28. Let x, y, and z denote the number (in thousands) of bottles of formula *I*, formula *II*, and formula *III*, respectively, produced. Then the profit function to be maximized is $P = 180x + 200y + 300z$. Next, the limitation on time implies that $2.5x + 3y + 4z \le 70$. Similarly, the restrictions on the amount of ingredients available imply that $x \le 9$, $y \le 12$ and $z \le 6$. Summarizing, we have the following linear programming problem:

Maximize $P = 180x + 200y + 300z$ subject to

$$\begin{aligned} \tfrac{5}{2}x + 3y + 4z &\le 70 \\ x &\le 9 \\ y &\le 12 \\ z &\le 6 \\ x \ge 0, y \ge 0, z &\ge 0. \end{aligned}$$

29. False. The objective function $P = xy$ is not a linear function in x and y.

30. True. It satisfies the definition of a linear programming problem.

3.3 CONCEPT QUESTIONS, page 179

1. a. The feasible set is the set of points satisfying the constraints associated with the linear programming problem.
   b. A feasible solution of a linear programming problem is a point in the feasible set.
   c. An optimal solution of a linear programming problem is a feasible solution that also optimizes (maximizes or minimizes) the objective function.

2. In the method of corners, the following steps are followed to find the solution of a linear programming problem.
   a. Graph the feasible set. b. Find the corner points. c. Evaluate the objective function at each corner point. d. Find the vertex or vertices that optimize the solution.

EXERCISES 3.3, page 179

1. Evaluating the objective function at each of the corner points we obtain the following table.

Vertex	$Z = 2x + 3y$
(1, 1)	$\boxed{5}$
(8, 5)	31
(4, 9)	$\boxed{35}$
(2, 8)	28

From the table, we conclude that the maximum value of Z is 35 and it occurs at the vertex (4, 9). The minimum value of Z is 5 and it occurs at the vertex (1, 1).

2. Evaluating the objective function at each of the corner points we obtain the following table.

Vertex	$Z = 3x - y$
(2, 2)	4
(10, 1)	$\boxed{29}$
(7, 9)	12
(2, 6)	$\boxed{0}$

From the table, we conclude that the maximum value of Z is 29 and it occurs at the vertex (10, 1). The minimum value of Z is 0 and it occurs at the vertex (2, 6).

3. Evaluating the objective function at each of the corner points we obtain the following table.

Vertex	$Z = 3x + 4y$
(0, 20)	80
(3, 10)	49
(4, 6)	36
(9, 0)	$\boxed{27}$

From the graph, we conclude that there is no maximum value since Z is unbounded. The minimum value of Z is 27 and it occurs at the vertex (9, 0).

4. Evaluating the objective function at each of the corner points we obtain the following table.

Vertex	$Z = 7x + 9y$
(0, 7)	63
(1, 5)	52
(4, 2)	$\boxed{46}$
(8, 0)	56

From the graph, we conclude that there is no maximum value since Z is unbounded. The minimum value of Z is 46 and it occurs at the vertex (4, 2).

5. Evaluating the objective function at each of the corner points we obtain the following table.

Vertex	$Z = x + 4y$
(0, 6)	24
(4, 10)	$\boxed{44}$
(12, 8)	$\boxed{44}$
(15, 0)	15

From the table, we conclude that the maximum value of Z is 44 and it occurs at every point on the line segment joining the points (4, 10) and (12, 8). The minimum value of Z is 15 and it occurs at the vertex (15, 0).

6. Evaluating the objective function at each of the corner points we obtain the following table.

Vertex	$Z = 3x + 2y$
(1, 4)	11
(3, 1)	11
(5, 4)	23
(3, 6)	21

From the table, we conclude that the maximum value of Z is 23 and it occurs at the vertex (5, 4). The minimum value of Z is 11 and it occurs at every point on the line segment joining the points (1, 4) and (3, 1).

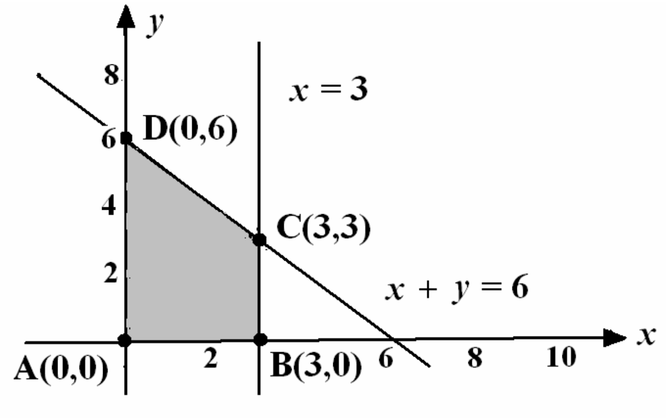
7. The problem is to maximize $P = 2x + 3y$ subject to

$$\begin{aligned} x + y &\le 6 \\ x &\le 3 \\ x \ge 0, y &\ge 0 \end{aligned}$$

The feasible set S for the problem is shown in the following figure, and the values of the function P at the vertices of S are summarized in the accompanying table.

Vertex	$P = 2x + 3y$
A(0, 0)	0
B(3, 0)	6
C(3, 3)	15
D(0, 6)	$\boxed{18}$

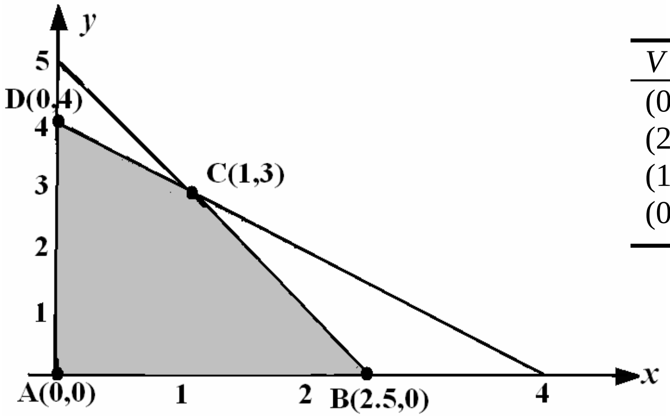
We conclude that P attains a maximum value of 18 when $x = 0$ and $y = 6$.



8. The problem is to maximize $P = x + 2y$ subject to

$$\begin{aligned} x + y &\le 4 \\ 2x + y &\le 5 \\ x \ge 0, y &\ge 0 \end{aligned}$$

The feasible set for the problem is shown in the following figure. From the table we conclude that the maximum value is 8 when $x = 0$ and $y = 4$.

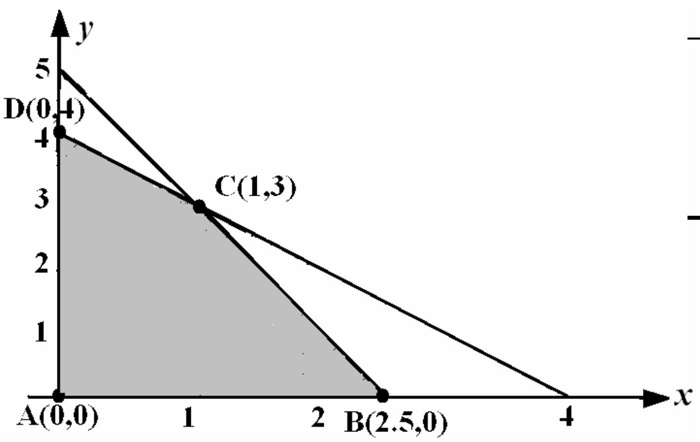


V	$P = x + 2y$
(0, 0)	0
(2.5, 0)	2.5
(1, 3)	7
(0, 4)	$\boxed{8}$

9. The problem is to maximize $P = 2x + y$ subject to

$$x + y \le 4$$
$$2x + y \le 5$$
$$x \ge 0, y \ge 0$$

Referring to the following figure and table



V	$P = 2x + y$
(0, 0)	0
(2.5, 0)	$\boxed{5}$
(1, 3)	$\boxed{5}$
(0, 4)	4

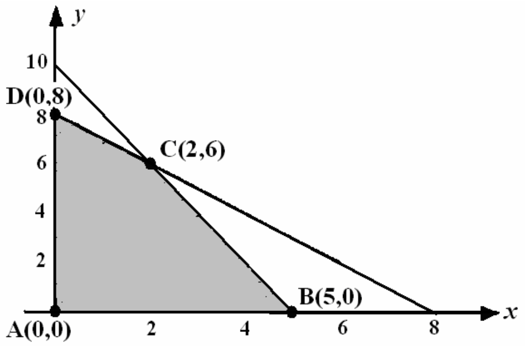
we conclude that P attains a maximum value of 5 at any point (x, y) lying on the line segment joining (1, 3) to (2.5, 0).

10. The problem is

Maximize $P = 4x + 2y$ subject to

$$x + y \le 8$$
$$2x + y \le 10$$
$$x \ge 0, y \ge 0$$

From the following figure and table



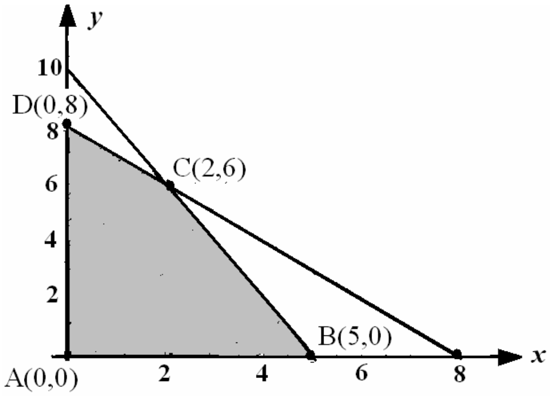
V	$P = 4x + 2y$
(0, 0)	0
(5, 0)	$\boxed{20}$
(2, 6)	$\boxed{20}$
(0, 8)	16

we conclude that *P* attains a maximum value of 20 at any point (*x*, *y*) lying on the line segment joining (2, 6) to (5, 0).

