b. $\begin{bmatrix} 56.4 & -85.2 & 72.9 & 12 & 86.7 \\ 131.8 & 24.3 & 189.4 & 165 & 186.8 \\ 100.8 & 26.8 & 135.1 & 112.2 & 124.1 \\ 137 & 79.4 & 197.1 & 181.3 & 172.3 \end{bmatrix}$

c. $\begin{bmatrix} 170 & 18.1 & 133.1 & -106.3 & 341.3 \\ 349 & 226.5 & 324.1 & 164 & 506.4 \\ 245.2 & 157.7 & 231.5 & 125.5 & 312.9 \\ 310 & 245.2 & 291 & 274.3 & 354.2 \end{bmatrix}$ d. Yes

2.6 CONCEPT QUESTIONS, page 135

1. The inverse of a square matrix A is the matrix A^{-1} satisfying the conditions $AA^{-1} = A^{-1}A = I$
2. The steps for finding the inverse of a matrix are given on page 130 of the text.
3. The formula for finding the inverse of a 2×2 matrix are given on page 131 of the text.
4. The method cannot be used to solve a system of m linear equations in n unknowns because we cannot find an inverse for the matrix of coefficients A where $Ax = B$.

EXERCISES 2.6, page 135

1. $\begin{bmatrix} 1 & -3 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} -2 & 3 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$; $\begin{bmatrix} -2 & 3 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

2. $\begin{bmatrix} 4 & 5 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} \frac{3}{2} & -\frac{5}{2} \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$; $\begin{bmatrix} \frac{3}{2} & -\frac{5}{2} \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

3. $\begin{bmatrix} 3 & 2 & 3 \\ 2 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}\begin{bmatrix} -\frac{1}{3} & -\frac{1}{3} & \frac{4}{3} \\ 0 & 1 & -1 \\ \frac{2}{3} & -\frac{1}{3} & -\frac{2}{3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and

$$\begin{bmatrix} -\frac{1}{3} & -\frac{1}{3} & \frac{4}{3} \\ 0 & 1 & -1 \\ \frac{2}{3} & -\frac{1}{3} & -\frac{2}{3} \end{bmatrix}\begin{bmatrix} 3 & 2 & 3 \\ 2 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

4. $$\begin{bmatrix} 2 & 4 & -2 \\ -4 & -6 & 1 \\ 3 & 5 & -1 \end{bmatrix}\begin{bmatrix} \frac{1}{2} & -3 & -4 \\ -\frac{1}{2} & 2 & 3 \\ -1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ and

$$\begin{bmatrix} \frac{1}{2} & -3 & -4 \\ -\frac{1}{2} & 2 & 3 \\ -1 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 4 & -2 \\ -4 & -6 & 1 \\ 3 & 5 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

5. Using Formula (13), we find $A^{-1} = \dfrac{1}{(2)(3)-(1)(5)}\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$.

6. Using Formula (13), we find $A^{-1} = \dfrac{1}{(2)(5)-(3)(3)}\begin{bmatrix} 5 & -3 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 5 & -3 \\ -3 & 2 \end{bmatrix}$.

7. Since $ad - bc = (3)(2) - (-2)(-3) = 6 - 6 = 0$, the inverse does not exist.

8. Since $ad - bc = (4)(3) - (6)(2) = 12 - 12 = 0$, the inverse does not exist.

9. $$\left[\begin{array}{ccc|ccc} 2 & -3 & -4 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 1 & -2 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 2 & -3 & -4 & 1 & 0 & 0 \end{array}\right] \xrightarrow{R_3 - 2R_1}$$

$$\left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & -6 & 1 & 0 & -2 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 0 & 0 & 1 \\ 0 & 1 & -6 & 1 & 0 & -2 \\ 0 & 0 & -1 & 0 & 1 & 0 \end{array}\right] \xrightarrow[-R_3]{R_1 + 2R_2}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -11 & 2 & 0 & -3 \\ 0 & 1 & -6 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 & -1 & 0 \end{array}\right] \xrightarrow[R_2 + 6R_3]{R_1 + 11R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -11 & -3 \\ 0 & 1 & 0 & 1 & -6 & -2 \\ 0 & 0 & 1 & 0 & -1 & 0 \end{array}\right].$$

Therefore, the required inverse is $\begin{bmatrix} 2 & -11 & -3 \\ 1 & -6 & -2 \\ 0 & -1 & 0 \end{bmatrix}$.

10. $$\left[\begin{array}{rrr|rrr} 1 & -1 & 3 & 1 & 0 & 0 \\ 2 & 1 & 2 & 0 & 1 & 0 \\ -2 & -2 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R_3+2R_1]{R_2-2R_1} \left[\begin{array}{rrr|rrr} 1 & -1 & 3 & 1 & 0 & 0 \\ 0 & 3 & -4 & -2 & 1 & 0 \\ 0 & -4 & 7 & 2 & 0 & 1 \end{array}\right] \xrightarrow{\frac{1}{3}R_2}$$

$$\left[\begin{array}{rrr|rrr} 1 & -1 & 3 & 1 & 0 & 0 \\ 0 & 1 & -\frac{4}{3} & -\frac{2}{3} & \frac{1}{3} & 0 \\ 0 & -4 & 7 & 2 & 0 & 1 \end{array}\right] \xrightarrow[R_3+4R_2]{R_1+R_2} \left[\begin{array}{rrr|rrr} 1 & 0 & \frac{5}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 1 & -\frac{4}{3} & -\frac{2}{3} & \frac{1}{3} & 0 \\ 0 & 0 & \frac{5}{3} & -\frac{2}{3} & \frac{4}{3} & 1 \end{array}\right] \xrightarrow{\frac{3}{5}R_3}$$

$$\left[\begin{array}{rrr|rrr} 1 & 0 & \frac{5}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 1 & -\frac{4}{3} & -\frac{2}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 1 & -\frac{2}{5} & \frac{4}{5} & \frac{3}{5} \end{array}\right] \xrightarrow[R_2+\frac{4}{3}R_3]{R_1-\frac{5}{3}R_3} \left[\begin{array}{rrr|rrr} 1 & 0 & 0 & 1 & -1 & -1 \\ 0 & 1 & 0 & -\frac{6}{5} & \frac{7}{5} & \frac{4}{5} \\ 0 & 0 & 1 & -\frac{2}{5} & \frac{4}{5} & \frac{3}{5} \end{array}\right].$$

11. $$\left[\begin{array}{rrr|rrr} 4 & 2 & 2 & 1 & 0 & 0 \\ -1 & -3 & 4 & 0 & 1 & 0 \\ 3 & -1 & 6 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1-R_3} \left[\begin{array}{rrr|rrr} 1 & 3 & -4 & 1 & 0 & -1 \\ -1 & -3 & 4 & 0 & 1 & 0 \\ 3 & -1 & 6 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2+R_1} \left[\begin{array}{rrr|rrr} 1 & 3 & -4 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 & -1 \\ 3 & -1 & 6 & 0 & 0 & 1 \end{array}\right]$$

Because there is a row of zeros to the left of the vertical line, we see that the inverse does not exist.

12. $$\left[\begin{array}{rrr|rrr} 1 & 2 & 0 & 1 & 0 & 0 \\ -3 & 4 & -2 & 0 & 1 & 0 \\ -5 & 0 & -2 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R_3+5R_1]{R_2+3R_1} \left[\begin{array}{rrr|rrr} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 10 & -2 & 3 & 1 & 0 \\ 0 & 10 & -2 & 5 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_3-R_2} \left[\begin{array}{rrr|rrr} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 10 & -2 & 3 & 1 & 0 \\ 0 & 0 & 0 & 2 & -1 & 1 \end{array}\right].$$

Since the last row of the 3 × 3 submatrix comprising the left-hand side of the last augmented matrix is comprised of all zero entries, we conclude that the given matrix is singular and does not possess an inverse.

