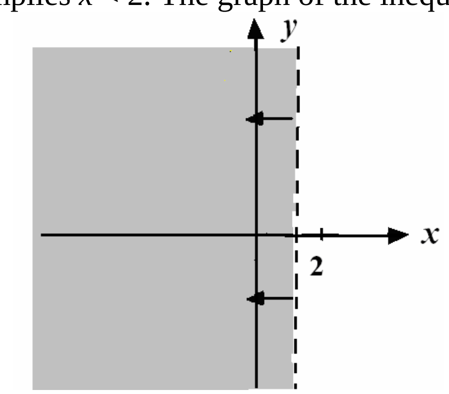
CHAPTER 3

3.1 CONCEPT QUESTIONS, p. 161

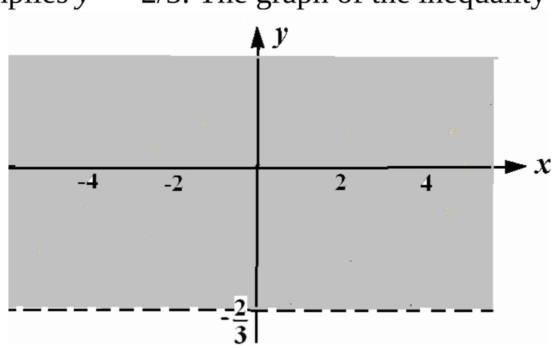
1. a. The solution set of $ax + by < c$ is a half-plane that does not include the line with equation $ax + by = c$. The solution of the set $ax + by \le c$, on the other hand, includes the line.
   b. Its the line with equation $ax + by = c$.
2. a. Its the set that is the intersection of the half-planes determined by the inequalities comprising the system.
   b. We graph each half-plane determined by an inequality in the system. Then shade the part common to all the half-planes.

EXERCISES 3.1, page 161

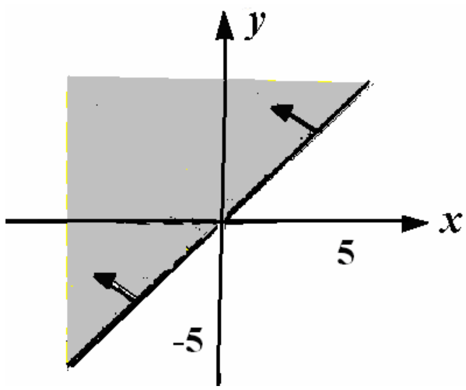
1. $4x - 8 < 0$ implies $x < 2$. The graph of the inequality follows.



2. $3y + 2 > 0$ implies $y > -2/3$. The graph of the inequality follows.



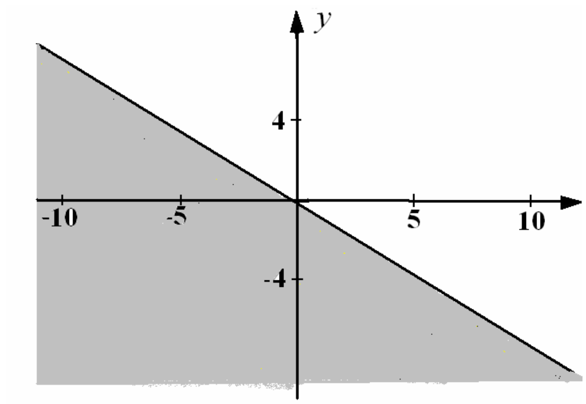
3. $x - y \le 0$ implies $x \le y$. The graph of the inequality follows.



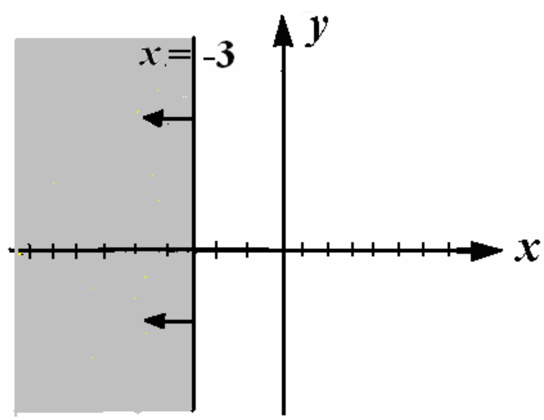
4. We first sketch the straight line with equation $3x + 4y = -2$. Next, picking (0, 0) as the test point, we see that

