maximized; whereas, in a minimization problem the objective function is to be minimized.

EXERCISES 3.2, page 168

1. We tabulate the given information:

	Product A	*Product B*	*Time Available*
Machine I	6	9	300
Machine II	5	4	180
Profit per unit ($)	3	4	

Let x and y denote the number of units of Product A and Product B to be produced. Then the required linear programming problem is:

Maximize $P = 3x + 4y$ subject to the constraints

$$6x + 9y \leq 300$$
$$5x + 4y \leq 180$$
$$x \geq 0, y \geq 0 \quad .$$

2. Let x and y denote the number of model A and model B fax machines produced each shift. Then the restriction on manufacturing costs implies

$$100x + 150y \leq 600,000$$

and the limitation of the number produced implies

$$x + y \leq 2500$$

The total profit is $P = 30x + 40y$. Summarizing, we have the following linear programming problem.

Maximize $P = 30x + 40y$ subject to

$$100x + 150y \leq 600,000$$
$$x + \quad y \leq \quad 2,500$$
$$x \geq 0, \ y \geq 0$$

3. Let x denote the number of model A grates to be produced and y denote the number of model B grates to be produced. Since only 1000 pounds of cast iron are available, we must have

$$3x + 4y \leq 1000.$$

The restriction that only 20 hours of labor are available per day implies that

$6x + 3y \leq 1200.$ (time in minutes)

Then the profit on the production of these grates is given by

$P = 2x + 1.5y.$

Summarizing, we have the following linear programming problem:

Maximize $P = 2x + 1.5y$ subject to

$$3x + 4y \leq 1000$$
$$6x + 3y \leq 1200$$
$$x \geq 0, y \geq 0$$

4. Let x denote the number of model A grates to be produced and y denote the number of model B grates to be produced. Since only 1000 pounds of cast iron are available, we must have

$3x + 4y \leq 1000.$

The restriction that only 20 hours of labor are available per day implies that

$6x + 3y \leq 1200.$ (time in minutes)

Then the profit on the production of these grates is given by

$P = 2x + 1.5y.$

The additional restriction that at least 150 model A grates be produced each day implies that $x \geq 150$. Summarizing, we have the following linear programming problem:

Maximize $P = 2x + 1.5y$ subject to

$$3x + 4y \leq 1000$$
$$6x + 3y \leq 1200$$
$$x \geq 150, y \geq 0$$

5. Let x denote the number of tables and y denote the number of chairs to be manufactured. Since 3200 board feet are available, we have

$$40x + 16y \leq 3200$$

Next, since 520 hours of labor are available, we have

$$3x + 4y \leq 520$$

Then the profit for the production of tables and chairs is given by

$$P = 45x + 20y$$

Summarizing, we have the following linear programming problem:

Maximize $P = 45x + 20y$ subject to

$$40x + 16y \leq 3200$$
$$3x + 4y \leq 520$$
$$x \geq 0, y \geq 0$$

6. Refer to Exercise 5. Let x denote the number of tables and y denote the number of chairs to be manufactured. Then the linear programming problem is

Maximize $P = 50x + 18y$ subject to

$$\begin{aligned} 40x + 16y &\le 3200 \\ 3x + 4y &\le 520 \\ x \ge 0, y &\ge 0 \end{aligned}$$

7. Suppose the company extends x million dollars in homeowner loans and y million dollars in automobile loans. Then, the returns on these loans are given by $P = 0.1x + 0.12y$ million dollars. Since the company has a total of \$20 million for these loans, we have $x + y \le 20$. Furthermore, since the total amount of homeowner loans should be greater than or equal to four times the total amount of automobile loans, we have $x \ge 4y$. Therefore, the required linear programming problem is

Maximize $P = 0.1x + 0.12y$ subject to

$$\begin{aligned} x + y &\le 20 \\ x - 4y &\ge 0 \\ x \ge 0, y &\ge 0 \end{aligned}$$

8. Let x and y denote the amount (in thousands of dollars) to be invested in project A and project B, respectively. Since the amount available for investment is up to \$500,000, we have $x + y \le 500$. Next, the condition on the allocation of the funds implies that

$$y \le 0.4(x + y), \ -0.4x + 0.6y \le 0, \text{ or } -2x + 3y \le 0$$

The linear programming problem at hand is

Maximize $P = 0.1x + 0.15y$ subject to

$$\begin{aligned} x + y &\le 500 \\ -2x + 3y &\le 0 \\ x \ge 0, y &\ge 0 \end{aligned}$$