11. The problem is

$$\text{Maximize } P = x + 8y \text{ subject to}$$
$$x + y \leq 8$$
$$2x + y \leq 10$$
$$x \geq 0, y \geq 0$$

From the following figure and table



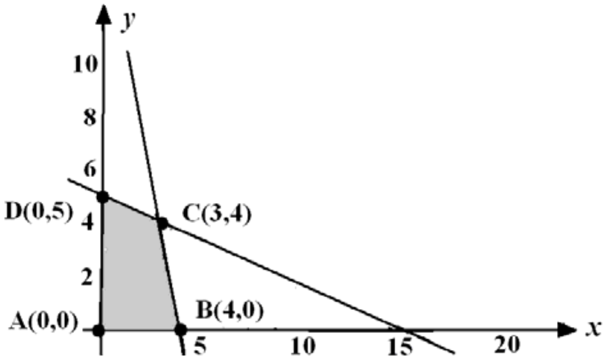
V	*P* = *x* + 8*y*
A(0, 0)	0
B(5, 0)	5
C(2, 6)	50
D(0, 8)	$\boxed{64}$

we conclude that *P* attains a maximum value of 64 when *x* = 0 and *y* = 8.

12. The problem is to maximize *P* = 3*x* - 4*y* subject to

$$x + 3y \leq 15$$
$$4x + y \leq 16$$
$$x \geq 0, \ \ y \geq 0$$

The feasible set *S* for the problem is shown in the following figure, and the values of the function *P* at the vertices of *S* are summarized in the accompanying table.



Vertex	$P = 3x - 4y$
A(0, 0)	0
B(4, 0)	12
C(3, 4)	-7
D(0, 5)	-20

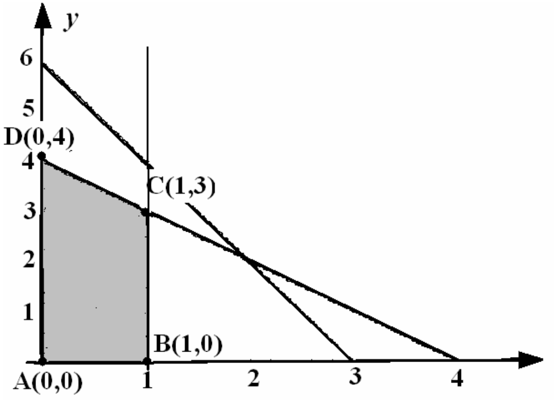
We conclude that P attains a maximum value of 12 when $x = 4$ and $y = 0$.

13. The linear programming problem is

Maximize $P = x + 3y$ subject to

$$\begin{aligned} 2x + y &\le 6 \\ x + y &\le 4 \\ x &\le 1 \\ x \ge 0, y &\ge 0 \end{aligned}$$

From the following figure and table,



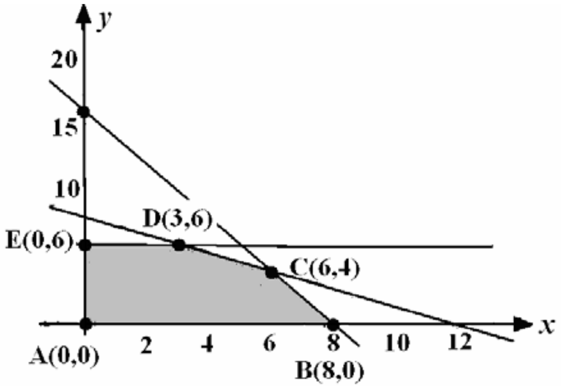
V	$P = x + 3y$
A(0, 0)	0
B(1, 0)	1
C(1, 3)	10
D(0, 4)	12

we conclude that P attains a maximum value of 12 when $x = 0$ and $y = 4$.

14. The problem is to maximize $P = 2x + 5y$ subject to

$$\begin{aligned} 2x + y &\le 16 \\ 2x + 3y &\le 24 \\ y &\le 6 \\ x \ge 0, y &\ge 0 \end{aligned}$$

The feasible set S for the problem is shown in the following figure, and the values of the function P at the vertices of S are summarized in the accompanying table.



Vertex	$P = 2x + 5y$
A(0, 0)	0
B(8, 0)	16
C(6, 4)	32
D(3, 6)	$\boxed{36}$
E(0, 6)	30

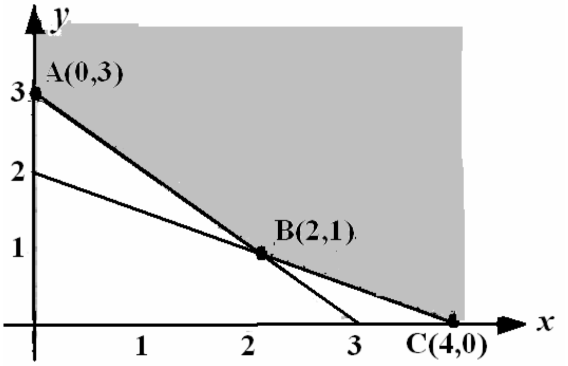
We conclude that P attains a maximum value of 36 when $x = 3$ and $y = 6$.

15. The linear programming problem is

Minimize $C = 3x + 4y$ subject to

$$\begin{aligned} x + y &\ge 3 \\ x + 2y &\ge 4 \\ x \ge 0, y &\ge 0 \end{aligned}$$

From the following figure and table,



V	$C = 3x + 4y$
A(0, 3)	12
B(2, 1)	$\boxed{10}$
C(4, 0)	12

we conclude that C attains a minimum value of 10 when $x = 2$ and $y = 1$.

16. The linear programming problem is

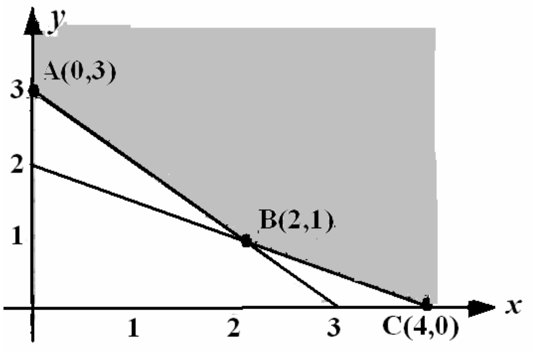
$$\text{Minimize } C = 2x + 4y \text{ subject to}$$

$$x + y \geq 3$$

$$x + 2y \geq 4$$

$$x \geq 0, y \geq 0$$

From the following figure and table,



V	$C = 2x + 4y$
A(0, 3)	12
B(2, 1)	$\boxed{8}$
C(4, 0)	$\boxed{8}$

we conclude that C attains a minimum value of 8 at any point on the line segment joining (2, 1) to (4, 0).

17. The linear programming problem is

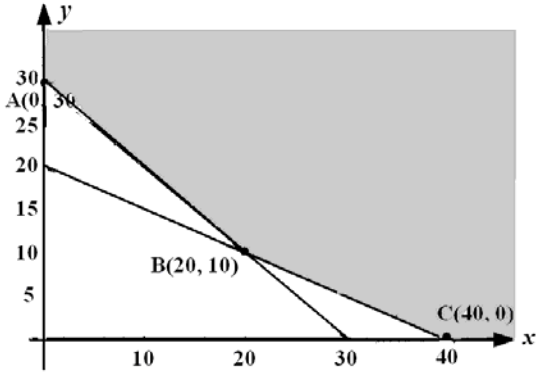
$$\text{Minimize } C = 3x + 6y \text{ subject to}$$

$$x + 2y \geq 40$$

$$x + y \geq 30$$

$$x \geq 0, y \geq 0$$

From the following figure and table,



V	$C = 3x + 6y$
A(0, 30)	180
B(20, 10)	120
C(40, 0)	120

we conclude that C attains a minimum value of 120 at any point on the line segment joining (20, 10) to (40, 0).

18. Refer to the figure in Exercise 17. From the following table,

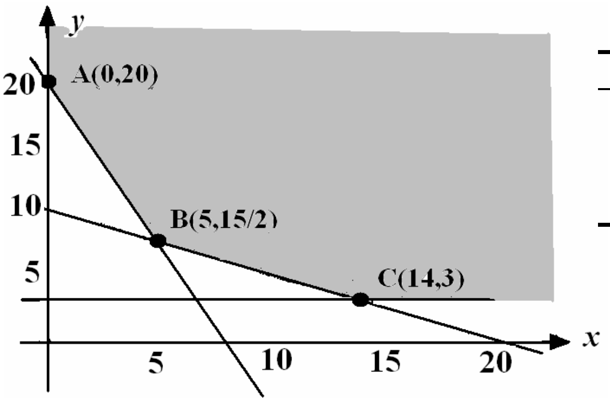
V	$C = 3x + y$
A(0, 30)	$\boxed{30}$
B(20, 10)	70
C(40, 0)	120

we conclude that C attains a minimum value of 30 when $x = 0$ and $y = 30$.

