frac{2}{5}$	0	$-\frac{6}{5}$	$-\frac{8}{5}$	0	4	--
1	0	0	$-\frac{2}{5}$	1	$\boxed{\frac{6}{5}}$	$\frac{8}{5}$	0	5	$\frac{25}{6}$
0	1	0	0	0	1	0	0	12	12
0	0	1	0	0	0	1	0	6	--
0	0	0	72	0	−16	12	1	4920	

$\xrightarrow{\frac{5}{6}R_2}$

x	y	z	s	t	u	v	P	Const.
1	0	0	$\frac{2}{5}$	0	$-\frac{6}{5}$	$-\frac{8}{5}$	0	4
0	0	0	$-\frac{1}{3}$	$\frac{5}{6}$	1	$\frac{4}{3}$	0	$\frac{25}{6}$
0	1	0	0	0	1	0	0	12
0	0	1	0	0	0	1	0	6
0	0	0	72	0	−16	12	1	4920

$\xrightarrow[R_5+16R_2]{R_1+\frac{6}{5}R_2,\ R_3-R_2}$

x	y	z	s	t	u	v	P	Const.
1	0	0	0	1	0	0	0	9
0	0	0	$-\frac{1}{3}$	$\frac{5}{6}$	1	$\frac{4}{3}$	0	$\frac{25}{6}$
0	1	0	$\frac{1}{3}$	$-\frac{5}{6}$	0	$-\frac{4}{3}$	0	$\frac{47}{6}$
0	0	1	0	0	0	1	0	6
0	0	0	$\frac{200}{3}$	$\frac{40}{3}$	0	$\frac{100}{3}$	1	$4986\frac{2}{3}$

Therefore, $x = 9$, $y = 47/6$, $z = 6$, $s = 0$, $t = 0$, $u = \frac{25}{6}$ and $P \approx 4986.67$; that is, the company should manufacture 9000 bottles of formula *I*, 7833 bottles of formula *II*, and 6000 bottles of formula *III* for a maximum profit of \$4986.60. Yes, ingredients for 4167 bottles of formula *II*.

42. Refer to Section 3.2, Exercise 27, of this manual. Let u, v, w, and s be slack variables. The initial tableau is

x	y	z	u	v	w	s	P	Const.
$\boxed{8}$	0	4	1	0	0	0	0	16000
8	12	8	0	1	0	0	0	24000
0	4	4	0	0	1	0	0	5000
0	0	1	0	0	0	1	0	800
-1	$-\frac{4}{5}$	$-\frac{9}{10}$	0	0	0	0	1	0

Using the following sequence of row operations

1. $\frac{1}{8}R_1$ 2. $R_2 - 8R_1$; $R_5 + R_1$ 3. $\frac{1}{12}R_2$ 4. $R_3 - 4R_2$; $R_5 + \frac{4}{5}R_2$

5. $R_1 - \frac{1}{2}R_4$; $R_2 - \frac{1}{3}R_4$; $R_3 - \frac{8}{3}R_4$; $R_5 + \frac{2}{15}R_4$

we obtain the final tableau

x	y	z	u	v	w	s	P	Const.
1	0	0	$\frac{1}{8}$	0	0	-1	0	1600
0	1	0	$-\frac{1}{12}$	$\frac{1}{12}$	0	$-\frac{1}{3}$	0	400
0	0	0	$\frac{1}{3}$	$-\frac{1}{3}$	1	$-\frac{8}{3}$	0	200
0	0	1	0	0	0	1	0	800
0	0	0	$\frac{7}{120}$	$\frac{1}{15}$	0	1	1	2640

We see that $x = 1600$, $y = 400$, $z = 800$, and $P = 2640$. Therefore, Cal-Juice should produce 1600 cartons of pineapple-orange juice, 400 cartons of orange-banana juice, and 800 cartons of pineapple-orange-banana juice, respectively. The maximum profit is \$2640. Since w = 200, we see that 200 oz of banana pulp are left over.