$$3(0) + 4(0) = 0 \not< -2.$$

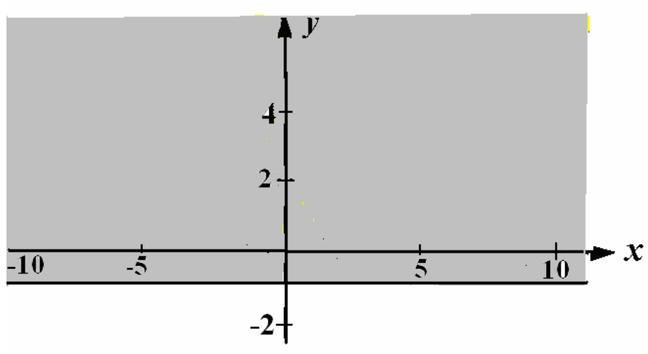
Therefore, the half-plane not containing the origin is the required half-plane. The graph of the inequality is at the right.



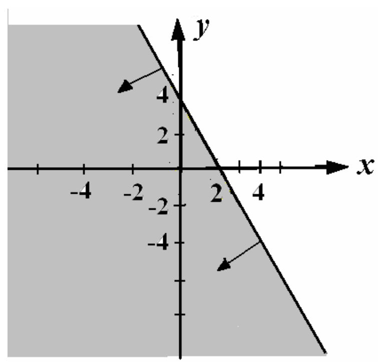
5. The graph of the inequality $x \leq -3$ follows.



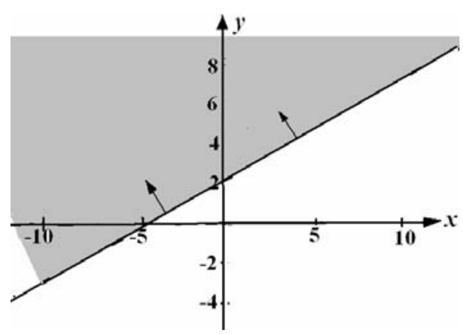
6. The graph of the inequality $y \geq -1$ follows.



7. We first sketch the straight line with equation $2x + y = 4$. Next, picking the test point (0, 0), we have $2(0) + (0) = 0 \leq 4$. We conclude that the half-plane containing the origin is the required half-plane.

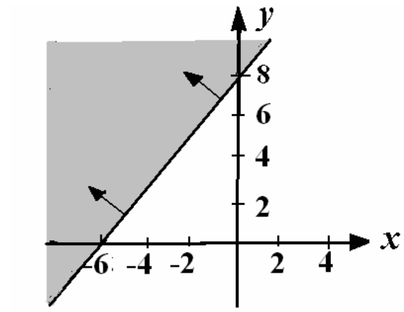


8. We first sketch the straight line with equation $-3x + 6y = 12$. Next, picking the test point (0, 0), we see that $-3(0) + 6(0) = 0 \not\geq 12$. Therefore, the half-plane not containing the origin is the required half-plane. The graph of this inequality is at the right.

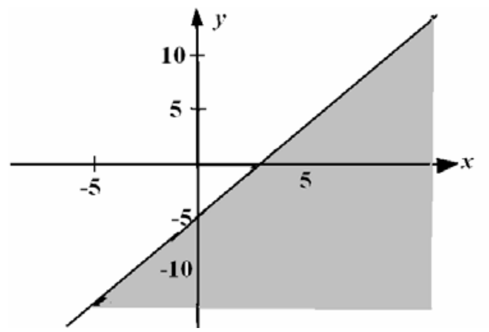


9. We first sketch the graph of the straight line $4x - 3y = -24$. Next, picking the test point (0, 0), we see that $4(0) - 3(0) = 0 \not\leq -24$. We conclude that the half-plane not

containing the origin is the required half-plane. The graph of this inequality follows.



10. We first sketch the graph of the straight line $5x - 3y = 15$. Next we pick the test point (0, 0). Since $5(0) - 3(0) = 0 \not> 15$, we conclude that the half-plane not containing the origin is the required half-plane.