19. The problem is

$$\text{Minimize } C = 2x + 10y \text{ subject to}$$
$$5x + 2y \ge 40$$
$$x + 2y \ge 20$$
$$y \ge 3,\ \ x \ge 0$$

The feasible set S for the problem is shown in the following figure and the values of the function C at the vertices of S are summarized in the accompanying table.



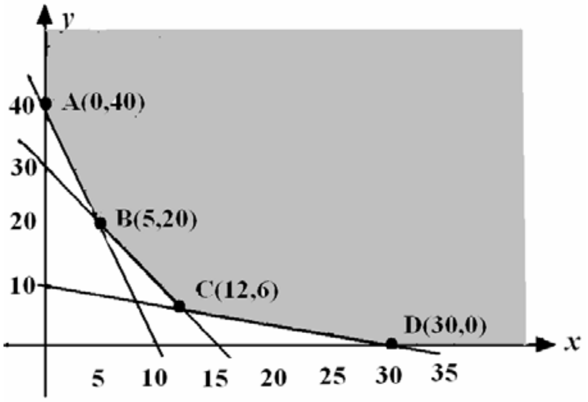
Vertex	$C = 2x + 10y$
A(0, 20)	200
B(5, $\frac{15}{2}$)	85
C(14, 3)	$\boxed{58}$

We conclude that C attains a minimum value of 58 when $x = 14$ and $y = 3$.

20. The problem is to minimize $C = 2x + 5y$ subject to

$$\begin{aligned} 4x + y &\geq 40 \\ 2x + y &\geq 30 \\ x + 3y &\geq 30 \\ x \geq 0, y &\geq 0 \end{aligned}$$

The feasible set *S* is shown in the following figure, and the values of *C* at each of the vertices of *S* are shown in the accompanying table.



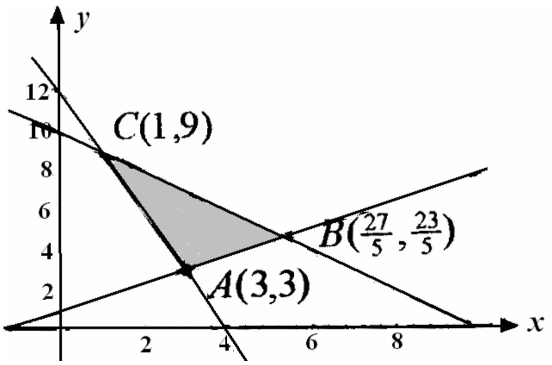
Vertex	$C = 2x + 5y$
A(0, 40)	200
B(5, 20)	110
C(12, 6)	$\boxed{54}$
D(30, 0)	60

We conclude that *C* attains a minimum value of 54 when $x = 12$ and $y = 6$.

21. The problem is to minimize $C = 10x + 15y$ subject to

$$\begin{aligned} x + y &\leq 10 \\ 3x + \quad y &\geq 12 \\ -2x + 3y &\geq 3 \\ x \geq 0,\ y &\geq 0 \end{aligned}$$

The feasible set is shown in the following figure, and the values of *C* at each of the vertices of *S* are shown in the accompanying table.



Vertex	$C = 10x + 15y$
A(3, 3)	$\boxed{75}$
B($\frac{27}{5}$, $\frac{23}{5}$)	123
C(1, 9)	145

We conclude that C attains a minimum value of 75 when $x = 3$ and $y = 3$.

22. The problem is to maximize $P = 2x + 5y$ subject to the constraints of Exercise 21. Since the graph of the feasible set S has already been sketched (see Exercise 21), we need only compute the value of P at each of the vertices of the feasible set S. Thus,

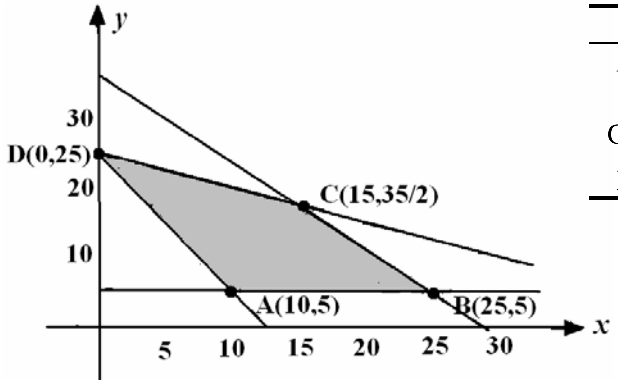
Vertex	$P = 2x + 5y$
A(3, 3)	21
B($\frac{27}{5}$, $\frac{23}{5}$)	33.8
C(1, 9)	$\boxed{47}$

We conclude that P attains a maximum value of 47 when $x = 1$ and $y = 9$.

23. The problem is to maximize $P = 3x + 4y$ subject to

$$\begin{aligned} x + 2y &\le 50 \\ 5x + 4y &\le 145 \\ 2x + \ y &\ge 25 \\ y \ge 5, x &\ge 0 \end{aligned}$$

The feasible set S is shown in the figure that follows, and the values of P at each of the vertices of S are shown in the accompanying table.



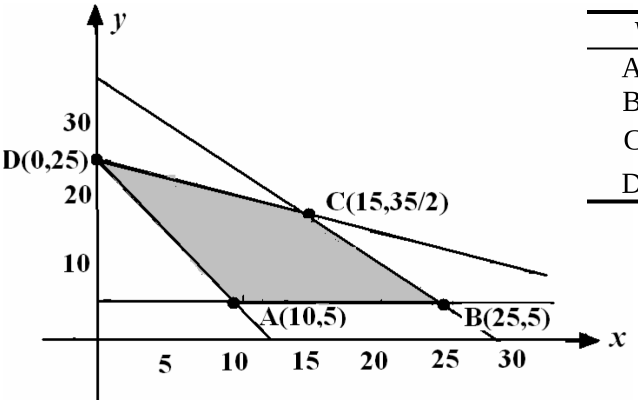
Vertex	$P = 3x + 4y$
A(10, 5)	50
B(25, 5)	95
C(15, $\frac{35}{2}$)	$\boxed{115}$
D(0, 25)	100

We conclude that P attains a maximum value of 115 when $x = 15$ and $y = 35/2$.

24. The problem is to maximize $P = 4x - 3y$ subject to

$$\begin{aligned} x + 2y &\le 50 \\ 5x + 4y &\le 145 \\ 2x + \ \ y &\ge 25 \\ y \ge 5, x &\ge 0 \end{aligned}$$

The feasible set S is shown in the following figure, and the values of P at each of the vertices of S are shown in the accompanying table. We conclude that P attains a maximum value of 85 when $x = 25$ and $y = 5$.

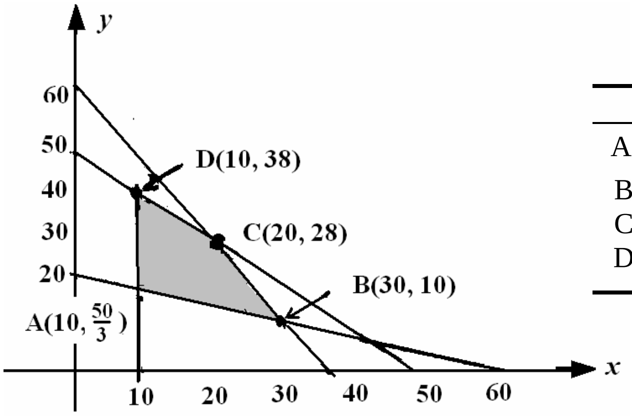


Vertex	$P = 4x - 3y$
A(10, 5)	25
B(25, 5)	85
C(15, $\frac{35}{2}$)	$\frac{15}{2}$
D(0, 25)	-75

25. The problem is to maximize $P = 2x + 3y$
subject to

$$\begin{aligned} x + \ \ y &\le \ 48 \\ x + 3y &\ge \ 60 \\ 9x + 5y &\le 320 \\ x \ge 10, y &\ge 0 \end{aligned}$$

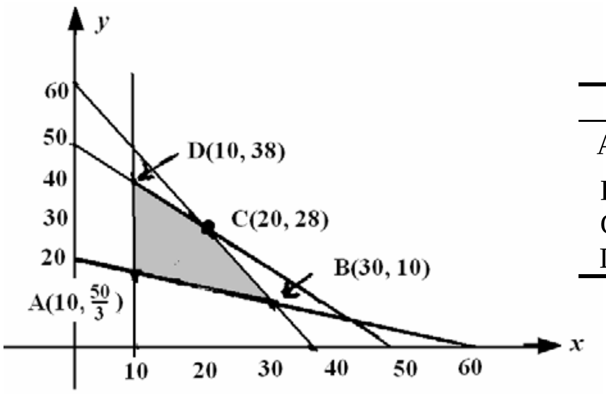
The feasible set S is shown in the figure that follows, and the values of P at each of the vertices of S are shown in the accompanying table.



Vertex	$P = 2x + 3y$
A(10, $\frac{50}{3}$)	70
B(30, 10)	90
C(20, 28)	124
D(10, 38)	$\boxed{134}$

We conclude that P attains a maximum value of 134 when $x = 10$ and $y = 38$.

26. The problem is to minimize $C = 5x + 3y$ subject to the constraints of Exercise 25. The feasible set is below. The table of values for the objective function follows.