43. Refer to Section 3.2, Exercise 19, of this manual. Let u, v, w, and s be slack variables. We obtain the following tableaus.

	x	y	z	u	v	w	s	P	*Const.*	*Ratio*
	1	1	1	1	0	0	0	0	2000000	2000000
p.r. →	–2	–2	$\boxed{8}$	0	1	0	0	0	0	0
	–6	4	4	0	0	1	0	0	0	0
	–10	6	6	0	0	0	1	0	0	0
	$-\frac{1}{10}$	$-\frac{3}{20}$	$-\frac{1}{5}$	0	0	0	0	1	0	
			↑ *p.c.*							

$\xrightarrow{\frac{1}{8}R_2}$

x	y	z	u	v	w	s	P	*Const.*
1	1	1	1	0	0	0	0	2000000
$-\frac{1}{4}$	$-\frac{1}{4}$	$\boxed{1}$	0	$\frac{1}{8}$	0	0	0	0
–6	4	4	0	0	1	0	0	0
–10	6	6	0	0	0	1	0	0
$-\frac{1}{10}$	$-\frac{3}{20}$	$-\frac{1}{5}$	0	0	0	0	1	0

$\xrightarrow[\substack{R_3-4R_2 \\ R_4-6R_2 \\ R_5+\frac{1}{5}R_2}]{R_1-R_2}$

x	y	z	u	v	w	s	P	*Const.*	*Ratio*
$\frac{5}{4}$	$\frac{5}{4}$	0	1	$-\frac{1}{8}$	0	0	0	2000000	1600000
$-\frac{1}{4}$	$-\frac{1}{4}$	1	0	$\frac{1}{8}$	0	0	0	0	– –
–5	$\boxed{5}$	0	0	$-\frac{1}{2}$	1	0	0	0	0
$-\frac{17}{2}$	$\frac{15}{2}$	0	0	$-\frac{3}{4}$	0	1	0	0	0
$-\frac{3}{20}$	$-\frac{1}{5}$	0	0	$\frac{1}{40}$	0	0	1	0	

$\xrightarrow{\frac{1}{5}R_3}$

x	y	z	u	v	w	s	P	*Const.*
$\frac{5}{4}$	$\frac{5}{4}$	0	1	$-\frac{1}{8}$	0	0	0	2000000
$-\frac{1}{4}$	$-\frac{1}{4}$	1	0	$\frac{1}{8}$	0	0	0	0
–1	$\boxed{1}$	0	0	$-\frac{1}{10}$	$\frac{1}{5}$	0	0	0
$-\frac{17}{2}$	$\frac{15}{2}$	0	0	$-\frac{3}{4}$	0	1	0	0
$-\frac{3}{20}$	$-\frac{1}{5}$	0	0	$\frac{1}{40}$	0	0	1	0

$\xrightarrow[\substack{R_2+\frac{1}{4}R_3 \\ R_4-\frac{15}{2}R_3 \\ R_5+\frac{1}{5}R_3}]{R_1-\frac{5}{4}R_3}$

x	y	z	u	v	w	s	P	*Const.*	*Ratio*
$\boxed{\frac{5}{2}}$	0	0	1	$-\frac{1}{4}$	0	0	0	2000000	800000
$-\frac{1}{2}$	0	1	0	$\frac{1}{10}$	$\frac{1}{20}$	0	0	0	– –
-1	1	0	0	$-\frac{1}{10}$	$\frac{1}{5}$	0	0	0	0
-1	0	0	0	0	$-\frac{3}{2}$	1	0	0	0
$-\frac{7}{20}$	0	0	0	$\frac{1}{200}$	$\frac{1}{25}$	0	1	0	

$\xrightarrow{\frac{2}{5}R_1}$

x	y	z	u	v	w	s	P	*Const.*
$\boxed{1}$	0	0	$\frac{2}{5}$	0	$-\frac{1}{10}$	0	0	800000
$-\frac{1}{2}$	0	1	0	$\frac{1}{10}$	$\frac{1}{20}$	0	0	0
-1	1	0	0	$-\frac{1}{10}$	$\frac{1}{5}$	0	0	0
-1	0	0	0	0	$-\frac{3}{2}$	1	0	0
$-\frac{7}{20}$	0	0	0	$\frac{1}{200}$	$\frac{1}{25}$	0	1	0

$\xrightarrow[\substack{R_3+R_1 \\ R_4+R_1 \\ R_5+\frac{7}{20}R_1}]{R_2+\frac{1}{2}R_1}$