13. $$\left[\begin{array}{rrr|rrr} 1 & 4 & -1 & 1 & 0 & 0 \\ 2 & 3 & -2 & 0 & 1 & 0 \\ -1 & 2 & 3 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R_3+R_1]{R_2-2R_1} \left[\begin{array}{rrr|rrr} 1 & 4 & -1 & 1 & 0 & 0 \\ 0 & -5 & 0 & -2 & 1 & 0 \\ 0 & 6 & 2 & 1 & 0 & 1 \end{array}\right] \xrightarrow{R_2+R_3}$$

$$\left[\begin{array}{rrr|rrr} 1 & 4 & -1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 1 & 1 \\ 0 & 6 & 2 & 1 & 0 & 1 \end{array}\right] \xrightarrow[R_3-6R_2]{R_1-4R_2} \left[\begin{array}{rrr|rrr} 1 & 0 & -9 & 5 & -4 & -4 \\ 0 & 1 & 2 & -1 & 1 & 1 \\ 0 & 0 & -10 & 7 & -6 & -5 \end{array}\right] \xrightarrow{-\frac{1}{10}R_3}$$

$$\left[\begin{array}{rrr|rrr} 1 & 0 & -9 & 5 & -4 & -4 \\ 0 & 1 & 2 & -1 & 1 & 1 \\ 0 & 0 & 1 & -\frac{7}{10} & \frac{3}{5} & \frac{1}{2} \end{array}\right] \xrightarrow[R_2-2R_3]{R_1+9R_3} \left[\begin{array}{rrr|rrr} 1 & 0 & 0 & -\frac{13}{10} & \frac{7}{5} & \frac{1}{2} \\ 0 & 1 & 0 & \frac{2}{5} & -\frac{1}{5} & 0 \\ 0 & 0 & 1 & -\frac{7}{10} & \frac{3}{5} & \frac{1}{2} \end{array}\right]$$

So $A^{-1} = \begin{bmatrix} -\frac{13}{10} & \frac{7}{5} & \frac{1}{2} \\ \frac{2}{5} & -\frac{1}{5} & 0 \\ -\frac{7}{10} & \frac{3}{5} & \frac{1}{2} \end{bmatrix}$.

14. Starting with the matrix

$$\left[\begin{array}{rrr|rrr} 3 & -2 & 7 & 1 & 0 & 0 \\ -2 & 1 & 4 & 0 & 1 & 0 \\ 6 & -5 & 8 & 0 & 0 & 1 \end{array}\right]$$

we use the sequence of row operations

1. $R_1 + R_2$ 2. $R_2 + 2R_1,\ R_3 - 6R_1$ 3. $-R_2$

4. $R_1 + R_2,\ R_3 - R_2$ 5. $-\frac{1}{32}R_3$ 6. $R_1 + 15R_3,\ R_2 + 26R_3$

to find that $A^{-1} = \begin{bmatrix} \frac{7}{8} & -\frac{19}{32} & -\frac{15}{32} \\ \frac{5}{4} & -\frac{9}{16} & -\frac{13}{16} \\ \frac{1}{8} & \frac{3}{32} & -\frac{1}{32} \end{bmatrix}$.

15. $$\left[\begin{array}{cccc|cccc} 1 & 1 & -1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 2 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 2 & -1 & -1 & 3 & 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R_4-2R_1]{\substack{R_2-2R_1 \\ R_3-2R_1}} \left[\begin{array}{cccc|cccc} 1 & 1 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 3 & -2 & -2 & 1 & 0 & 0 \\ 0 & -1 & 2 & -1 & -2 & 0 & 1 & 0 \\ 0 & -3 & 1 & 1 & -2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{-R_2}$$

$$\left[\begin{array}{cccc|cccc} 1 & 1 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -3 & 2 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & -2 & 0 & 1 & 0 \\ 0 & -3 & 1 & 1 & -2 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R_4+3R_2]{\substack{R_1-R_2 \\ R_3+R_2}} \left[\begin{array}{cccc|cccc} 1 & 0 & 2 & -1 & -1 & 1 & 0 & 0 \\ 0 & 1 & -3 & 2 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & -8 & 7 & 4 & -3 & 0 & 1 \end{array}\right] \xrightarrow{-R_3}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 2 & -1 & -1 & 1 & 0 & 0 \\ 0 & 1 & -3 & 2 & 2 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 & -1 & 0 \\ 0 & 0 & -8 & 7 & 4 & -3 & 0 & 1 \end{array}\right] \xrightarrow[R_4+8R_3]{\substack{R_1-2R_3 \\ R_2+3R_3}} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 1 & -1 & -1 & 2 & 0 \\ 0 & 1 & 0 & -1 & 2 & 2 & -3 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 4 & 5 & -8 & 1 \end{array}\right]$$

$$\xrightarrow[-R_4]{\substack{R_1+R_4 \\ R_2-R_4 \\ R_3-R_4}} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 3 & 4 & -6 & 1 \\ 0 & 1 & 0 & 0 & -2 & -3 & 5 & -1 \\ 0 & 0 & 1 & 0 & -4 & -4 & 7 & -1 \\ 0 & 0 & 0 & 1 & -4 & -5 & 8 & -1 \end{array}\right].$$

So the required inverse is

$$A^{-1} = \begin{bmatrix} 3 & 4 & -6 & 1 \\ -2 & -3 & 5 & -1 \\ -4 & -4 & 7 & -1 \\ -4 & -5 & 8 & -1 \end{bmatrix}.$$

We can verify our result by showing that $A^{-1}A = A.$ Thus,

$$\begin{bmatrix} 3 & 4 & -6 & 1 \\ -2 & -3 & 5 & -1 \\ -4 & -4 & 7 & -1 \\ -4 & -5 & 8 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 & -1 & 1 \\ 2 & 1 & 1 & 0 \\ 2 & 1 & 0 & 1 \\ 2 & -1 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

16. Starting with the matrix

$$\left[\begin{array}{rrrr|rrrr} 1 & 1 & 2 & 3 & 1 & 0 & 0 & 0 \\ 2 & 3 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 2 & -1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 2 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

we use the sequence of row operations

1. $R_2 - 2R_1,\ R_4 - R_1$ 2. $R_1 - R_2,\ R_3 - 2R_2,\ R_4 - R_2$ 3. $R_3 - 2R_4$
4. $R_1 - 6R_3,\ R_2 + 4R_3,\ R_4 - 3R_3$ 5. $-\frac{1}{10}R_4$
6. $R_1 + 20R_4,\ R_2 - 13R_4,\ R_3 - 5R_4$

to find that

$$A^{-1} = \begin{bmatrix} 1 & 1 & 0 & -2 \\ -\frac{1}{2} & -\frac{3}{10} & \frac{1}{10} & \frac{11}{10} \\ -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{3}{2} \\ \frac{1}{2} & \frac{1}{10} & \frac{3}{10} & -\frac{7}{10} \end{bmatrix}.$$

17. a. $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \end{bmatrix},\ B = \begin{bmatrix} 3 \\ 2 \end{bmatrix};$

b. $X = A^{-1}B = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix};$

18. a. $A = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \end{bmatrix},\ B = \begin{bmatrix} 5 \\ 8 \end{bmatrix};$

b. $X = A^{-1}B = \begin{bmatrix} 5 & -3 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ 8 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

19. a. $A = \begin{bmatrix} 2 & -3 & -4 \\ 0 & 0 & -1 \\ 1 & -2 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 4 \\ 3 \\ -8 \end{bmatrix}$

$$X = A^{-1}B = \begin{bmatrix} 2 & -11 & -3 \\ 1 & -6 & -2 \\ 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} 4 \\ 3 \\ -8 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ -3 \end{bmatrix}$$

20. a. $A = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 1 & 2 \\ -2 & -2 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$; b. $X = A^{-1}B = \begin{bmatrix} 1 & -1 & -1 \\ -\frac{6}{5} & \frac{7}{5} & \frac{4}{5} \\ -\frac{2}{5} & \frac{4}{5} & \frac{3}{5} \end{bmatrix}\begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -3 \\ \frac{14}{5} \\ \frac{13}{5} \end{bmatrix}$

21. a. $A = \begin{bmatrix} 1 & 4 & -1 \\ 2 & 3 & -2 \\ -1 & 2 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ 1 \\ 7 \end{bmatrix}$; b. $X = A^{-1}B = \begin{bmatrix} -\frac{13}{10} & \frac{7}{5} & \frac{1}{2} \\ \frac{2}{5} & -\frac{1}{5} & 0 \\ -\frac{7}{10} & \frac{3}{5} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} 3 \\ 1 \\ 7 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$.