Vertex	$C = 5x + 3y$
A(10, $\frac{50}{3}$)	$\boxed{100}$
B(30, 10)	180
C(20, 28)	184
D(10, 38)	164

We conclude that C attains a minimum value of 100 when $x = 10$ and $y = 50/3$.

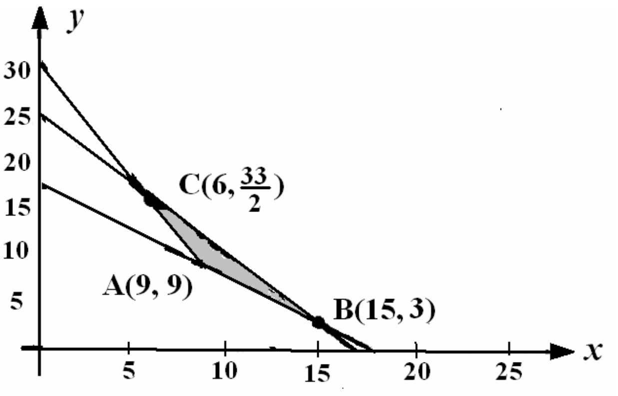
27. The problem is to find the maximum and minimum value of $P = 10x + 12y$ subject to

$$\begin{aligned} 5x + 2y &\geq 63 \\ x + y &\geq 18 \\ 3x + 2y &\leq 51 \\ x \geq 0, y &\geq 0 \end{aligned}$$

The feasible set is shown in the table that follows and the values of P at each of the vertices of S are shown in the accompanying table.

Vertex	*P* = 10*x* + 12*y*
A(9, 9)	198
B(15,3)	186
C(6, $\frac{33}{2}$)	258

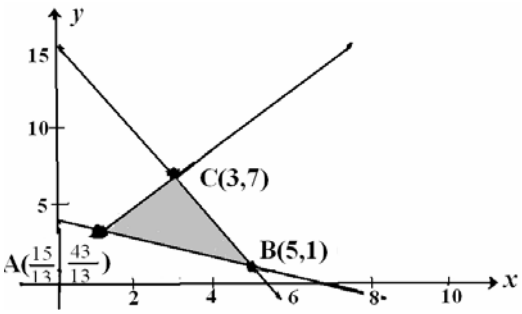
P attains a maximum value of 258 when $x = 6$ and $y = 33/2$. The minimum value of *P* is 186. It is attained when $x = 15$ and $y = 3$.



28. The problem is to find the maximum and minimum value of $P = 4x + 3y$ subject to

$$\begin{aligned} 3x+5y &\ge 20 \\ 3x+\ y &\le 16 \\ -2x+\ y &\le\ \ 1 \\ x \ge 0, y &\ge 0 \end{aligned}$$

The feasible set is shown in the following figure at the right and the values of *P* at each of the vertices of *S* are shown in the accompanying table.



Vertex	*P* = 4*x* + 3*y*
A($\frac{15}{13}$, $\frac{43}{13}$)	$14\frac{7}{13}$
B(5, 1)	23
C(3, 7)	33

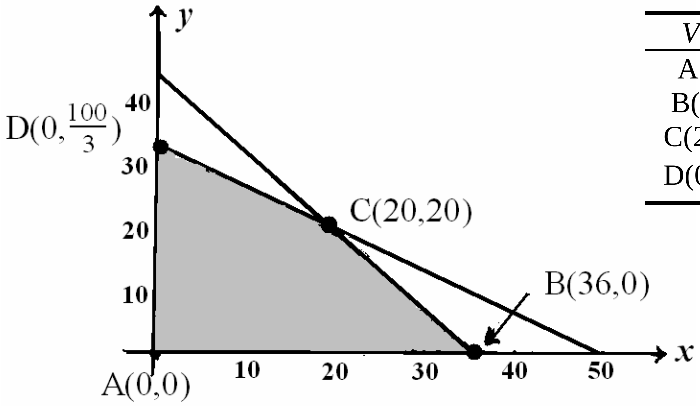
P attains a maximum value of 33 when $x = 3$ and $y = 7$. The minimum value of *P* is $14\frac{7}{13}$. It is attained when $x = 15/13$ and $y = 43/13$.

29. Refer to the solution of Exercise 1, Section 3.2, The problem is

Maximize $P = 3x + 4y$ subject to

$$6x + 9y \le 300$$
$$5x + 4y \le 180$$
$$x \ge 0, y \ge 0$$

The graph of the feasible set S and the associated table of values of P follow.



Vertex	$C = 3x + 4y$
A(0, 0)	0
B(36, 0)	108
C(20, 20)	140
D(0, $\frac{100}{3}$)	$133\frac{1}{3}$

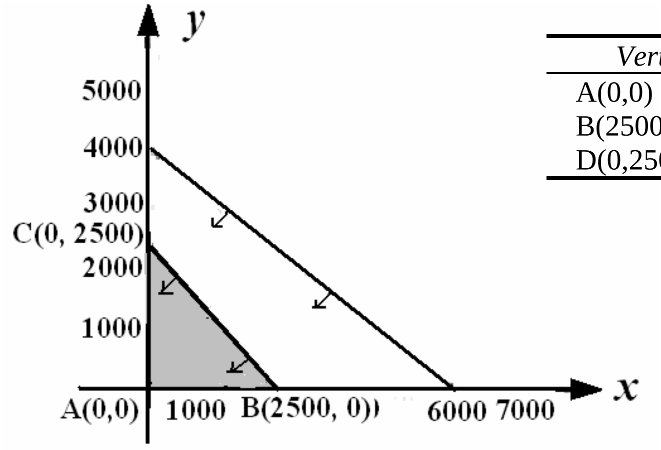
P attains a maximum value of 140 when $x = y = 20$. Thus, by producing 20 units of each product in each shift, the company will realize an optimal profit of $140.

30. Let x and y denote the number of model *A* and model *B* fax machines produced in each shift. Then the restriction on manufacturing costs implies $100x + 150y \le 600{,}000$, and the limitation on the number produced implies $x + y \le 2{,}500$. The total profit is $P = 30x + 40y$. Summarizing, we have the following linear programming problem.

Maximize $P = 30x + 40y$ subject to

$$\begin{aligned} 100x + 150y &\le 600{,}000 \\ x + y &\le 2{,}500 \\ x \ge 0,\ y &\ge 0 \end{aligned}$$

The graph of the feasible set S and the associated table of values of P follow.



Vertex	$C = 30x + 40y$
A(0,0)	0
B(2500,0)	75,000
D(0,2500)	100,000

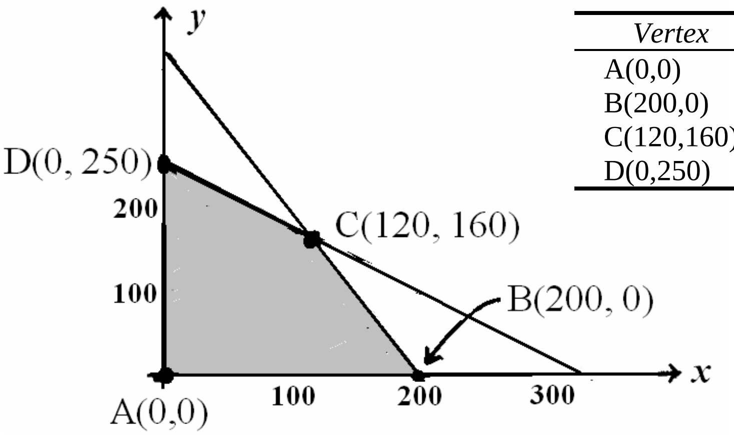
P attains a maximum value of 100,000 when $x = 0$ and $y = 2500$. Thus, by producing 2500 model *B* fax machines in each shift, the company will realize an optimal profit of $100,000.

31. Refer to the solution of Exercise 3, Section 3.2, The problem is

Maximize $P = 2x + 1.5y$ subject to

$$3x + 4y \le 1000$$
$$6x + 3y \le 1200$$
$$x \ge 0,\ y \ge 0$$

The graph of the feasible set *S* and the associated table of values of *P* follow.



Vertex	$P = 2x + 1.5y$
A(0,0)	0
B(200,0)	400
C(120,160)	480
D(0,250)	375

P attains a maximum value of 480 when $x = 120$ and $y = 160$. Thus, by producing

120 model *A* grates and 160 model *B* grates in each shift, the company will realize an optimal profit of $480.

32. Refer to the solution of Exercise 4, Section 3.2. The problem is

Maximize $P = 2x + 1.5y$ subject to

$$3x + 4y \le 1000$$

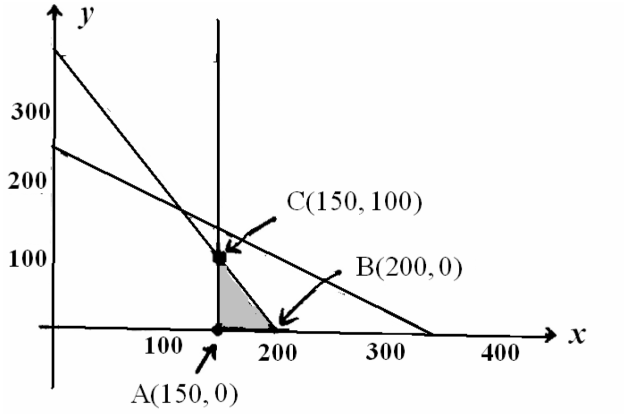
$$6x + 3y \le 1200$$

$$x \ge 150,\ y \ge 0$$

The graph of the feasible set S and the associated table of values of P follow.