11. The system of linear inequalities that describes the shaded region is

$$x \geq 1,\ x \leq 5,\ y \geq 2, \text{ and } y \leq 4.$$

We may also combine the first and second inequalities and the third and fourth inequalities and write

$$1 \leq x \leq 5 \quad \text{and} \quad 2 \leq y \leq 4.$$

12. The system of linear inequalities that describes the shaded region is

$$x + y \geq 3,\quad y - x \geq 0, \text{ and } y \leq 4.$$

13. The system of linear inequalities that describes the shaded region is

$$2x - y \geq 2,\ 5x + 7y \geq 35, \text{ and } x \leq 4.$$

14. The system of linear inequalities that describes the shaded region is

$$5x + 2y \leq 20,\ x + 2y \leq 8,\ x \geq 0, \text{ and } y \geq 0.$$

15. The system of linear inequalities that describes the shaded region is

$$7x + 4y \leq 140,\ x + 3y \geq 30, \text{ and } x - y \geq -10.$$

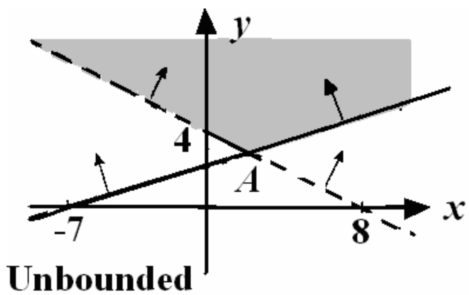
16. The system of linear inequalities that describes the shaded region is

$x + y \geq 2$, $9x + 5y \leq 90$, $3x + 5y \leq 60$, $x \geq 0$, and $y \geq 0$.

17. The system of linear inequalities that describes the shaded region is
    $x + y \geq 7$, $x \geq 2$, $y \geq 3$, and $y \leq 7$.

18. The system of linear inequalities that describes the shaded region is
    $5x + 4y \geq 40$, $x + 5y \geq 20$, and $4x + y \geq 16$, $x \geq 0$, and $y \geq 0$.

19. The required solution set is shown below.



To find the coordinates of A, we solve the system

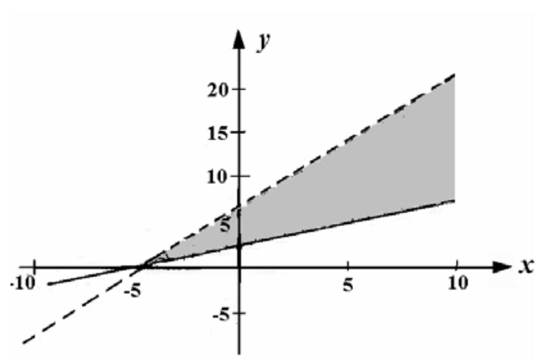
$$2x + 4y = 16$$
$$-x + 3y = 7,$$

giving $A = (2, 3)$. Observe that a dotted line is used to show that no point on the line constitutes a solution to the given problem. Observe also that this is an unbounded solution set.

20. The required solution set is shown at the right. To find the coordinates of A, we solve

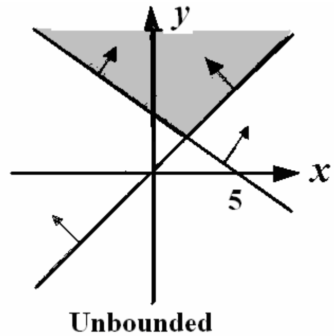
$$3x - 2y = -13$$
$$-x + 2y = 5$$

giving $A = (-4, \frac{1}{2})$. Observe that the dotted lines are shown in the figure to indicate that no point on the line constitutes a solution to the problem. Also observe that the solution set is unbounded.