9. Let x denote the number of fully assembled units to be produced daily and let y denote the number of kits to be produced. Then the fraction of the day the fabrication department works on the fully assembled cabinets is $\frac{1}{200}x$. Similarly the fraction of the day the fabrication department works on kits is $\frac{1}{200}y$. Since the fraction of the day during which the fabrication department is busy cannot exceed one, we must have

$$\frac{1}{200}x+\frac{1}{200}y\le 1.$$

Similarly, the restrictions place on the assembly department leads to the inequality

$$\frac{1}{100}x+\frac{1}{300}y\le 1.$$

The profit (objective) function is $P=50x+40y$. Summarizing, the required linear programming problem is

Maximize $P=50x+40y$ subject to

$$\frac{1}{200}x+\frac{1}{200}y\le 1$$

$$\frac{1}{100}x+\frac{1}{300}y\le 1$$

$$x\ge 0,\ y\ge 0.$$

10. Let x denote the number of acres of crop A that will be planted and y the number of acres of crop B that will be planted. Then the restriction on the amount of money available for land cultivation implies $40x + 60y \le 7400$. Similarly, the restriction regarding the amount of time available for labor implies that $20x + 25y \le 3300$. Since the profit on crop A is \$150 per acre and the profit on crop B is \$200 per acre, we have $P = 150x + 200y$. Summarizing, we have the following linear programming problem: Maximize $P = 150x + 200y$ subject to

$$40x+60y\le 7400$$

$$20x+25y\le 3300\ .$$

$$x\ge 0, y\ge 0$$

11. Let x and y denote the number of days the Saddle Mine and the Horseshoe Mine are operated, respectively. Then the operating cost is $C = 14{,}000x + 16{,}000y$. The amount of gold produced in the two mines is $(50x+75y)$ oz, and this amount must be at least 650 oz. So we have $50x + 75y \ge 650$. Similarly, the requirement for silver production leads to the inequality $3000x + 1000y \ge 18{,}000$. So the problem is

Minimize $C = 14{,}000x + 16{,}000y$ subject to

$$50x+\quad 75y\ge 650$$

$$3000x+1000y\ge 18{,}000$$

$$x\ge 0,\ y\ge 0$$

12. Let x and y denote the number of type-A and type-B vessels to be used, respectively.

Then the problem is

Minimize $C = 44{,}000x + 54{,}000y$ subject to

$$60x + 80y \geq 360$$
$$160x + 120y \geq 680$$
$$x \geq 0,\ y \geq 0$$

13. Let x denote the number of gallons of water in millions obtained from the local reservoir per day and let y denote the number of gallons of water in millions obtained from the pipeline. The requirement that at least 10 million gal of water be supplied per day implies that

$$x + y \geq 10$$

Next, since the maximum yield of the local reservoir is 5 million gallons per day, we have

$$x \leq 5$$

Since the maximum yield of the pipeline is 10 million gal per day and the pipeline has been contracted to supply at least 6 million gal per day, we have

$$6 \leq y \leq 10$$

Then the cost function is given by $C = 300x + 500y$.

Summarizing, we have the following linear programming problem

Minimize $C = 300x + 500y$ subject to

$$x + y \geq 10$$
$$x \leq 5$$
$$6 \leq y \leq 10$$
$$x \geq 0$$

14. Let x and y denote the number of Pandas and Saint Bernards produced. Since there are 3600 sq yd of plush available, we have

$$1.5x + 2y \leq 3{,}600$$

Next, since there are 66,000 cu ft of stuffing available and 13,600 pieces of trim, we have

$$30x + 35y \leq 66{,}000$$

and

$$5x + 8y \leq 13{,}600$$

Finally, the profit is given by

$$P = 10x + 15y$$

Summarizing, we have the following linear programming problem:

Maximize $P = 10x + 15y$ subject to

$$\begin{aligned} 1.5x + \ \ 2y &\le \ \ 3,600 \\ 30x + 35y &\le 66,000 \\ 5x + \ \ 8y &\le 13,600 \\ x \ge 0, y &\ge 0 \end{aligned}$$