Vertex	*P* = 2x + 1.5y
A(150,0)	300
B(200,0)	400
C(150,100)	450

P attains a maximum value of 450 when x = 150 and y = 100. Thus, by producing 150 model A grates and 100 model B grates in each shift, the company will realize an optimal profit of $450.



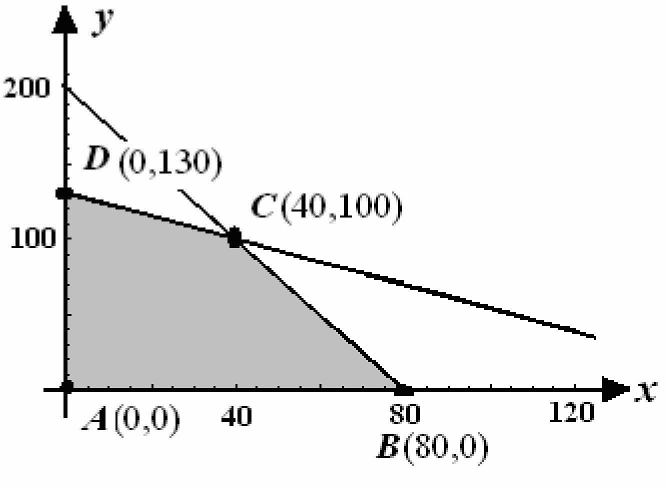
33. Let *x* denote the number of tables and *y* denote the number of chairs to be manufactured. Then the linear programming problem is

Maximize $P = 45x + 20y$ subject to

$$40x + 16y \le 3200$$

$$3x + 4y \le 520$$

$$x \ge 0,\ y \ge 0$$



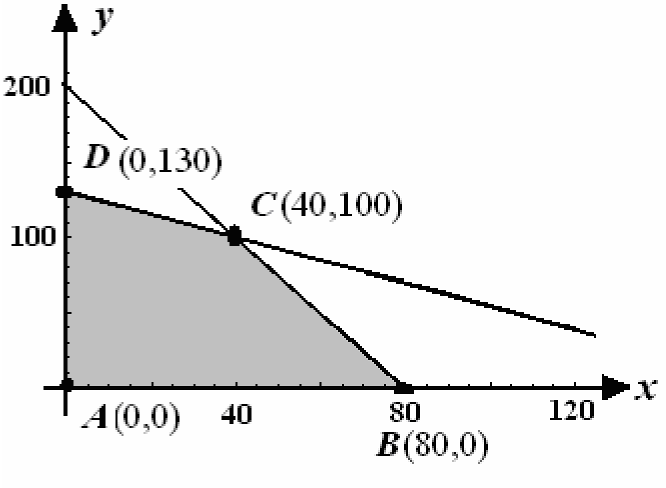
Vertex	$P=45x+20y$
$A(0,0)$	0
$B(80,0)$	3600
$C(40,100)$	3800
$D(0,130)$	2600

We see that Winston should manufacture 40 tables and 100 chairs for a maximum profit of \$3800.

34. Let x denote the number of tables and y denote the number of chairs to be manufactured. Then the linear programming problem is

Maximize $P=50x+18y$ subject to

$$40x+16y\le 3200$$
$$3x+4y\le 520$$
$$x\ge 0,\ y\ge 0$$



Vertex	$P=50x+18y$
$A(0,0)$	0
$B(80,0)$	4000
$C(40,100)$	3800
$D(0,130)$	2340

We see that Winston should manufacture 80 tables for a maximum profit of \$4000.

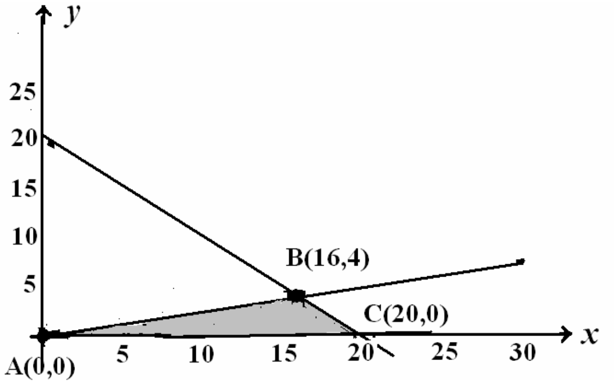
35. Refer to the solution of Exercise 5, Section 3.2. The linear programming problem is

Maximize $P = 0.1x + 0.12y$ subject to

$$\begin{aligned} x + \quad y &\le 20 \\ x - 4y &\ge 0 \\ x \ge 0,\ y &\ge 0 \end{aligned}$$

The feasible set S for the problem is shown in the figure at the right, and the value of P at each of the vertices of S is shown in the accompanying table.

Vertex	$C = 0.1x + 0.12y$
A(0,0)	0
B(16,4)	2.08
C(20,0)	2.00



The maximum value of P is attained when $x = 16$ and $y = 4$.
Thus, by extending \$16 million in housing loans and \$4 million in automobile loans, the company will realize a return of \$2.08 million on its loans.

36. Let x and y denote the amount (in thousands of dollars) to be invested in project A and project B, respectively. Since the amount available for investment is up to \$500,000, we have $x + y \le 500$. Next, the condition on the allocation of the funds implies that

$$y \le 0.4(x + y),\ -0.4x + 0.6y \le 0, \text{ or } -2x + 3y \le 0$$

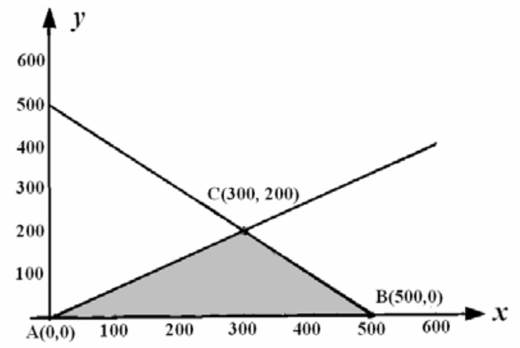
The linear programming problem at hand is

Maximize $P = 0.1x + 0.15y$

subject to

$$\begin{aligned} x + \quad y &\le 500 \\ -2x + 3y &\le \quad 0 \\ x \ge 0, y &\ge 0 \end{aligned}$$

From the following figure and table



Vertex	$P = 0.1x + 0.15y$
A(0, 0)	0
B(500, 0)	50
C(300, 200)	$\boxed{60}$

we see that the maximum of *P* occurs at C(300, 200) and has a value of 60. So, she should invest \$300,000 in project *A* and \$200,000 in project *B*. The maximum return is \$60,000.

37. Refer to Exercise 7, Section 3.2. The problem is

Maximize $P = 50x + 40y$ subject to

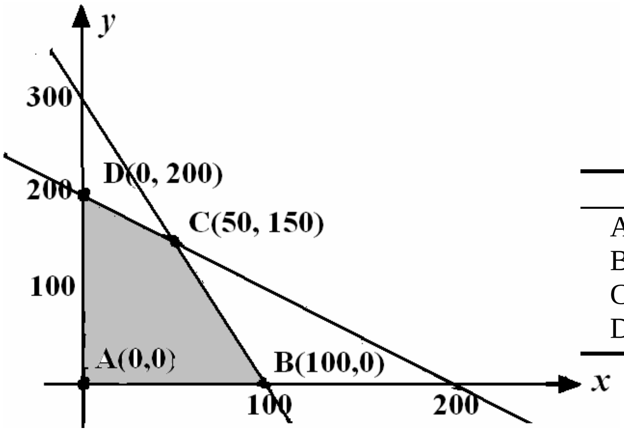
$$\frac{1}{200}x + \frac{1}{200}y \le 1$$
$$\frac{1}{100}x + \frac{1}{300}y \le 1$$
$$x \ge 0,\ y \ge 0.$$

This system may be rewritten in the equivalent form

Maximize $P = 50x + 40y$ subject to

$$x + y \le 200$$
$$3x + y \le 300$$
$$x \le 0,\ y \le 0$$

The graph of the feasible set S and the associated table of values of *P* follow.



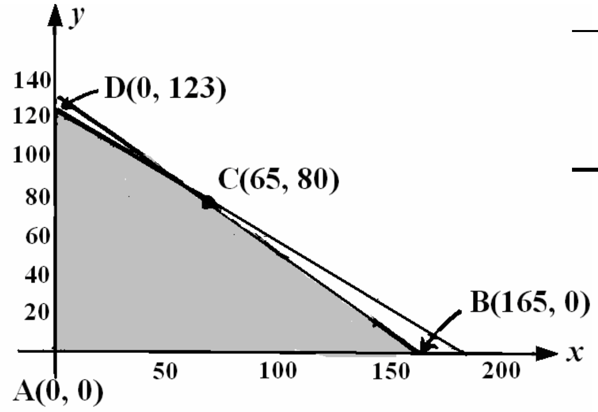
Vertex	$P = 50x + 40y$
A(0,0)	0
B(100,0)	5000
C(50, 150)	8500
D(0, 200)	8000

We conclude that the company should produce 50 fully assembled units and 150 kits daily in order to realize a profit of $8500.