21. The solution set is shown in the figure below. Observe that the set is unbounded.

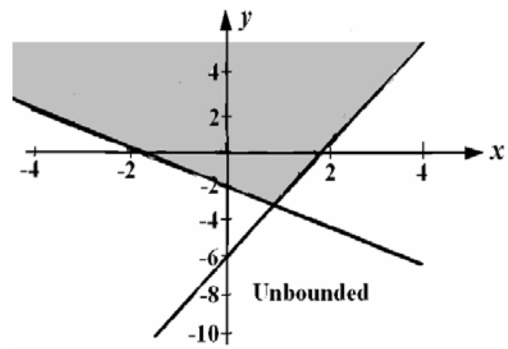
To find the coordinates of A, we solve the system $\begin{cases} x - \ \ y = 0 \\ 2x + 3y = 10 \end{cases}$ giving $A = (2, 2)$.



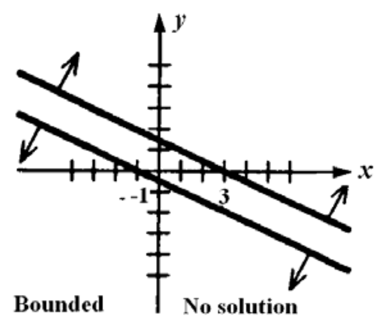
22. The solution set is shown in the figure at the right. Observe that the set is unbounded. To find the coordinates of A, we solve the system

$$x + y = -2$$
$$3x - y = \ \ 6,$$

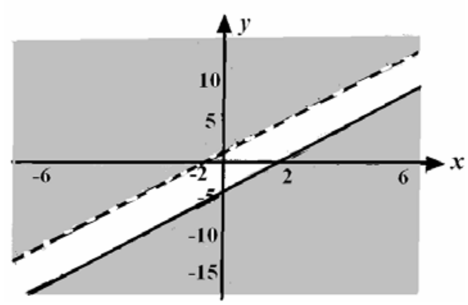
giving $A = (1, -3)$.



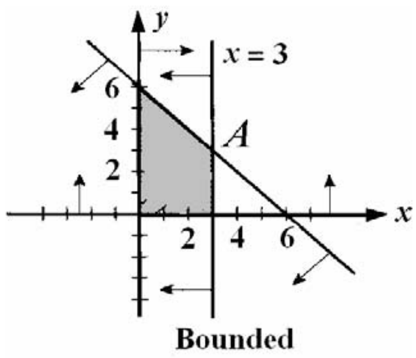
23. The half-planes defined by the two inequalities are shown in the figure at the right. Since the two half-planes have no points in common, we conclude that the given system of inequalities has no solution. (The empty set is a bounded set.)



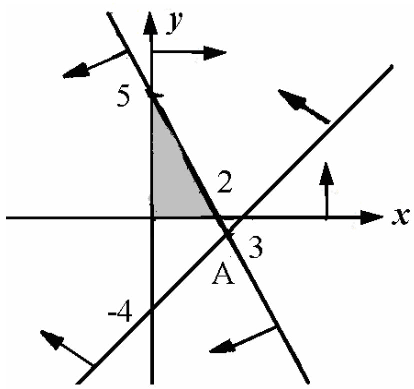
24. The half-planes defined by the two inequalities are shown at the right. Since the two half-planes have no points in common, we conclude that the given system of inequalities has no solution. The (empty) set is bounded.



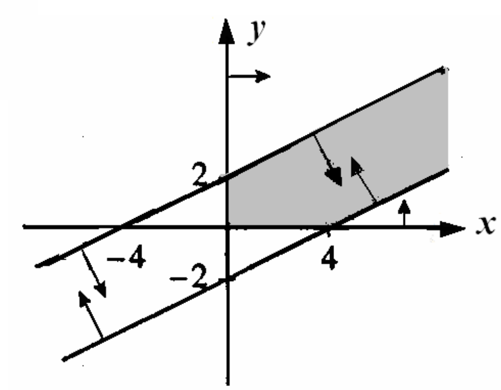
25. The half-planes defined by the three inequalities are shown below. The point *A* is found by solving the system $\begin{cases} x + y = 6 \\ x \quad\;\; = 3 \end{cases}$ giving *A* = (3, 3). Observe that this is a bounded solution set.



26. The half-planes defined by the given inequalities are shown in the figure below. The coordinates of *A* are found by solving the system $\begin{cases} 4x - 3y = 12 \\ 5x + 2y = 10 \end{cases}$ giving $A = (\frac{54}{23}, -\frac{20}{23})$. The solution set is bounded.