15. Let x and y denote the amount of food A and food B, respectively, used to prepare a meal. Then the requirement that the meal contain a minimum of 400 mg of calcium implies $30x + 25y \ge 400$. Similarly, the requirements that the meal contain at least 10 mg of iron and 40 mg of vitamin C imply that $\begin{cases} x + 0.5y \ge 10 \\ 2x + 5y \ge 40 \end{cases}$. The cholesterol content is given by $C = 2x + 5y$. Therefore, the linear programming problem is

Minimize $C = 2x + 5y$ subject to

$$\begin{aligned} 30x + 25y &\ge 400 \\ x + 0.5y &\ge 10 \\ 2x + \ \ 5y &\ge 40 \\ x \ge 0, y &\ge 0 \end{aligned}.$$

16. Let x and y denote the amount of money given in aid to country A and country B, respectively. Then the requirement that country A receives between \$1 and \$1.5 million, inclusive, in aid, implies $1 \le x \le 1.5$. Similarly, the requirement that country B receive at least \$0.75 million in aid implies $y \ge 0.75$.The condition that between \$2 and \$2.5 million dollars in aid has been earmarked for these two countries implies that

$$\begin{aligned} x + y &\le 2.5 \\ x + y &\ge 2 \end{aligned}.$$

Therefore, we have the following linear programming problem:

Maximize $P = 0.6x + 0.8y$ subject to

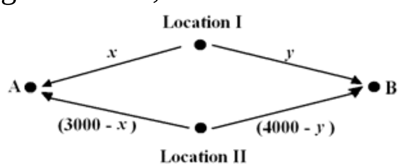
$$\begin{aligned} x + y &\le 2.5 \\ x + y &\ge 2 \\ 1 \le x &\le 1.5 \\ y &\ge 0.75 \end{aligned}$$

17. Let x and y denote the number of advertisements to be placed in newspaper I and newspaper II, respectively. Then the problem is

Minimize $C = 1000x + 800y$ subject to

$$70,000x + 10,000y \geq 2,000,000$$
$$40,000x + 20,000y \geq 1,400,000$$
$$20,000x + 40,000y \geq 1,000,000$$
$$x \geq 0, y \geq 0$$

18. Let x denote the number of LCD TVs shipped from location I to city A and let y denote the number of LCD TVs shipped from location I to city B. Since the number of LCD TVs required by the two factories in city A and city B are 3000 and 4000, respectively, the number of LCD TVs shipped from location II to city A and city B, are (3000 - x) and (4000 - y), respectively. These numbers are shown in the following schematic,



Referring to the schematic and the shipping schedule, we find that the total shipping costs incurred by the company are given by

$$C = 6x + 4y + 8(3000 - x) + 10(4000 - y)$$
$$= 64000 - 2x - 6y$$

The production constraints on Location I and II lead to the inequalities

$$x + \qquad y \leq 6000$$
$$(3000 - x) + (4000 - y) \leq 5000$$

This last inequality simplifies to $x + y \geq 2000$.

The requirements of the two factories lead to the inequalities

$x \geq 0$, $y \geq 0$, $3000 - x \geq 0$, and $4000 - y \geq 0$.

These last two inequalities may be written as $x \leq 3000$ and $y \leq 4000$.

Summarizing, we have the following linear programming problem:

Minimize $C = 64000 - 2x - 6y$ subject to

$$x + y \leq 6000$$
$$x + y \geq 2000$$
$$x \leq 3000$$

$$y \le 4000$$
$$x \ge 0, \quad y \ge 0$$

19. Let x, y, and z denote the amount of money she invests in project A, project B, and project C, respectively. Since she plans to invest up to \$2 million, we must have $x + y + z \le 2,000,000$. Because she decides to put not more than 20% of her total investment in project C, we have

$$z \le 0.2(x + y + z) \quad \text{or} \quad -0.2x - 0.2y + 0.8z \le 0$$

Since her investments in project B and C should not exceed 60% of her total investment, we have

$$y + z \le 0.6(x + y + z) \quad \text{or} \quad -0.6x + 0.4y + 0.4z \le 0$$

Also, since her investment in project A should be at least 60% of her investments in projects B and C, we have

$$x \ge 0.6(y + z) \quad \text{or} \quad -x + 0.6y + 0.6z \le 0$$

Finally, the returns on her investments are given by $P = 0.1x + 0.15y + 0.2z$.