38. Refer to Exercise 8, Section 3.2. The problem is

$$\text{Maximize } P = 150x + 200y \text{ subject to}$$
$$40x + 60y \le 7400$$
$$20x + 25y \le 3300$$
$$x \ge 0,\ y \ge 0$$

The graph of the feasible set S and the associated table of values of P follow.



Vertex	*P = 150x + 200y*
A(0,0)	0
B(165,0)	24,750
C(65,80)	25,750
D(0,123)	24,600

P attains a maximum value of 25,750 when $x = 65$ and $y = 80$. Thus, by producing

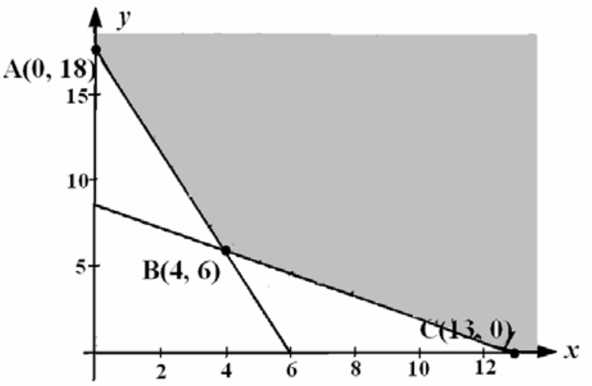
65 acres of crop *A* and 80 acres of crop *B*, the farmer will realize a maximum profit of $25,750.

39. Let x and y denote the number of days the Saddle Mine and the Horseshoe Mine are operated, respectively. Then the operating cost is $C = 14{,}000x + 16{,}000y$. The amount of gold produced in the two mines is $(50x + 75y)$ oz, and this amount must be at least 650 oz. So we have $50x + 75y \geq 650$. Similarly, the requirement for silver production leads to the inequality $3000x + 1000y \geq 18{,}000$. So the problem is

Minimize $C = 14{,}000x + 16{,}000y$ subject to

$$\begin{aligned} 50x + \quad 75y &\geq 650 \\ 3000x + 1000y &\geq 18{,}000 \\ x \geq 0,\ y &\geq 0 \end{aligned}$$

The feasible set is shown in the accompanying figure. From the table, we see that the minimum value of $C = 152{,}000$ is attained at $x = 4$ and $y = 6$. So, the Saddle Mine should be operated for 4 days and the Horseshoe Mine should be operated for 6 days at a minimum cost of $152,000/day.

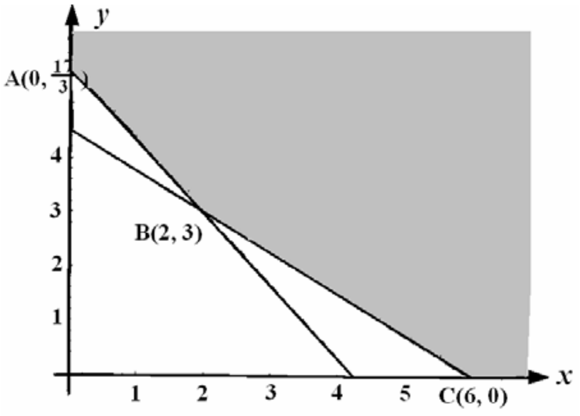


Vertex	$C = 14{,}000x + 16{,}000y$
A(0, 18)	288,000
B(4, 6)	$\boxed{152{,}000}$
C(13, 0)	182,000

40. Let x and y denote the number of type-A and type-B vessels to be used, respectively. Then the problem is

Minimize $C = 44{,}000x + 54{,}000y$ subject to

$$\begin{aligned} 60x + \ 80y &\geq 360 \\ 160x + 120y &\geq 680 \\ x \geq 0,\ y &\geq 0 \end{aligned}$$



Vertex	$C = 44{,}000x + 54{,}000y$
A(0, $\frac{17}{3}$)	306,000
B(2, 3)	$\boxed{250{,}000}$
C(6, 0)	264,000

The feasible set is shown in the accompanying figure. From the table, we see that the minimum value of C is 250,000 attained at $x = 2$ and $y = 3$. Thus, Deluxe River Cruises should use 2 type-A vessels and 3 type-B vessels. The minimum operating cost is $250,000.

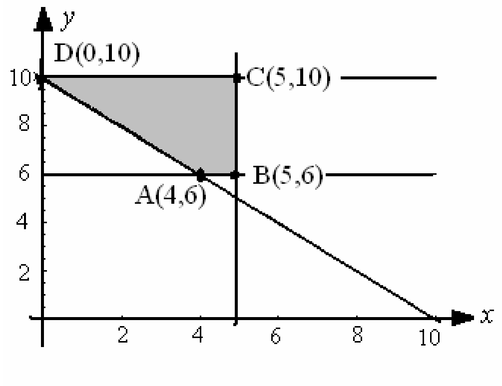
41. Let x denote the number of gallons of water in millions obtained from the local reservoir per day and let y denote the number of gallons of water in millions obtained from the pipeline. Then, we have the following linear programming problem

$$\text{Minimize } C = 300x + 500y \text{ subject to}$$
$$\begin{aligned} x + y &\geq 10 \\ x &\leq 5 \\ 6 \leq y &\leq 10 \\ x &\geq 0 \end{aligned}$$

The feasible set is shown in the accompanying figure. From the table, we see that the minimum value of C is 4200 and it is attained at $x = 4$ and $y = 6$. Thus, 400 million gallons should be obtained from the reservoir and 6 million gallons from the pipeline at a minimum cost of $4200.

Vertex	$C = 300x + 500y$
A(4,6)	4200
B(5,6)	4500
C(5,10)	6500
D(0,10)	5000

.



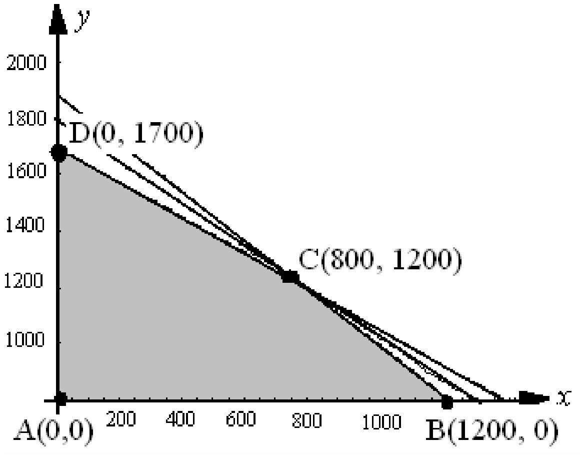
42. Let x and y denote the number of pandas and Saint Bernards produced, respectively. Then the linear programming problem is

Maximize $P = 10x + 15y$ subject to

$$\begin{aligned} 1.5x + 2y &\leq 3600 \\ 30x + 35y &\leq 6600 \\ 5x + 8y &\leq 13,600 \\ x \geq 0,\ y &\geq 0 \end{aligned}$$

The feasible set is shown in the accompanying figure. From the table, we see that the maximum value of P is 26,000 and it is attained at x = 800 and y = 1200. Thus, 800 pandas and 1200 Saint Bernards should be produced for a maximum profit of $26,000.

Vertex	$P = 10x + 15y$
A(0,0)	0
B(1200, 0)	1200
C(800, 1200)	26,000
D(0, 1700)	25,500



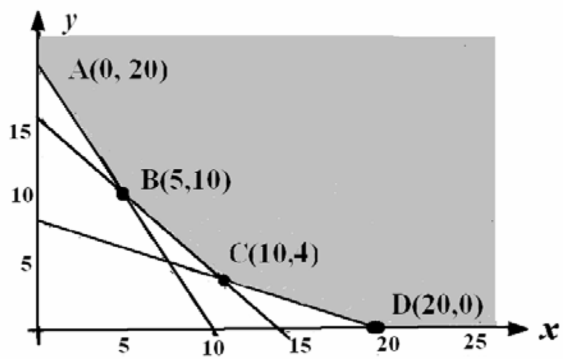
43. Refer to Exercise 11, Section 3.2. The problem is

Minimize $C = 2x + 5y$ subject to

$$\begin{aligned} 30x + 25y &\geq 400 \\ x + 0.5y &\geq 10 \\ 2x + 5y &\geq 40 \\ x \geq 0,\ y &\geq 0 \end{aligned}$$

The graph of the feasible set S and the associated table of values of C follow.

Vertex	$C = 2x + 5y$
A(0,20)	100
B(5,10)	60
C(10,4)	40
D(20,0)	40



C attains a minimum value of 40 when $x = 10$ and $y = 4$ and $x = 20$, and $y = 0$. This means that any point lying on the line joining the points (10,4) and (20,0) will satisfy these constraints. For example, we could use 10 ounces of food *A* and 4 ounces of food *B*, or we could use 20 ounces of food *A* and zero ounces of food *B*.

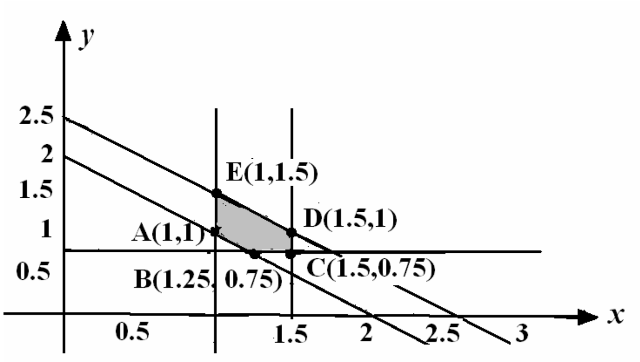
44. Refer to the solution of Exercise 12, Section 3.2. The problem is

Maximize $P = 0.6x + 0.8y$ subject to

$$\begin{aligned} x + y &\leq 2.5 \\ x + y &\geq 2 \\ 1 \leq x &\leq 1.5 \\ y &\geq 0.75 \end{aligned}$$

The feasible set S for the problem is shown in the figure at the right, and the value of P at each of the vertices of S is given in the table that follows.