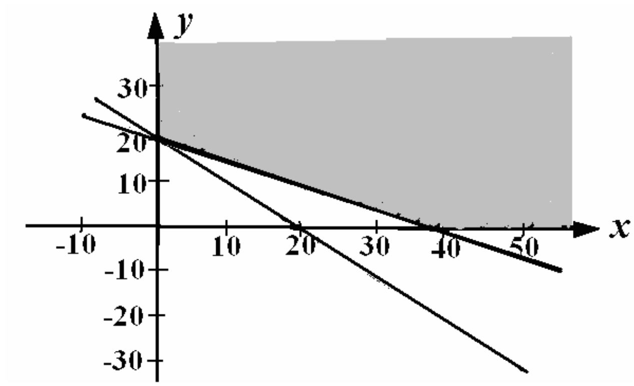


27. The half-planes defined by the given inequalities are shown at the right. Observe that the two lines described by the equations 3*x* – 6*y* = 12 and –*x* + 2*y* = 4 do not intersect because they are parallel. The solution set is unbounded.



28. The half-planes defined by the given

inequalities are shown in the figure that follows.

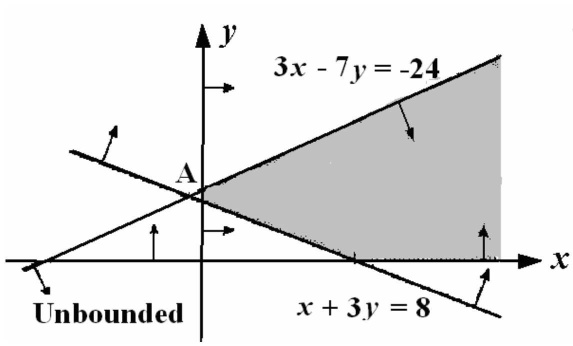


The vertex $A(0, 20)$ is found by solving the system

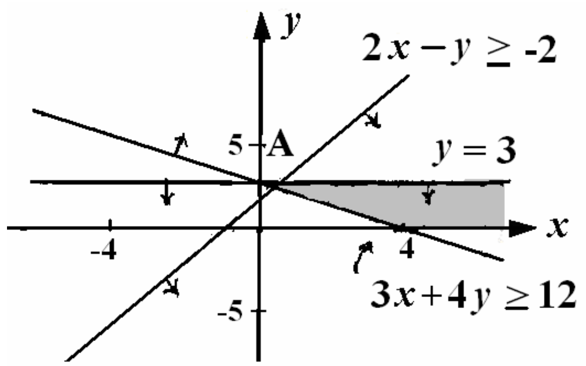
$$x + 2y = 40$$
$$x + \quad y = 20$$

The solution set is unbounded.

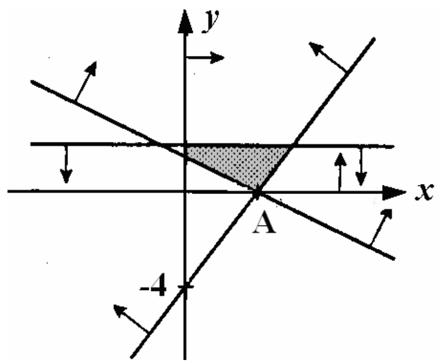
29. The required solution set is shown in the figure below. The coordinates of A are found by solving the system $\begin{cases} 3x - 7y = -24 \\ x + 3y = \quad 8 \end{cases}$ giving $(-1, 3)$. The solution set is unbounded.



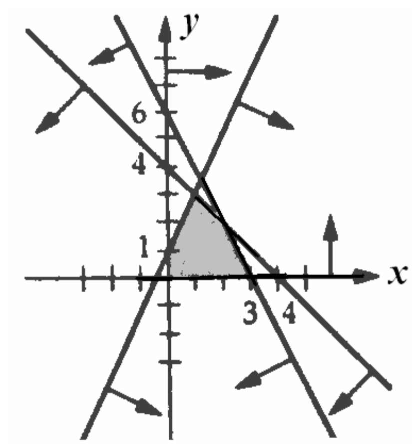
30. The required solution set is shown in the figure below. The coordinates of A are $(\frac{4}{11}, \frac{30}{11})$. The solution set is unbounded.