22. a. $A = \begin{bmatrix} 3 & -2 & 7 \\ -2 & 1 & 4 \\ 6 & -5 & 8 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 4 \\ 4 \end{bmatrix}$; b. $X = A^{-1}B = \begin{bmatrix} \frac{7}{8} & -\frac{19}{32} & -\frac{15}{32} \\ \frac{5}{4} & -\frac{9}{16} & -\frac{13}{16} \\ \frac{1}{8} & \frac{3}{32} & -\frac{1}{32} \end{bmatrix}\begin{bmatrix} 6 \\ 4 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$.

23. a. $A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 2 & 1 & 1 & 0 \\ 2 & 1 & 0 & 1 \\ 2 & -1 & -1 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 4 \\ 7 \\ 9 \end{bmatrix}$.

b. $X = A^{-1}B = \begin{bmatrix} 3 & 4 & -6 & 1 \\ -2 & -3 & 5 & -1 \\ -4 & -4 & 7 & -1 \\ -4 & -5 & 8 & -1 \end{bmatrix}\begin{bmatrix} 6 \\ 4 \\ 7 \\ 9 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 3 \end{bmatrix}$.

24. a. $A = \begin{bmatrix} 1 & 1 & 2 & 3 \\ 2 & 3 & 0 & -1 \\ 0 & 2 & -1 & 1 \\ 1 & 2 & 1 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$, $B = \begin{bmatrix} 4 \\ 11 \\ 7 \\ 6 \end{bmatrix}$.

b. $X = A^{-1}B = \begin{bmatrix} 1 & 1 & 0 & -2 \\ -\frac{1}{2} & -\frac{3}{10} & \frac{1}{10} & \frac{11}{10} \\ -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{3}{2} \\ \frac{1}{2} & \frac{1}{10} & \frac{3}{10} & -\frac{7}{10} \end{bmatrix} \begin{bmatrix} 4 \\ 11 \\ 7 \\ 6 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -2 \\ 1 \end{bmatrix}$.

25. a. $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$;

b (i). $X = A^{-1}B = \begin{bmatrix} 0.2 & 0.4 \\ 0.4 & -0.2 \end{bmatrix} \begin{bmatrix} 14 \\ 5 \end{bmatrix} = \begin{bmatrix} 4.8 \\ 4.6 \end{bmatrix}$ and we conclude that $x = 4.8$ and $y = 4.6$.

(ii). $X = A^{-1}B = \begin{bmatrix} 0.2 & 0.4 \\ 0.4 & -0.2 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \end{bmatrix} = \begin{bmatrix} 0.4 \\ 1.8 \end{bmatrix}$ and we conclude that $x = 0.4$ and $y = 1.8$.

26. a. $A = \begin{bmatrix} 3 & -2 \\ 4 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$;

b. (i) $X = A^{-1}B = \begin{bmatrix} \frac{3}{17} & \frac{2}{17} \\ -\frac{4}{17} & \frac{3}{17} \end{bmatrix} \begin{bmatrix} -6 \\ 10 \end{bmatrix} = \begin{bmatrix} \frac{2}{17} \\ \frac{54}{17} \end{bmatrix}$, and we conclude that $x = 2/17$ and $y = 54/17$

(ii) $X = A^{-1}B = \begin{bmatrix} \frac{3}{17} & \frac{2}{17} \\ -\frac{4}{17} & \frac{3}{17} \end{bmatrix} \begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} \frac{5}{17} \\ -\frac{18}{17} \end{bmatrix}$, and we conclude that $x = 5/17$ and $y = -18/17$.

27. a. First we find A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R_3-3R_1]{R_2-R_1} \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & -5 & -2 & -3 & 0 & 1 \end{array}\right] \xrightarrow{-R_2}$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & -5 & -2 & -3 & 0 & 1 \end{array}\right] \xrightarrow[R_3+5R_2]{R_1-2R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & -1 & 2 & 0 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & -2 & 2 & -5 & 1 \end{array}\right] \xrightarrow{-\frac{1}{2}R_3}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & -1 & 2 & 0 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & \frac{5}{2} & -\frac{1}{2} \end{array}\right] \xrightarrow{R_1-R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & \frac{5}{2} & -\frac{1}{2} \end{array}\right]$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

b. (i). $$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{2} & \frac{1}{2} \\ 1 & -1 & 0 \\ -1 & \frac{5}{2} & -\frac{1}{2} \end{bmatrix}\begin{bmatrix} 7 \\ 4 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}$$

and we conclude that $x = -1$, $y = 3$, and $z = 2$

(ii). $$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{2} & \frac{1}{2} \\ 1 & -1 & 0 \\ -1 & \frac{5}{2} & -\frac{1}{2} \end{bmatrix}\begin{bmatrix} 5 \\ -3 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \\ -12 \end{bmatrix}$$

and we conclude that $x = 1$, $y = 8$, and $z = -12$.

28. a. $$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & -2 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}.$$

b. (i) $$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{3}{4} & -\frac{1}{4} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{4} & \frac{3}{4} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 5 \\ -3 \\ -1 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ -3 \end{bmatrix},$$

and we conclude that $x_1 = 4$, $x_2 = 4$, and $x_3 = -3$.

(ii) $$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{3}{4} & -\frac{1}{4} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{4} & \frac{3}{4} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix} = \begin{bmatrix} -\frac{5}{4} \\ -\frac{3}{2} \\ \frac{15}{4} \end{bmatrix}$$

and we conclude that $x_1 = -\frac{5}{4}$, $x_2 = -\frac{3}{2}$, and $x_3 = \frac{15}{4}$.

29. a. $$\left[\begin{array}{ccc|ccc} 3 & 2 & -1 & 1 & 0 & 0 \\ 2 & -3 & 1 & 0 & 1 & 0 \\ 1 & -1 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & -1 & -1 & 0 & 0 & 1 \\ 2 & -3 & 1 & 0 & 1 & 0 \\ 3 & 2 & -1 & 1 & 0 & 0 \end{array}\right] \xrightarrow[R_3 - 3R_1]{R_2 - 2R_1}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & -1 & 0 & 0 & 1 \\ 0 & -1 & 3 & 0 & 1 & -2 \\ 0 & 5 & 2 & 1 & 0 & -3 \end{array}\right] \xrightarrow{-R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & -1 & 0 & 0 & 1 \\ 0 & 1 & -3 & 0 & -1 & 2 \\ 0 & 5 & 2 & 1 & 0 & -3 \end{array}\right] \xrightarrow[R_3 - 5R_2]{R_1 + R_2}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -4 & 0 & -1 & 3 \\ 0 & 1 & -3 & 0 & -1 & 2 \\ 0 & 0 & 17 & 1 & 5 & -13 \end{array}\right] \xrightarrow{\frac{1}{17}R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & -4 & 0 & -1 & 3 \\ 0 & 1 & -3 & 0 & -1 & 2 \\ 0 & 0 & 1 & \frac{1}{17} & \frac{5}{17} & -\frac{13}{17} \end{array}\right]$$

$$\xrightarrow[R_2 + 3R_3]{R_1 + 4R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{4}{17} & \frac{3}{17} & -\frac{1}{17} \\ 0 & 1 & 0 & \frac{3}{17} & -\frac{2}{17} & -\frac{5}{17} \\ 0 & 0 & 1 & \frac{1}{17} & \frac{5}{17} & -\frac{13}{17} \end{array}\right].$$ Therefore $$A^{-1} = \begin{bmatrix} \frac{4}{17} & \frac{3}{17} & -\frac{1}{17} \\ \frac{3}{17} & -\frac{2}{17} & -\frac{5}{17} \\ \frac{1}{17} & \frac{5}{17} & -\frac{13}{17} \end{bmatrix}.$$