Vertex	$P = 0.6x + 0.8y$
A(1,1)	1.4
B(1.25,0.75)	1.35
C(1.5,0.75)	1.5
D(1.5,1)	1.7
E(1,1.5)	1.8



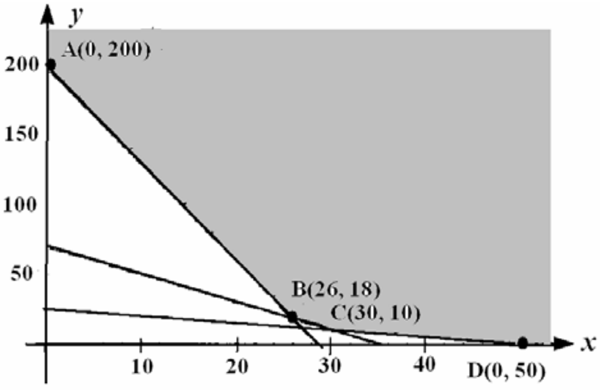
We conclude that the maximum value of P occurs when $x = 1$ and $y = 1.5$. Therefore, AntiFam should give $1 million in aid to country *A* and $1.5 million in aid to country *B*.

45. Let x and y denote the number of advertisements to be placed in newspaper I and newspaper II, respectively. Then the problem is

Minimize $C = 1000x + 800y$ subject to

$$70,000x + 10,000y \geq 2,000,000$$
$$40,000x + 20,000y \geq 1,400,000$$
$$20,000x + 40,000y \geq 1,000,000$$
$$x \geq 0, y \geq 0$$

The feasible set is shown in the accompanying figure.



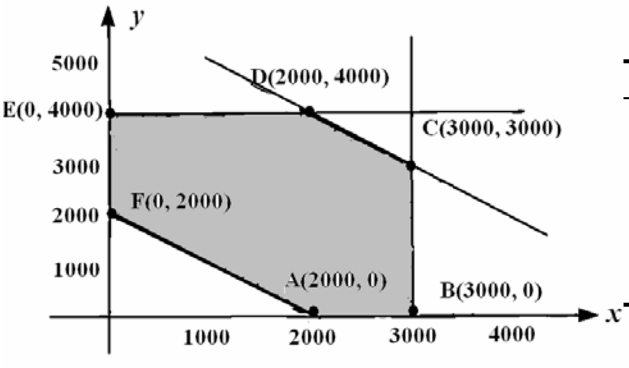
V	$C = 1000x + 800y$
A(0,200)	200,000
B(26,18)	40,400
C(30,10)	$\boxed{38,000}$
D(0,50)	40,000

From the table, we see that the minimum value of C of 38,000 is attained at $x = 30$ and $y = 10$. Thus, Everest Deluxe World Travel should place 30 advertisements in newspaper I, and 10 advertisements in newspaper II at a total (minimum) cost of $38,000.

46. Refer to the solution of Exercise 18, Section 3.2.

Minimize $C = 64{,}000 - 2x - 6y$ subject to

$$\begin{aligned} x + y &\le 6000 \\ x + y &\ge 2000 \\ x &\le 3000 \\ y &\le 4000 \\ x \ge 0,\ y &\ge 0 \end{aligned}$$



Vertex	$C = 64000 - 2x - 6y$
A(2000,0)	60,000
B(3000,0)	58,000
C(3000,3000)	40,000
D(2000,4000)	36,000
E(0,4000)	40,000
F(0,2000)	52,000

Since x denotes the number of televisions shipped from location *I* to city *A* and y denotes the number of televisions shipped from location *I* to city *B*, we see that the company should ship 2000 tubes from location I to city *A* and 4000 tubes from location *I* to city *B*. Since the number of televisions required by the two factories in city *A* and city *B* are 3000 and 4000, respectively, the number of televisions shipped from location *II* to city *A* and city *B*, are

$(3000 - x) = 3000 - 2000 = 1000$ and $(4000 - y) = 4000 - 4000 = 0$

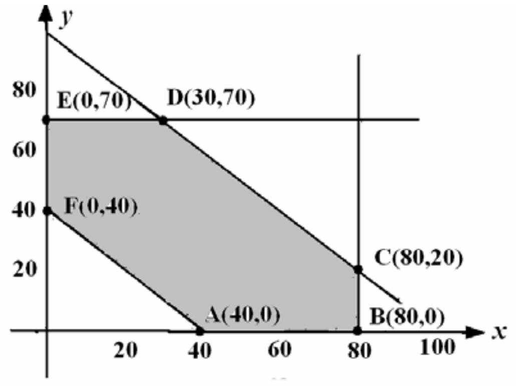
respectively. The minimum shipping cost will then be $36,000.

47. The problem is

Minimize $C = 14{,}500 - 20x - 10y$ subject to

$$\begin{aligned} x + y &\ge 40 \\ x + y &\le 100 \\ 0 \le x &\le 80 \\ 0 \le y &\le 70 \end{aligned}$$

The feasible set S for the problem is shown in the figure at the right, and the value of C at each of the vertices of S is given in the accompanying table.



Vertex	$C = 14{,}500 - 20x - 10y$
A(40,0)	13,700
B(80,0)	12,900
C(80,20)	12,700
D(30,70)	13,200
E(0,70)	13,800
F(0,40)	14,100

We conclude that the minimum value of C occurs when $x = 80$ and $y = 20$. Thus, 80 engines should be shipped from plant *I* to assembly plant *A*, and 20 engines should be shipped from plant *I* to assembly plant *B*; whereas

$$(80 - x) = 80 - 80 = 0, \quad \text{and} \quad (70 - y) = 70 - 20 = 50$$

engines should be shipped from plant *II* to assembly plants *A* and *B*, respectively, at a total cost of $12,700.

48. Let x and y denote the number of luxury and standard model steppers, respectively, produced each day. Then we have the following linear programming problem:

Maximize $P = 40x + 30y$ subject to

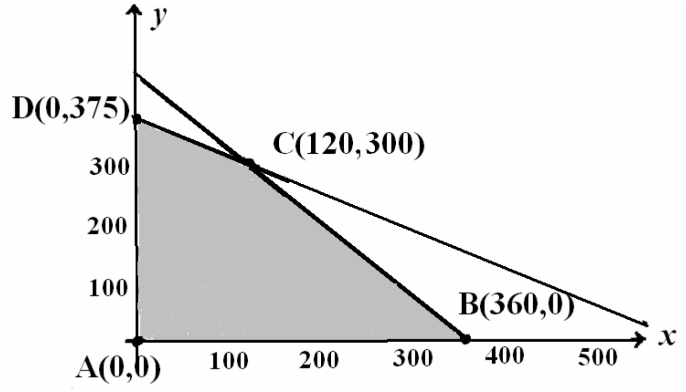
$$10x + 16y \le 6000$$

$$10x + 8y \le 3600$$

$$x \ge 0,\ y \ge 0$$

The graph of the feasible set S and the associated table of values of P follow.

Vertex	$C = 40x + 30y$
A(0,0)	0
B(360,0)	14,400
C(120,300)	13,800
D(0,375)	11,250



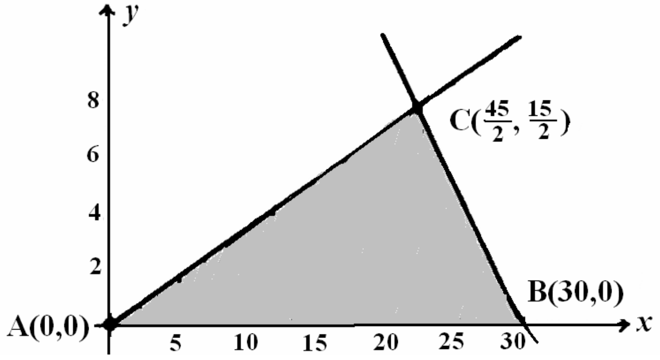
P attains a maximum value of 14400 when $x = 360$ and $y = 0$. Therefore, Bata should produce 360 luxury steppers per day in order to maximize its profits.

49. Let x denote Patricia's investment in growth stocks and y denote the value of her investment in speculative stocks, where both x and y are measured in thousands of dollars. Then the return on her investments is given by $P = 0.15x + 0.25y$. Since her investment may not exceed \$30,000, we have the constraint $x + y \leq 30$. The condition that her investment in growth stocks be at least 3 times as much as her investment in speculative stocks translates into the inequality $x \geq 3y$. Thus, we have the following linear programming problem:

$$\text{Maximize } P = 0.15x + 0.25y \text{ subject to}$$
$$\begin{aligned} x + y &\leq 30 \\ x - 3y &\geq 0 \\ x \geq 0,\ y &\geq 0 \end{aligned}$$

The graph of the feasible set S is shown in the figure at the right and the value of P at each of the vertices of S is shown in the accompanying table.