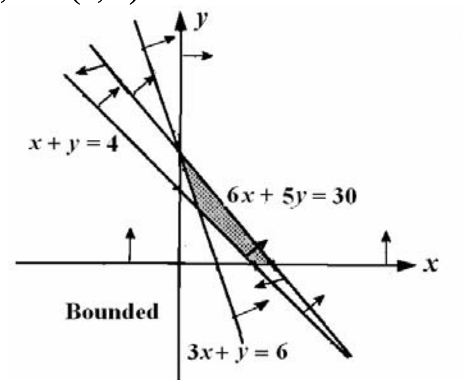
31. The required solution set is shown in the figure below. The solution set is bounded.



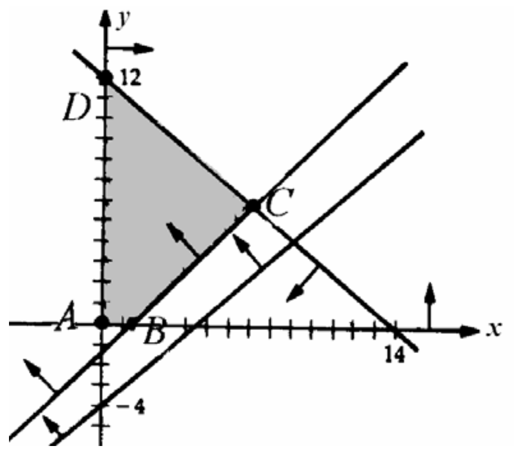
32. The required solution set is shown in the figure at the right. The corners of this solution set are (0, 0), (0, 1), (1, 3), (2, 2), and (3, 0). The solution set is bounded.



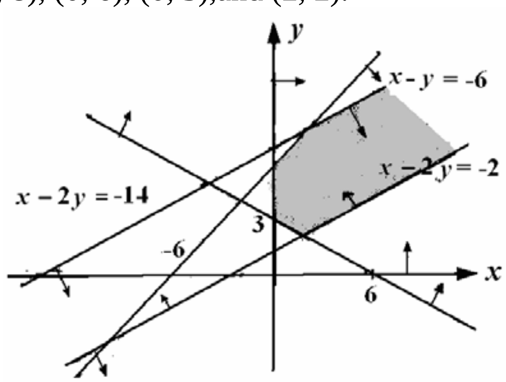
33. The required solution set is shown in the figure below. The solution set has vertices at (0, 6), (5, 0), (4, 0), and (1, 3). The solution set is bounded.



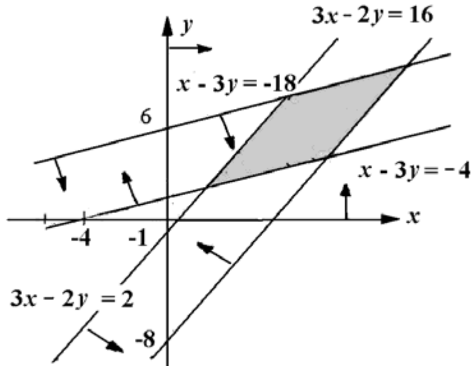
34. The required solution set is shown in the figure at the right. The bounded solution set has vertices at (0, 0), (0, 12), (7, 6) and (1.5, 0).



35. The required solution set is shown in the figure below. The unbounded solution set has vertices at (2, 8), (0, 6), (0, 3),and (2, 2).



36. The required solution set is shown in the figure that follows. The bounded solution set has vertices at (2,2), (8,4), (12,10),and (6,8).



37. False. It is always a half-plane. A straight line is the graph of a linear equation and vice-versa.

38. False. The graph of $-2x-3y+6 \le 0$ is the upper or right-half plane.

39. True. Since a circle can always be enclosed by a rectangle, the solution set of such a system is bounded if it can be enclosed by a rectangle.

40. True. The first two inequalities define regions in the plane that are bounded on the right by the lines with equations $ax+by=e$ and $cx+dy=f$. These lines have positive x- and y- intercepts. The conditions $x \ge 0$, and $y \ge 0$ dictate that the solution set lies in the first quadrant. Thus the solution set is bounded.