Next, $$\begin{bmatrix} 3 & 2 & -1 \\ 2 & -3 & 1 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

b. (i) $$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{4}{17} & \frac{3}{17} & -\frac{1}{17} \\ \frac{3}{17} & -\frac{2}{17} & -\frac{5}{17} \\ \frac{1}{17} & \frac{5}{17} & -\frac{13}{17} \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix} = \begin{bmatrix} -\frac{2}{17} \\ -\frac{10}{17} \\ -\frac{60}{17} \end{bmatrix}$$

We conclude that $x = -2/17$, $y = -10/17$, and $z = -60/17$.

(ii) $$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{4}{17} & \frac{3}{17} & -\frac{1}{17} \\ \frac{3}{17} & -\frac{2}{17} & -\frac{5}{17} \\ \frac{1}{17} & \frac{5}{17} & -\frac{13}{17} \end{bmatrix} \begin{bmatrix} 8 \\ -3 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -5 \end{bmatrix}.$$ We conclude that $x = 1$, $y = 0$, and $z = -5$.

30. a. $$\left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 1 & -3 & 4 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{ccc|ccc} -1 & 0 & 1 & 0 & 0 & 1 \\ 1 & -3 & 4 & 0 & 1 & 0 \\ 2 & 1 & 1 & 1 & 0 & 0 \end{array}\right] \xrightarrow{-R_1}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 0 & 0 & -1 \\ 1 & -3 & 4 & 0 & 1 & 0 \\ 2 & 1 & 1 & 1 & 0 & 0 \end{array}\right] \xrightarrow[R_3 - 2R_1]{R_2 - R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 0 & 0 & -1 \\ 0 & -3 & 5 & 0 & 1 & 1 \\ 0 & 1 & 3 & 1 & 0 & 2 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_3}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 0 & 0 & -1 \\ 0 & 1 & 3 & 1 & 0 & 2 \\ 0 & -3 & 5 & 0 & 1 & 1 \end{array}\right] \xrightarrow{R_3 + 3R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 0 & 0 & -1 \\ 0 & 1 & 3 & 1 & 0 & 2 \\ 0 & 0 & 14 & 3 & 1 & 7 \end{array}\right] \xrightarrow{\frac{1}{14}R_3}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 0 & 0 & -1 \\ 0 & 1 & 3 & 1 & 0 & 2 \\ 0 & 0 & 1 & \frac{3}{14} & \frac{1}{14} & \frac{1}{2} \end{array}\right] \xrightarrow[R_2 - 3R_3]{R_1 + R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{3}{14} & \frac{1}{14} & -\frac{1}{2} \\ 0 & 1 & 0 & \frac{5}{14} & -\frac{3}{14} & \frac{1}{2} \\ 0 & 0 & 1 & \frac{3}{14} & \frac{1}{14} & \frac{1}{2} \end{array}\right].$$

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & -3 & 4 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}.$$

b. (i) $$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{3}{14} & \frac{1}{14} & -\frac{1}{2} \\ \frac{5}{14} & -\frac{3}{14} & \frac{1}{2} \\ \frac{3}{14} & \frac{1}{14} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ -3 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix}.$$ (ii). $$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{3}{14} & \frac{1}{14} & -\frac{1}{2} \\ \frac{5}{14} & -\frac{3}{14} & \frac{1}{2} \\ \frac{3}{14} & \frac{1}{14} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 2 \\ -5 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{14} \\ \frac{25}{14} \\ \frac{1}{14} \end{bmatrix}.$$

31. a. $AX = B_1$ and $AX = B_2$, where

$$A=\begin{bmatrix}1&1&1&1\\1&-1&-1&1\\0&1&2&2\\1&2&1&-2\end{bmatrix},\ X=\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix},\ B_1=\begin{bmatrix}1\\-1\\4\\0\end{bmatrix}\text{ and }B_2=\begin{bmatrix}2\\8\\4\\-1\end{bmatrix}.$$

We first find A^{-1}.

$$\left[\begin{array}{rrrr|rrrr}1&1&1&1&1&0&0&0\\1&-1&-1&1&0&1&0&0\\0&1&2&2&0&0&1&0\\1&2&1&-2&0&0&0&1\end{array}\right]\xrightarrow[R_4-R_1]{R_2-R_1}\left[\begin{array}{rrrr|rrrr}1&1&1&1&1&0&0&0\\0&-2&-2&0&-1&1&0&0\\0&1&2&2&0&0&1&0\\0&1&0&-3&-1&0&0&1\end{array}\right]$$

$$\xrightarrow{R_2\leftrightarrow R_3}\left[\begin{array}{rrrr|rrrr}1&1&1&1&1&0&0&0\\0&1&2&2&0&0&1&0\\0&-2&-2&0&-1&1&0&0\\0&1&0&-3&-1&0&0&1\end{array}\right]\xrightarrow[\substack{R_3+2R_2\\R_4-R_2}]{R_1-R_2}$$

$$\left[\begin{array}{rrrr|rrrr}1&0&-1&-1&1&0&-1&0\\0&1&2&2&0&0&1&0\\0&0&2&4&-1&1&2&0\\0&0&-2&-5&-1&0&-1&1\end{array}\right]\xrightarrow{\frac{1}{2}R_3}\left[\begin{array}{rrrr|rrrr}1&0&-1&-1&1&0&-1&0\\0&1&2&2&0&0&1&0\\0&0&1&2&-\frac{1}{2}&\frac{1}{2}&1&0\\0&0&-2&-5&-1&0&-1&1\end{array}\right]$$

$$\xrightarrow[\substack{R_2-2R_3\\R_4+2R_3}]{R_1+R_3}\left[\begin{array}{rrrr|rrrr}1&0&0&1&\frac{1}{2}&\frac{1}{2}&0&0\\0&1&0&-2&1&-1&-1&0\\0&0&1&2&-\frac{1}{2}&\frac{1}{2}&1&0\\0&0&0&-1&-2&1&1&1\end{array}\right]\xrightarrow[\substack{R_2-2R_4\\R_3+2R_4\\-R_4}]{R_1+R_4}$$

$$\left[\begin{array}{rrrr|rrrr}1&0&0&0&-\frac{3}{2}&\frac{3}{2}&1&1\\0&1&0&0&5&-3&-3&-2\\0&0&1&0&-\frac{9}{2}&\frac{5}{2}&3&2\\0&0&0&1&2&-1&-1&-1\end{array}\right].\quad\text{So}\quad A^{-1}=\begin{bmatrix}-\frac{3}{2}&\frac{3}{2}&1&1\\5&-3&-3&-2\\-\frac{9}{2}&\frac{5}{2}&3&2\\2&-1&-1&-1\end{bmatrix}.$$

b. (i). $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -\frac{3}{2} & \frac{3}{2} & 1 & 1 \\ 5 & -3 & -3 & -2 \\ -\frac{9}{2} & \frac{5}{2} & 3 & 2 \\ 2 & -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \\ 5 \\ -1 \end{bmatrix}$

and we conclude that $x_1 = 1$, $x_2 = -4$, $x_3 = 5$, and $x_4 = -1$.