To summarize, the problem is

Maximize $P = 0.1x + 0.15y + 0.2z$ subject to

$$x + y + z \le 2,000,000$$
$$-2x - 2y + 8z \le 0$$
$$-6x + 4y + 4z \le 0$$
$$-10x + 6y + 6z \le 0$$
$$x \ge 0, y \ge 0, z \ge 0$$

20. Suppose Ashley invests x, y, and z dollars in the money market fund, the international equity fund, and the growth-and-income fund, respectively. Then the objective function is $P = 0.06x + 0.1y + 0.15z$. The constraints are

$$x + y + z \le 250,000; \quad z \le 0.25(x + y + z); \text{and} \quad y \le 0.5(x + y + z).$$

The last two inequalities simplify to

$$-\tfrac{1}{4}x - \tfrac{1}{4}y + \tfrac{3}{4}z \le 0 \quad \text{or} \quad -x - y + 3z \le 0$$

and $$-\tfrac{1}{2}x + \tfrac{1}{2}y - \tfrac{1}{2}z \le 0, \quad \text{or} \quad -x + y - z \le 0.$$

So the required linear programming problem is

Maximize $P = 0.06x + 0.1y + 0.15z = \frac{3}{50}x + \frac{1}{10}y + \frac{3}{20}z$ subject to

$$x + y + z \le 250,000$$
$$-x - y + 3z \le 0$$

$$-x + y - z \le 0$$
$$x \ge 0, y \ge 0, z \ge 0$$

21. Let x, y, and z denote the number of units produced of products A, B, and C, respectively. From the given information, we formulate the following linear programming problem:

Maximize $P = 18x + 12y + 15z$ subject to

$$2x + y + 2z \le 900$$
$$3x + y + 2z \le 1080$$
$$2x + 2y + z \le 840$$
$$x \ge 0, y \ge 0, z \ge 0$$

22. Let x denote the number of minutes of morning television advertising time, y denote the number of minutes of afternoon advertising time, and z denote the number of minutes of evening advertising time that Excelsior should buy. Then the budget restrictions imply that $3000x + 1000y + 12{,}000z \le 102{,}000$. Next, the restrictions regarding the availability of time at Station KAOS imply that $x + y + z \le 25$ and $z \le 6$. The data regarding the exposure of these commercials imply that the function we wish to maximize is

$$P = 200{,}000x + 100{,}000y + 600{,}000z.$$

Summarizing, we have the following linear programming problem:

Maximize $P = 200{,}000x + 100{,}000y + 600{,}000z$ subject to

$$3000x + 1000y + 12{,}000z \le 102{,}000$$
$$x + y + z \le 25$$
$$z \le 6$$
$$x \ge 0, y \ge 0, z \ge 0$$

23. We first tabulate the given information:

Department	*Model A*	*Model B*	*Model C*	*Time Available*
Fabrication	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{3}{2}$	310
Assembly	1	1	$\frac{3}{4}$	205
Finishing	1	1	$\frac{1}{2}$	190

Let x, y, and z denote the number of units of model *A*, model *B*, and model *C* to be produced, respectively. Then the required linear programming problem is

Maximize $P = 26x + 28y + 24z$ subject to

$$\frac{5}{4}x + \frac{3}{2}y + \frac{3}{2}z \le 310$$
$$x + y + \frac{3}{4}z \le 205$$
$$x + y + \frac{1}{2}z \le 190$$
$$x \ge 0, y \ge 0, z \ge 0$$

24. The shipping costs per loudspeaker system in dollars are given in the following table.

WAREHOUSE

	A	*B*	*C*
Plant I	16	20	22
Plant II	18	16	14

Letting x_1 denote the number of loudspeaker systems shipped from plant *I* to warehouse *A*, x_2 the number of loudspeaker systems shipped from plant *I* to warehouse *B*, and so on we have

WAREHOUSE

	A	*B*	*C*	*Maximum Production*
Plant I	x_1	x_2	x_3	800
Plant II	x_4	x_5	x_6	600
Minimum Requirement	500	400	400	

From the two tables we see that the total monthly shipping cost incurred by Acrosonic is given by

$$C = 16x_1 + 20x_2 + 22x_3 + 18x_4 + 16x_5 + 14x_6.$$

Next, the production constraints on plants *I* and *II* lead to the inequalities

$$x_1 + x_2 + x_3 \le 800$$
$$x_4 + x_5 + x_6 \le 600.$$

Also, the minimum requirements of each warehouse leads to the three inequalities

$$x_1 + x_4 \ge 500$$
$$x_2 + x_5 \ge 400$$
$$x_3 + x_6 \ge 400$$

Summarizing we have the following linear programming problem:

Minimize $C = 16x_1 + 20x_2 + 22x_3 + 18x_4 + 16x_5 + 14x_6$.

$$\tfrac{5}{2}x + 3y + 4z \le 70$$
$$x \le 9$$
$$y \le 12$$
$$z \le 6$$
$$x \ge 0, y \ge 0, z \ge 0.$$

25. The shipping costs are tabulated in the following table.

	Warehouse A	*Warehouse B*	*Warehouse C*
Plant I	60	60	80
Plant II	80	70	50

Letting x_1 denote the number of pianos shipped from plant *I* to warehouse *A*, x_2 the number of pianos shipped from plant *I* to warehouse *B*, and so we have

	Warehouse A	*Warehouse B*	*Warehouse C*	*Maximum Production*
Plant I	x_1	x_2	x_3	300
Plant II	x_4	x_5	x_6	250
Minimum Requirement	200	150	200	

From the two tables we see that the total monthly shipping cost is given by

$C = 60x_1 + 60x_2 + 80x_3 + 80x_4 + 70x_5 + 50x_6$.

Next, the production constraints on plants *I* and *II* lead to the inequalities

$$x_1 + x_4 \ge 200$$
$$x_2 + x_5 \ge 150$$
$$x_3 + x_6 \ge 200$$

Summarizing we have the following linear programming problem:

Minimize $C = 60x_1 + 60x_2 + 80x_3 + 80x_4 + 70x_5 + 50x_6$ subject to

$$x_1 + x_2 + x_3 \le 300$$
$$x_4 + x_5 + x_6 \le 250$$
$$x_1 + x_4 \ge 200$$
$$x_2 + x_5 \ge 150$$
$$x_3 + x_6 \ge 200$$
$$x_1 \ge 0,\ x_2 \ge 0,\ \ldots,\ x_6 \ge 0$$

26. Let *x*, *y*, and *z* denote the number of standard, deluxe, and luxury models to be completed. Then, the required linear programming problem is

Maximize $P = 3400x + 4000y + 5000z$ subject to

$$6x + 8y + 10z \le 8,200$$
$$24x + 22y + 20z \le 21,800$$
$$18x + 21y + 30z \le 23,700$$
$$x \ge 0, y \ge 0, z \ge 0$$

(Note that we have divided the first inequality by 1000 and the second and third inequalities by 10 in order to simplify our work later on.

27. The given data can be summarized as follows:

	Concentrates			
	Pineapple	*Orange*	*Banana*	*Profit ($)*
Pineapple-orange	8	8	0	1
Orange-banana	0	12	4	0.80
Pineapple-orange-banana	4	8	4	0.90
Maximum available (oz)	16,000	24,000	5,000	

Suppose *x*, *y*, and *z* cartons of pineapple-orange, orange-banana, and pineapple-orange-banana juice are to be produced, respectively. The linear programming problem is

Maximize $P = x + 0.8y + 0.9z$ subject to

$$8x + 4z \le 16,000$$
$$8x + 12y + 8z \le 24,000$$
$$4y + 4z \le 5,000$$
$$z \le 800$$
$$x \ge 0, y \ge 0,\ z \le 0$$

28. Let x, y, and z denote the number (in thousands) of bottles of formula *I*, formula *II*, and formula *III*, respectively, produced. Then the profit function to be maximized is $P = 180x + 200y + 300z$. Next, the limitation on time implies that $2.5x + 3y + 4z \leq 70$. Similarly, the restrictions on the amount of ingredients available imply that $x \leq 9$, $y \leq 12$ and $z \leq 6$. Summarizing, we have the following linear programming problem:

Maximize $P = 180x + 200y + 300z$ subject to

$$\begin{aligned} \tfrac{5}{2}x + 3y + 4z &\leq 70 \\ x &\leq 9 \\ y &\leq 12 \\ z &\leq 6 \\ x \geq 0, y \geq 0, z &\geq 0. \end{aligned}$$

29. False. The objective function $P = xy$ is not a linear function in x and y.

30. True. It satisfies the definition of a linear programming problem.