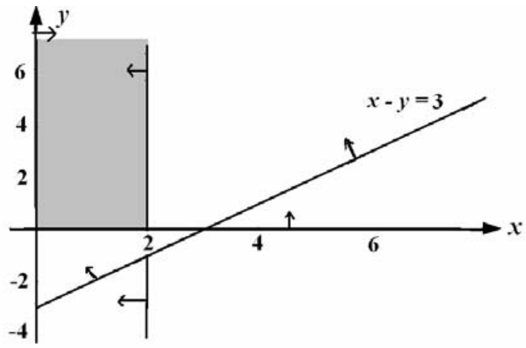
x	y	z	u	v	w	s	P	*Const.*
1	0	0	$\frac{2}{5}$	0	$-\frac{1}{10}$	0	0	800000
0	0	1	$\frac{1}{5}$	$\frac{1}{10}$	0	0	0	400000
0	1	0	$\frac{2}{5}$	$-\frac{1}{10}$	$\frac{1}{10}$	0	0	800000
0	0	0	$\frac{2}{5}$	0	$-\frac{8}{5}$	1	0	800000
0	0	0	$\frac{7}{50}$	$\frac{1}{200}$	$\frac{1}{200}$	0	1	280000

The last tableau is in final form, and we see that $x = 800{,}000$, $y = 800{,}000$, $z = 400{,}000$, and $P = 280{,}000$. Thus, the financier should invest \$800,000 each in Projects A and B, and \$400,000 in Project C. The maximum returns are \$280,000.

44. The problem is Maximize $P = 3x + 2y$ subject to

$$x - y \le 3$$
$$x \le 2$$
$$x \ge 0, y \ge 0$$

a.



b. This is evident from part (a).
c. Introduce the slack variables u and v. We have the following tableaus:

	x	y	u	v	P	Const	Ratio
p.r. →	1	−1	1	0	0	3	3
	[1]	0	0	1	0	2	2
	−3	−2	0	0	1	0	
	↑ p.c.						

$\xrightarrow[R_3+3R_2]{R_1-R_2}$

x	y	u	v	P	Const.
0	−1	1	−1	0	1
1	0	0	1	0	2
0	−2	0	3	1	6
	↑ p.c.				

Since the entries in the pivot column are 0, and –1, the ratios cannot be computed.
d. The first line in the second tableau in part (c) tells us that $-y+u-v=1$ or $y=u-v-1$. The last equation shows that y can be made as large as we please by taking u sufficiently large. Thus, P can be made as large as we please. So, no solution exists for this problem.

45. False. Consider the linear programming problem
Maximize $P = 2x + 3y$ subject to

$$-x+y\le 0$$
$$x\ge 0,\ y\ge 0$$

46. True. See the explanation of the simplex method on page 207 in the text.

47. True. Consider the objective function $P=c_1x+c_2x_2+\cdots c_nx_n$, which may be written in the form

$$-c_1x_1-c_2x_2-\cdots-c_nx_n+P=0$$

Observe that the most negative of the numbers $-c_1, -c_2, \cdots, -c_n$ (which are the numbers comprising the last row of the simplex tableau) is just the largest coefficient of x_i in the expression for *P*. Thus, moving in the direction of the variable with this coefficient ensures that *P* increases most.

48. True.

USING TECHNOLOGY EXERCISES 4.1, page 226

1. $x = 1.2, y = 0, z = 1.6, w = 0; P = 8.8$

2. $x = 5.33, y = 0, z = 2.67, w = 0; P = 21.33.$

3. $x = 1.6, y = 0, z = 0, w = 3.6; P = 12.4$

4. $x = 0, y = 3.33, z = 1.33, w = 0; P = 17.33.$

4.2 CONCEPT QUESTIONS, page 234

1. Maximize $P = -C = 3x + 5y$
subject to
$$5x + 2y \le 30$$
$$x + 3y \le 21$$
$$x \ge 0,\ y \ge 0$$

2. a. The objective function is to be minimized.
b. All the variables involved are nonnegative.
c. Each linear constraint may be written so that the expression involving the variables is greater than or equal to a constant.

3. The primal problem is the linear programming (maximization) problem associated with a minimization linear programming problem. The dual problem is the linear programming (minimization) problem associated with the maximization linear programming problem.

4. a. The primal problem has a solution if and only if the corresponding dual problem has a solution.