(ii). $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -\frac{3}{2} & \frac{3}{2} & 1 & 1 \\ 5 & -3 & -3 & -2 \\ -\frac{9}{2} & \frac{5}{2} & 3 & 2 \\ 2 & -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 8 \\ 4 \\ -1 \end{bmatrix} = \begin{bmatrix} 12 \\ -24 \\ 21 \\ -7 \end{bmatrix}$

and we conclude that $x_1 = 12$, $x_2 = -24$, $x_3 = 21$, and $x_4 = -7$.

32. a. $\begin{bmatrix} 1 & 1 & 2 & 1 \\ 4 & 5 & 9 & 1 \\ 3 & 4 & 7 & 1 \\ 2 & 3 & 4 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$

b. (i) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 & 5 & -7 & 1 \\ -2 & 0 & 0 & 1 \\ 1 & -2 & 3 & -1 \\ 1 & -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 6 \\ 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -1 \\ 2 \end{bmatrix}$

and we conclude that $x_1 = 2$, $x_2 = 1$, $x_3 = -1$, and $x_4 = 2$.

(ii). $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 & 5 & -7 & 1 \\ -2 & 0 & 0 & 1 \\ 1 & -2 & 3 & -1 \\ 1 & -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0 \\ -4 \end{bmatrix} = \begin{bmatrix} -9 \\ -6 \\ 7 \\ 2 \end{bmatrix}$

and we conclude that $x_1 = -9$, $x_2 = -6$, $x_3 = 7$, and $x_4 = 2$.

33. a. Using Formula (13), we find $A^{-1} = \frac{1}{(2)(-5)-(-4)(3)} \begin{bmatrix} -5 & -3 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} -\frac{5}{2} & -\frac{3}{2} \\ 2 & 1 \end{bmatrix}$.

b. Using Formula (13) once again, we find

$$\left(A^{-1}\right)^{-1} = \frac{1}{(-\frac{5}{2})(1) - 2(-\frac{3}{2})}\begin{bmatrix} 1 & \frac{3}{2} \\ -2 & -\frac{5}{2} \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ -4 & -5 \end{bmatrix} = A.$$

34. a. $AB = \begin{bmatrix} 6 & -4 \\ -4 & 3 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ 4 & -7 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 0 & -1 \end{bmatrix}$

Using Formula (13), we find

$$A^{-1} = \frac{1}{(6)(3) - (-4)(-4)}\begin{bmatrix} 3 & 4 \\ 4 & 6 \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & 2 \\ 2 & 3 \end{bmatrix}$$

$$B^{-1} = \frac{1}{(3)(-7) - (4)(-5)}\begin{bmatrix} -7 & 5 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} 7 & -5 \\ 4 & -3 \end{bmatrix}.$$

b. Using Formula (13),

$$(AB)^{-1} = \frac{1}{(2)(-1) - 0}\begin{bmatrix} -1 & 2 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -1 \\ 0 & -1 \end{bmatrix}.$$

Also, $B^{-1}A^{-1} = \begin{bmatrix} 7 & -5 \\ 4 & -3 \end{bmatrix}\begin{bmatrix} \frac{3}{2} & 2 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -1 \\ 0 & -1 \end{bmatrix} = (AB)^{-1}$.

35. a. $ABC = \begin{bmatrix} 2 & -5 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} 4 & 3 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -5 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} 2 & 15 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 10 \\ 2 & 3 \end{bmatrix}$.

Using the formula for finding the inverse of a 2 × 2 matrix, we find

$$A^{-1} = \begin{bmatrix} 3 & -5 \\ 1 & -2 \end{bmatrix},\ B^{-1} = \begin{bmatrix} 1 & -3 \\ -1 & 4 \end{bmatrix},\ C^{-1} = \begin{bmatrix} \frac{1}{8} & -\frac{3}{8} \\ \frac{1}{4} & \frac{1}{4} \end{bmatrix}.$$

b. Using the formula for finding the inverse of a 2 × 2 matrix, we find

$$(ABC)^{-1} = \begin{bmatrix} -\frac{3}{8} & \frac{5}{4} \\ \frac{1}{4} & -\frac{1}{2} \end{bmatrix}$$

$$C^{-1}B^{-1}A^{-1} = \begin{bmatrix} \frac{1}{8} & -\frac{3}{8} \\ \frac{1}{4} & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 1 & -3 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 3 & -5 \\ 1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{8} & -\frac{3}{8} \\ \frac{1}{4} & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} -\frac{3}{8} & \frac{5}{4} \\ \frac{1}{4} & -\frac{1}{2} \end{bmatrix}.$$

Therefore, $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$.

36. Multiply both sides of the equation on the left by $\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}^{-1}$, we obtain

$$\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix} A = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$$

$$A = \tfrac{1}{7} \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} \frac{8}{7} & \frac{2}{7} \\ \frac{5}{7} & \frac{10}{7} \end{bmatrix}$$

37. Multiply both sides of the equation on the right by

$\begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}^{-1}$, we obtain

$$A \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}^{-1} = \begin{bmatrix} 2 & 1 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}^{-1}$$

$$A = \begin{bmatrix} 2 & 1 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} \frac{1}{7} & \frac{2}{7} \\ \frac{3}{7} & -\frac{1}{7} \end{bmatrix} = \begin{bmatrix} \frac{5}{7} & \frac{3}{7} \\ -\frac{3}{7} & \frac{8}{7} \end{bmatrix}$$

38. Let x denote the number of adults who took the cruise and y the number of children who took the cruise. Then the number of adults and children who took the cruise on Saturday is found by solving the system

$$\begin{aligned} x + y &= 1000 \\ 16x + 8y &= 12800 \end{aligned}$$

and the number of adults and children who took the cruise on Sunday is found by solving the system

$$\begin{aligned} x + y &= 800 \\ 16x + 8y &= 9600. \end{aligned}$$

These systems may be written in the form $AX = B_1$ and $AX = B_2$ where

$$A = \begin{bmatrix} 1 & 1 \\ 16 & 8 \end{bmatrix}, \ B_1 = \begin{bmatrix} 1000 \\ 12800 \end{bmatrix}, \text{ and } \ B_2 = \begin{bmatrix} 800 \\ 9600 \end{bmatrix}.$$

Using the formula for finding the inverse of a 2×2 matrix, we find

$$A^{-1} = \begin{bmatrix} -1 & \frac{1}{8} \\ 2 & -\frac{1}{8} \end{bmatrix}.$$

Then the number of adults and children who took the cruise on Saturday may be found by computing

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 & \frac{1}{8} \\ 2 & -\frac{1}{8} \end{bmatrix} \begin{bmatrix} 1000 \\ 12800 \end{bmatrix} = \begin{bmatrix} 600 \\ 400 \end{bmatrix}.$$

We conclude that 600 adults and 400 children took the cruise on Saturday. Similarly, the number of adults and children who took the cruise on Sunday may be found by computing

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 & \frac{1}{8} \\ 2 & -\frac{1}{8} \end{bmatrix} \begin{bmatrix} 800 \\ 9600 \end{bmatrix} = \begin{bmatrix} 400 \\ 400 \end{bmatrix}.$$

We conclude that there were 400 adults and 400 children on the Sunday cruise.