Vertex	$C = 0.15x + 0.25y$
A(0,0)	0
B(30,0)	4.5
$C(\frac{45}{2}, \frac{15}{2})$	5.25



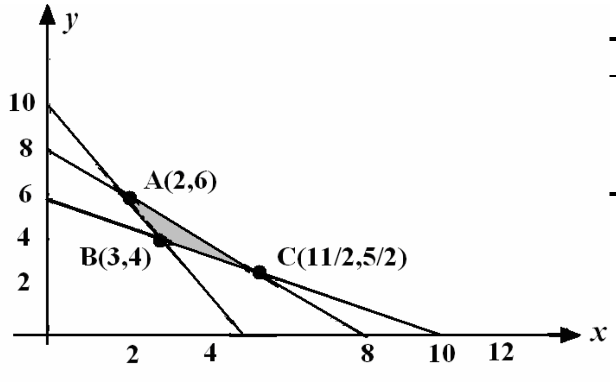
The maximum value of P occurs when $x = 22.5$ and $y = 7.5$. Thus, by investing \$22,500 in growth stocks and \$7,500 in speculative stocks. Patricia will realize a return of \$5250 on her investments.

50. Let x and y denote the number of ounces of brand A and brand B dog food to be used in each serving. Then the cost of each serving is given by $C = 3x + 4y$ cents. Since the size of each serving must not exceed 8 ounces, we have $x + y \leq 8$. Furthermore, since each serving must contain at least 29 units of nutrient I and at least 20 units of nutrient II, we have $3x + 5y \geq 29$ and $4x + 2y \geq 20$. Therefore, we have the following linear programming problem.

Minimize $C = 3x + 4y$ subject to

$$\begin{aligned} x + \ \ y &\le \ 8 \\ 3x + 5y &\ge 29 \\ 4x + 2y &\ge 20 \\ x \ge 0,\ y &\ge 0 \end{aligned}$$

The graph of the feasible set *S* is shown figure that follows and the value of *C* at each of the vertices of *S* is given in the accompanying table.



Vertex	$P = 3x + 4y$
A(2,6)	30
B(3,4)	25
C($\frac{11}{2}$, $\frac{5}{2}$)	$26\frac{1}{2}$

We conclude that the cost will be minimized when $x = 3$ and $y = 4$. Therefore, 3 ounces of brand *A* dog food and 4 ounces of brand *B* dog food can be prepared with the required nutrients at a minimum cost of 25 cents per serving.

51. Let *x* denote the number of urban families and let *y* denote the number of suburban families interviewed by the company. Then, the amount of money paid to Trendex will be

$$P = 6000 + 8(x + y) – 4.4x – 5y = 6000 + 3.6x + 3y.$$

Since a maximum of 1500 families are to be interviewed, we have

$$x + y \le 1500.$$

Next, the condition that at least 500 urban families are to be interviewed translates into the condition $x \ge 500$. Finally the condition that at least half of the families interviewed must be from the suburban area gives

$$y \ge \tfrac{1}{2}(x + y) \quad \text{or} \quad y - x \ge 0$$

Thus, we are led to the following programming problem:

Maximize $P = 6000 + 3.6x + 3y$ subject to

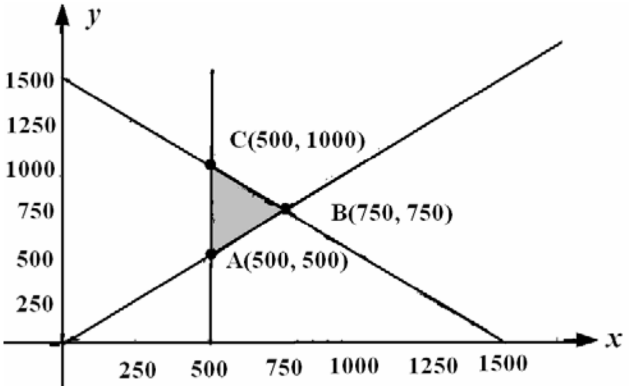
$$x + y \le 1500$$

$$y - x \ge 0$$

$$x \ge 500, y \ge 0$$

The graph of the feasible set *S* for this problem follows and the value of *P* at each of the vertices of *S* is given in the accompanying table.

Vertex	*P* = 6000 + 3.6*x* + 3*y*
A(500,500)	9,300
B(750,750)	10,950
C(500,1000)	10,800



Using the method of corners, we conclude that the profit will be maximized when $x = 750$ and $y = 750$. Thus, a maximum profit of \$10,950 will be realized when 750 urban and 750 suburban families are interviewed.

52. True. The optimal solution of a linear programming problem is the largest of all the feasible solutions.

53. False. It can have either one or infinitely many solutions.

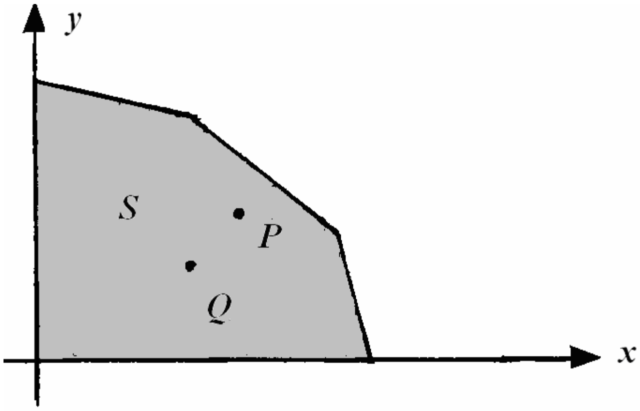
54. False. The feasible set could be empty, and therefore, bounded.

55. a. True. Since $a > 0$, the term ax can be made as large as we please by taking x sufficiently large (because *S* is unbounded) and therefore *P* is unbounded as well.
    b. True. Maximizing $P = ax + by$ on *S* is the same as minimizing

$$Q = -P = -(ax + by) = -ax - by = Ax + By,$$

where $A \geq 0$ and $B \geq 0$. Since $x \geq 0$ and $y \geq 0$, the linear function Q, and therefore P, has at least one optimal solution.

56. Since the point $Q(x_1, y_1)$ lies in the interior of the feasible set S, it is possible to find another point $P(x_1, y_2)$ lying to the right and above the point Q and contained in S. (See figure.) Clearly, $x_2 > x_1$ and $y_2 > y_1$. Therefore, $ax_2 + by_2 > ax_1 + by_1$, since $a > 0$ and $b > 0$ and this shows that the objective function $P = ax + by$ takes on a larger value at P than it does at Q. Therefore, the optimal solution cannot occur at Q.



57. Let $A(x_1, y_1)$ and $B(x_2, y_2)$. Then you can verify that $Q(\bar{x}, \bar{y})$, where

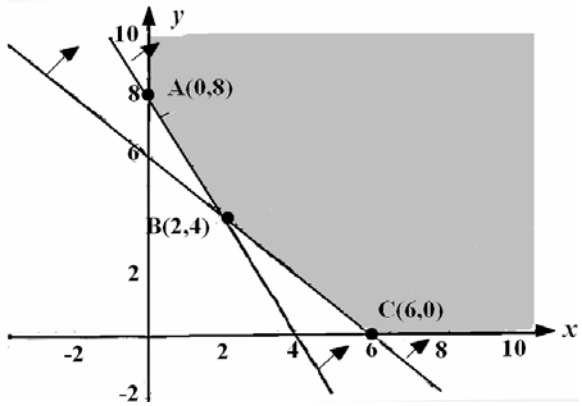
$$\bar{x} = x_1 + t(x_2 - x_1) \quad \text{and} \quad \bar{y} = y_1 + t(y_2 - y_1)$$

and t is a number satisfying $0 < t < 1$. Therefore, the value of P at Q is

$$\begin{aligned} P = a\bar{x} + b\bar{y} &= a[x_1 + t(x_2 - x_1)] + b[y_1 + t(y_2 - y_1)] \\ &= ax_1 + by_1 + [a(x_2 - x_1) + b(y_2 - y_1)]t. \end{aligned}$$

Now, if $c = a(x_2 - x_1) + b(y_2 - y_1) = 0$, then P has the (maximum) value $ax_1 + by_1$ on the line segment joining A and B; that is, the infinitely many solutions lie on this line segment. If $c > 0$, then a point a little to the right of Q will give a larger value of P. Thus, P is not maximal at Q. (Such a point can be found because Q lies in the interior of the line segment). A similar statement holds for the case $c < 0$. Thus, the maximum of P cannot occur at Q unless it occurs in every point on the line segment joining A and B.

58. a.

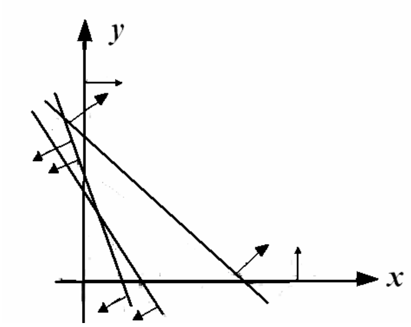


b-c.

Vertex	$P = 2x + 7y$
A(0,8)	42
B(2,4)	32
C(6,0)	12

d. Any point inside the feasible region will yield a larger value of *P*. For example, the point (6,6) will give $P = 2(6) + 7(6) = 54$. This does not contradict Theorem 1 because the set is not bounded.

59. a.



b. There is no point that satisfies all the given inequalities. Therefore, there is no solution.