39. Let x denote the number of copies of the deluxe edition and y the number of copies of the standard edition demanded per month when the unit prices are p and q dollars, respectively. Then the three systems of linear equations

$$\begin{aligned} 5x + y &= 20000 \\ x + 3y &= 15000 \end{aligned} \qquad \begin{aligned} 5x + y &= 25000 \\ x + 3y &= 15000 \end{aligned} \qquad \begin{aligned} 5x + y &= 25000 \\ x + 3y &= 20000 \end{aligned}$$

give the quantity demanded of each edition at the stated price. These systems may be written in the form $AX = B_1$, $AX = B_2$, and $AX = B_3$, where

$$A = \begin{bmatrix} 5 & 1 \\ 1 & 3 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 20000 \\ 15000 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 25000 \\ 15000 \end{bmatrix}, \quad \text{and} \quad B_3 = \begin{bmatrix} 25000 \\ 20000 \end{bmatrix}$$

Using the formula for finding the inverse of a 2×2 matrix, with $a = 5$, $b = 1$, $c = 1$, $d = 3$, and $D = ad - bc = (5)(3) - (1)(1) = 14$, we find that $A^{-1} = \begin{bmatrix} \frac{3}{14} & -\frac{1}{14} \\ -\frac{1}{14} & \frac{5}{14} \end{bmatrix}$.

a. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{3}{14} & -\frac{1}{14} \\ -\frac{1}{14} & \frac{5}{14} \end{bmatrix} \begin{bmatrix} 20{,}000 \\ 15{,}000 \end{bmatrix} = \begin{bmatrix} 3{,}214 \\ 3{,}929 \end{bmatrix}$ b. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{3}{14} & -\frac{1}{14} \\ -\frac{1}{14} & \frac{5}{14} \end{bmatrix} \begin{bmatrix} 25{,}000 \\ 15{,}000 \end{bmatrix} = \begin{bmatrix} 4{,}286 \\ 3{,}571 \end{bmatrix}$

c. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{3}{14} & -\frac{1}{14} \\ -\frac{1}{14} & \frac{5}{14} \end{bmatrix} \begin{bmatrix} 25{,}000 \\ 20{,}000 \end{bmatrix} = \begin{bmatrix} 3{,}929 \\ 5{,}357 \end{bmatrix}$.

40. Let x denote the number of ounces of food A, food B, and food C, respectively, in a meal. Then we can find the number of ounces of each food needed in Susan's and Tom's diets by solving the systems

$$\begin{aligned} 30x + 25y + 20z &= 400 \\ x + y + 2z &= 20 \\ 2x + 5y + 4z &= 50 \end{aligned} \qquad \begin{aligned} 30x + 25y + 20z &= 350 \\ x + y + 2z &= 15 \\ 2x + 5y + 4z &= 40 \end{aligned}$$

These systems may be written in the form $AX = B_1$ and $AX = B_2$, where

$$A = \begin{bmatrix} 30 & 25 & 20 \\ 1 & 1 & 2 \\ 2 & 5 & 4 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B_1 = \begin{bmatrix} 400 \\ 20 \\ 50 \end{bmatrix} \quad \text{and} \quad B_2 = \begin{bmatrix} 350 \\ 15 \\ 40 \end{bmatrix}.$$

To find A^{-1}, we compute

$$\left[\begin{array}{ccc|ccc} 30 & 25 & 20 & 1 & 0 & 0 \\ 1 & 1 & 2 & 0 & 1 & 0 \\ 2 & 5 & 4 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 0 & 1 & 0 \\ 30 & 25 & 20 & 1 & 0 & 0 \\ 2 & 5 & 4 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R_3 - 2R_1]{R_2 - 30R_1}$$

$$\left[\begin{array}{rrr|rrr} 1 & 1 & 2 & 0 & 1 & 0 \\ 0 & -5 & -40 & 1 & -30 & 0 \\ 0 & 3 & 0 & 0 & -2 & 1 \end{array}\right] \xrightarrow{-\frac{1}{5}R_2} \left[\begin{array}{rrr|rrr} 1 & 1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 8 & -\frac{1}{5} & 6 & 0 \\ 0 & 3 & 0 & 0 & -2 & 1 \end{array}\right] \xrightarrow[R_3-3R_2]{R_1-R_2}$$

$$\left[\begin{array}{rrr|rrr} 1 & 0 & -6 & \frac{1}{5} & -5 & 0 \\ 0 & 1 & 8 & -\frac{1}{5} & 6 & 0 \\ 0 & 0 & -24 & \frac{3}{5} & -20 & 1 \end{array}\right] \xrightarrow{-\frac{1}{24}R_3} \left[\begin{array}{rrr|rrr} 1 & 0 & -6 & \frac{1}{5} & -5 & 0 \\ 0 & 1 & 8 & -\frac{1}{5} & 6 & 0 \\ 0 & 0 & 1 & -\frac{1}{40} & \frac{5}{6} & -\frac{1}{24} \end{array}\right]$$

$$\xrightarrow[R_2-8R_3]{R_1+6R_3} \left[\begin{array}{rrr|rrr} 1 & 0 & 0 & \frac{1}{20} & 0 & -\frac{1}{4} \\ 0 & 1 & 0 & 0 & -\frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 1 & -\frac{1}{40} & \frac{5}{6} & -\frac{1}{24} \end{array}\right]. \text{ Therefore, } \quad A^{-1} = \begin{bmatrix} \frac{1}{20} & 0 & -\frac{1}{4} \\ 0 & -\frac{2}{3} & \frac{1}{3} \\ -\frac{1}{40} & \frac{5}{6} & -\frac{1}{24} \end{bmatrix}.$$

Next,

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{1}{20} & 0 & -\frac{1}{4} \\ 0 & -\frac{2}{3} & \frac{1}{3} \\ -\frac{1}{40} & \frac{5}{6} & -\frac{1}{24} \end{bmatrix} \begin{bmatrix} 400 \\ 20 \\ 50 \end{bmatrix} = \begin{bmatrix} \frac{15}{2} \\ \frac{10}{3} \\ \frac{55}{12} \end{bmatrix} \text{and} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{1}{20} & 0 & -\frac{1}{4} \\ 0 & -\frac{2}{3} & \frac{1}{3} \\ -\frac{1}{40} & \frac{5}{6} & -\frac{1}{24} \end{bmatrix} \begin{bmatrix} 350 \\ 15 \\ 40 \end{bmatrix} = \begin{bmatrix} \frac{15}{2} \\ \frac{10}{3} \\ \frac{25}{12} \end{bmatrix}.$$

We conclude that Susan's meals should contain $7\frac{1}{2}$ ounces of food *A*, $3\frac{1}{3}$ ounces of food *B*, and $4\frac{7}{12}$ ounces of food *C*.Tom's meals should contain $7\frac{1}{2}$ ounces of food *A*, 3 $\frac{1}{3}$ ounces of food *B*, and $2\frac{1}{12}$ ounces of food *C*.

41. Let *x*, *y*, and *z* denote the number of acres of soybeans, corn, and wheat to be cultivated, respectively. Furthermore, let *a*, *b*, and *c* denote the amount of land available; the amount of labor available, and the amount of money available for seeds, respectively. Then we have the system

$$\begin{aligned} x + y + z &= a \qquad \text{(land)} \\ 2x + 6y + 6z &= b \qquad \text{(labor)} \\ 12x + 20y + 8z &= c \qquad \text{(seeds)} \end{aligned}$$

The system can be written in the form $AX = B$, where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 6 & 6 \\ 12 & 20 & 8 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad \text{and} \quad B = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

Using the technique for finding A^{-1} developed in this section, we find

$$A^{-1} = \begin{bmatrix} \frac{3}{2} & -\frac{1}{4} & 0 \\ -\frac{7}{6} & \frac{1}{12} & \frac{1}{12} \\ \frac{2}{3} & \frac{1}{6} & -\frac{1}{12} \end{bmatrix}.$$

a. Here $a = 1000$, $b = 4400$, and $c = 13,200$. Therefore

$$X = A^{-1}B = \begin{bmatrix} \frac{3}{2} & -\frac{1}{4} & 0 \\ -\frac{7}{6} & \frac{1}{12} & \frac{1}{12} \\ \frac{2}{3} & \frac{1}{6} & -\frac{1}{12} \end{bmatrix} \begin{bmatrix} 1000 \\ 4400 \\ 13200 \end{bmatrix} = \begin{bmatrix} 400 \\ 300 \\ 300 \end{bmatrix}$$

So, Jackson Farms should cultivate 400, 300, and 300 acres of soybeans, corn, and wheat, respectively.

b. Here $a = 1200$, $b = 5200$, and $c = 16,400$. Therefore,

$$X = A^{-1}B = \begin{bmatrix} \frac{3}{2} & -\frac{1}{4} & 0 \\ -\frac{7}{6} & \frac{1}{12} & \frac{1}{12} \\ \frac{2}{3} & \frac{1}{6} & -\frac{1}{12} \end{bmatrix} \begin{bmatrix} 1200 \\ 5200 \\ 16400 \end{bmatrix} = \begin{bmatrix} 500 \\ 400 \\ 300 \end{bmatrix}$$

So, Jackson Farms should cultivate 500, 400, and 300 acres of soybeans, corn, and wheat, respectively.

42. Let x, y, and z denote the number of 100-lb bags of grade A, grade B, and grade C fertilizers to be produced. Next, let a, b, and c denote the amount of nitrogen, phosphate, and potassium available. Then we have

$$\begin{aligned} 18x + 20y + 24z &= a \\ 4x + 4y + 3z &= b \\ 5x + 4y + 6z &= c \end{aligned}$$

Writing the system in matrix form, we have $AX = B$, where

$$A=\begin{bmatrix}18 & 20 & 24\\ 4 & 4 & 3\\ 5 & 4 & 6\end{bmatrix},\quad X=\begin{bmatrix}x\\ y\\ z\end{bmatrix},\quad \text{and } B=\begin{bmatrix}a\\ b\\ c\end{bmatrix}.$$

We find

$$A^{-1}=\begin{bmatrix}-\frac{1}{5} & \frac{2}{5} & \frac{3}{5}\\ \frac{3}{20} & \frac{1}{5} & -\frac{7}{10}\\ \frac{1}{15} & -\frac{7}{15} & \frac{2}{15}\end{bmatrix}.$$

Here $a = 26{,}400$, $b = 4900$, and $c = 6200$. Therefore,

$$X=A^{-1}B=\begin{bmatrix}-\frac{1}{5} & \frac{2}{5} & \frac{3}{5}\\ \frac{3}{20} & \frac{1}{5} & -\frac{7}{10}\\ \frac{1}{15} & -\frac{7}{15} & \frac{2}{15}\end{bmatrix}\begin{bmatrix}26400\\ 4900\\ 6200\end{bmatrix}=\begin{bmatrix}400\\ 600\\ 300\end{bmatrix}.$$

So, Lawnco should produce 400, 600, and 300 bags of grade A, grade B, and grade C fertilizers.

b. Here $a = 21{,}800$, $b = 4200$, and $c = 5300$. We find

$$X=A^{-1}B=\begin{bmatrix}500\\ 400\\ 200\end{bmatrix}.$$

So, Lawnco should produce 500, 400, and 200 bags of grade A, grade B, and grade C fertilizers, respectively.

43. Let x, y, and z denote the amount to be invested in high-risk, medium-risk, and low-risk stocks, respectively. Next, let a denote the amount to be invested and let c denote the return on the investments. Then, we have the system

$$\begin{aligned} x + y + z &= a \\ x + y - z &= 0 \qquad (\text{since } z = x + y) \\ 0.15x + 0.1y + 0.06z &= c \end{aligned}$$

The system is equivalent to the matrix equation $AX = B$, where

$$A=\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & -1\\ .15 & .10 & .06\end{bmatrix},\quad X=\begin{bmatrix}x\\ y\\ z\end{bmatrix},\quad \text{and}\quad B=\begin{bmatrix}a\\ 0\\ c\end{bmatrix}.$$

We find $A^{-1} = \begin{bmatrix} -1.6 & -0.4 & 20 \\ 2.1 & 0.9 & -20 \\ 0.5 & -0.5 & 0 \end{bmatrix}$.

a. Here a = 200,000 and c = 20,000. Therefore,

$$X = A^{-1}B = \begin{bmatrix} -1.6 & -0.4 & 20 \\ 2.1 & 0.9 & -20 \\ 0.5 & -0.5 & 0 \end{bmatrix} \begin{bmatrix} 200,000 \\ 0 \\ 20,000 \end{bmatrix} = \begin{bmatrix} 80,000 \\ 20,000 \\ 100,000 \end{bmatrix}.$$

So, the club should invest \$80,000 in high-risk, \$20,000 in medium risk, and \$100,000 in low risk stocks.

b. Here a = 220,000 and c = 22,000. The solution is x = 88,000, y = 22,000, and z = 110,000; that is, the club should invest \$88,000 in high-risk, \$22,000 in medium-risk, and \$110,000 in low-risk stocks.

c. Here a = 240,000 and c = 22,000. The result is \$56,000 in high-risk stocks, \$64,000 in medium-risk stocks, and \$120,000 in low-risk stocks.

44. Let x, y, and z (in millions of dollars) be the amount awarded to organization I, II, and III, respectively. Then we have

$$\begin{aligned} 0.6x + 0.4y + 0.2z &= 9.2 \qquad (8.2) \\ 0.3x + 0.3y + 0.6z &= 9.6 \qquad (7.2) \\ 0.1x + 0.3y + 0.2z &= 5.2 \qquad (3.6). \end{aligned}$$

The quantities within the brackets are for part (b). We can rewrite the systems as

$AX = B_1$, and $AX = B_2$. Put

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \ A = \begin{bmatrix} 6 & 4 & 2 \\ 3 & 3 & 6 \\ 1 & 3 & 2 \end{bmatrix}, \ B_1 = \begin{bmatrix} 92 \\ 96 \\ 52 \end{bmatrix}, \text{ and } \ B_2 = \begin{bmatrix} 82 \\ 72 \\ 36 \end{bmatrix}.$$

To find A^{-1}, we use the Gauss-Jordan method:

$$\left[\begin{array}{ccc|ccc} 6 & 4 & 2 & 1 & 0 & 0 \\ 3 & 3 & 6 & 0 & 1 & 0 \\ 1 & 3 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & 3 & 2 & 0 & 0 & 1 \\ 3 & 3 & 6 & 0 & 1 & 0 \\ 6 & 4 & 2 & 1 & 0 & 0 \end{array}\right] \xrightarrow[R_3 - 6R_1]{R_2 - 3R_1}$$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 2 & 0 & 0 & 1 \\ 0 & -6 & 0 & 0 & 1 & -3 \\ 0 & -14 & -10 & 1 & 0 & -6 \end{array}\right] \xrightarrow{-\frac{1}{6}R_2} \left[\begin{array}{ccc|ccc} 1 & 3 & 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -\frac{1}{6} & \frac{1}{2} \\ 0 & -14 & -10 & 1 & 0 & -6 \end{array}\right] \xrightarrow[R_3 + 14R_2]{R_1 - 3R_2}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & 0 & 0 & -\frac{1}{6} & \frac{1}{2} \\ 0 & 0 & -10 & 1 & -\frac{7}{3} & 1 \end{array}\right] \xrightarrow{-\frac{1}{10}R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & 0 & 0 & -\frac{1}{6} & \frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{10} & \frac{7}{30} & -\frac{1}{10} \end{array}\right] \xrightarrow{R_1 - 2R_3}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{5} & \frac{1}{30} & -\frac{3}{10} \\ 0 & 1 & 0 & 0 & -\frac{1}{6} & \frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{10} & \frac{7}{30} & -\frac{1}{10} \end{array}\right].$$

a. $X = A^{-1}B_1 = \begin{bmatrix} \frac{1}{5} & \frac{1}{30} & -\frac{3}{10} \\ 0 & -\frac{1}{6} & \frac{1}{2} \\ -\frac{1}{10} & \frac{7}{30} & -\frac{1}{10} \end{bmatrix} \begin{bmatrix} 92 \\ 96 \\ 52 \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ 8 \end{bmatrix}$

that is, x = 6, y = 10, and z = 8, and Organization I will receive \$6 million, Organization II will receive \$10 million, and Organization III will receive \$8 million.

b. $X = A^{-1}B_1 = \begin{bmatrix} \frac{1}{5} & \frac{1}{30} & -\frac{3}{10} \\ 0 & -\frac{1}{6} & \frac{1}{2} \\ -\frac{1}{10} & \frac{7}{30} & -\frac{1}{10} \end{bmatrix} \begin{bmatrix} 82 \\ 72 \\ 36 \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \\ 5 \end{bmatrix}$

that is, $x = 8$, $y = 6$, and $z = 5$, and Organization I will receive \$8 million, Organization II will receive \$6 million, and Organization III will receive \$5 million.

45. In order for the inverse of A to exist, $D = ad - bc \neq 0$.
Here a = 1, b = 2, c = k, and d = 3. So (1)(3)-(2)(k) $\neq 0$, or $k \neq \frac{3}{2}$
So A^{-1} has an inverse provided $k \neq \frac{3}{2}$. Using Formula 13, we have

$$A^{-1} = \frac{1}{3-2k}\begin{bmatrix} 3 & -2 \\ -k & 1 \end{bmatrix}.$$

46.

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ -2 & 1 & k & 0 & 1 & 0 \\ -1 & 2 & k^2 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R_3+R_1]{R_2+2R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & k+2 & 2 & 1 & 0 \\ 0 & 2 & k^2+1 & 1 & 0 & 1 \end{array}\right] \xrightarrow{R_3-2R_2}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & k+2 & 2 & 1 & 0 \\ 0 & 0 & k^2-2k-3 & -3 & -2 & 1 \end{array}\right]$$

In order for A^{-1} to exist, $k^2-2k-3 \neq 0$. But $k^2-2k-3=(k-3)(k+1)$ if k = -1 or 3. So , A has an inverse provided $k \neq -1$ or 3.

47. True. Multiplying both sides of the equation by cA yields

$$I=(cA)(cA)^{-1}=(cA)\left[\frac{1}{c}(A^{-1})\right]=c\left(\frac{1}{c}\right)AA^{-1}=I\ .$$

48. False. The matrix A has a multiplicative inverse if and only if $ad-bc \neq 0$, in which case

$$A^{-1}=\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

49. True. $AX = B$ can have a unique solution only if A^{-1} exists, in which case the solution is found as follows:

$$\begin{aligned} A^{-1}(AX) &= A^{-1}B \\ (A^{-1}A)X &= A^{-1}B \\ IX &= A^{-1}B \\ X &= A^{-1}B \end{aligned}.$$

50. **Case 1:** $a \neq 0$

$$\left[\begin{array}{cc|cc} a & b & 1 & 0 \\ c & d & 0 & 1 \end{array}\right] \xrightarrow{\frac{1}{a}R_1} \left[\begin{array}{cc|cc} a & \frac{b}{a} & \frac{1}{a} & 0 \\ c & d & 0 & 1 \end{array}\right] \xrightarrow{R_2-cR_1} \left[\begin{array}{cc|cc} 1 & \frac{b}{a} & \frac{1}{a} & 0 \\ 0 & d-\frac{bc}{a} & -\frac{c}{a} & 1 \end{array}\right]$$

$$\xrightarrow{\frac{a}{ad-bc}R_2}\left[\begin{array}{cc|cc}1 & \frac{b}{a} & \frac{1}{a} & 0\\ 0 & 1 & -\frac{c}{ad-bc} & \frac{a}{ad-bc}\end{array}\right]\xrightarrow{R_1-\frac{b}{a}R_2}\left[\begin{array}{cc|cc}1 & 0 & \frac{d}{ad-bc} & -\frac{b}{ad-bc}\\ 0 & 1 & -\frac{c}{ad-bc} & \frac{a}{ad-bc}\end{array}\right]$$

since $\frac{1}{a}-\frac{b}{a}\left(-\frac{c}{ad-bc}\right)=\frac{ad-bc+bc}{a(ad-bc)}=\frac{d}{ad-bc}$ provided $ad-bc\neq 0$.

Case 2: $a = 0$

$$\left[\begin{array}{cc|cc}0 & b & 1 & 0\\ c & d & 0 & 1\end{array}\right]\xrightarrow{R_1\leftrightarrow R_2}\left[\begin{array}{cc|cc}c & d & 0 & 1\\ 0 & b & 1 & 0\end{array}\right]\xrightarrow[\frac{1}{b}R_2]{\frac{1}{c}R_1}$$

$$\left[\begin{array}{cc|cc}1 & \frac{d}{c} & 1 & \frac{1}{c}\\ 0 & 1 & \frac{1}{b} & 0\end{array}\right]\xrightarrow{R_1-\frac{d}{c}R_2}\left[\begin{array}{cc|cc}1 & 0 & -\frac{d}{bc} & \frac{1}{c}\\ 0 & 1 & \frac{1}{b} & 0\end{array}\right]$$ provided $bc\neq 0$.

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1. $\begin{bmatrix}0.36 & 0.04 & -0.36\\ 0.06 & 0.05 & 0.20\\ -0.19 & 0.10 & 0.09\end{bmatrix}$

2. $\begin{bmatrix}0.12 & 0.25 & -0.01\\ 0 & -0.10 & 0.11\\ 0.11 & -0.15 & -0.08\end{bmatrix}$

3. $\begin{bmatrix}0.01 & -0.09 & 0.31 & -0.11\\ -0.25 & 0.58 & -0.15 & -0.02\\ 0.86 & -0.42 & 0.07 & -0.37\\ -0.27 & 0.01 & -0.05 & 0.31\end{bmatrix}$

4. $\begin{bmatrix}-3.18 & -0.28 & 2.49 & -1.76\\ -2.06 & -0.32 & 1.84 & -1\\ 5.44 & 0.76 & -4.47 & 2.81\\ -6.84 & -0.84 & 5.42 & -3.38\end{bmatrix}$

5. $\begin{bmatrix}0.30 & 0.85 & -0.10 & -0.77 & -0.11\\ -0.21 & 0.10 & 0.01 & -0.26 & 0.21\\ 0.03 & -0.16 & 0.12 & -0.01 & 0.03\\ -0.14 & -0.46 & 0.13 & 0.71 & -0.05\\ 0.10 & -0.05 & -0.10 & -0.03 & 0.11\end{bmatrix}$

6. $\begin{bmatrix} -0.07 & 0.11 & 0.17 & 0.09 & -0.16 \\ 0.28 & 0.09 & -0.14 & 0.08 & -0.17 \\ 0.09 & -0.09 & 0.01 & -0.26 & 0.16 \\ 0.04 & -0.23 & 0.29 & -0.06 & 0.10 \\ -0.26 & 0.28 & -0.35 & 0.20 & 0.13 \end{bmatrix}$ 7. $x = 1.2$, $y = 3.6$, and $z = 2.7$.

8. $x = -0.76$, $y = -0.74$, and $z = 1.89$.

9. $x_1 = 2.50$, $x_2 = -0.88$, $x_3 = 0.70$, and $x_4 = 0.51$.

10. $x_1 = 2.32$, $x_2 = -6.46$, $x_3 = -2.43$, and $x_4 = 2$.