USING TECHNOLOGY EXERCISES 2.4, page 113

1. $\begin{bmatrix} 15 & 38.75 & -67.5 & 33.75 \\ 51.25 & 40 & 52.5 & -38.75 \\ 21.25 & 35 & -65 & 105 \end{bmatrix}$ 2. $\begin{bmatrix} -52.08 & 26.88 & -11.76 & 10.08 \\ -26.04 & -22.68 & 10.08 & -14.28 \\ -10.08 & 11.76 & 14.28 & -23.52 \end{bmatrix}$

3. $\begin{bmatrix} -5 & 6.3 & -6.8 & 3.9 \\ 1 & 0.5 & 5.4 & -4.8 \\ 0.5 & 4.2 & -3.5 & 5.6 \end{bmatrix}$ 4. $\begin{bmatrix} 5 & -6.3 & 6.8 & -3.9 \\ -1 & -0.5 & -5.4 & 4.8 \\ -0.5 & -4.2 & 3.5 & -5.6 \end{bmatrix}$

5. $\begin{bmatrix} 16.44 & -3.65 & -3.66 & 0.63 \\ 12.77 & 10.64 & 2.58 & 0.05 \\ 5.09 & 0.28 & -10.84 & 17.64 \end{bmatrix}$ 6. $\begin{bmatrix} -8.02 & 11.95 & -13.72 & 7.71 \\ 3.34 & 2.13 & 10.86 & -9.4 \\ 1.53 & 8.26 & -8.03 & 12.88 \end{bmatrix}$

7. $\begin{bmatrix} 22.2 & -0.3 & -12 & 4.5 \\ 21.6 & 17.7 & 9 & -4.2 \\ 8.7 & 4.2 & -20.7 & 33.6 \end{bmatrix}$ 8. $\begin{bmatrix} -12.142 & 26.091 & -32.968 & 17.979 \\ 12.584 & 8.983 & 25.974 & -21.606 \\ 5.473 & 19.11 & -22.633 & 36.4 \end{bmatrix}$

2.5 CONCEPT QUESTIONS, page 120

1. Scalar multiplication involves multiplying a matrix *A* by a scalar *c* (result: *cA*); whereas matrix multiplication involves the product of two matrices.
   Example:
$$3\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 6 & 9 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} 1 & 3 & 2 \\ 1 & 4 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 10 & 7 \\ 3 & 9 & 6 \end{bmatrix}.$$
2. a. Both matrices must be square with the same size.
   b. The number of columns of *A* must be equal to the number of rows of *C*.

EXERCISES 2.5, page 120

1. $(2 \times 3)(3 \times 5)$ so AB has order 2×5.
   $\uparrow\ \uparrow$
   $=$
   $(3 \times 5)(2 \times 3)$ so BA is not defined.
   $\uparrow\ \uparrow$
   $\neq$

2. $(3 \times 4)\,(4 \times 3)$ so AB has order 3×3.

$$\underset{=}{\uparrow\ \uparrow}$$

$(4 \times 3)\,(3 \times 4)$ so BA has order 4×4.

$$\underset{=}{\uparrow\ \uparrow}$$

3. $(1 \times 7)\,(7 \times 1)$ so AB has order 1×1.

$$\underset{=}{\uparrow\ \uparrow}$$

$(7 \times 1)\,(1 \times 7)$ so BA has order 7×7.

$$\underset{=}{\uparrow\ \uparrow}$$

4. $(4 \times 4)\,(4 \times 4)$ so AB has order 4×4.

$$\underset{=}{\uparrow\ \uparrow}$$

$(4 \times 4)\,(4 \times 4)$ so BA has order 4×4.

$$\underset{=}{\uparrow\ \uparrow}$$

5. If AB and BA are defined then $n = s$ and $m = t$.

6. A must be a square matrix. Then the order of A is $n \times n$ so that the order of A^2 is also $n \times n$.

7. $$\begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

8. $$\begin{bmatrix} -1 & 3 \\ 5 & 0 \end{bmatrix}\begin{bmatrix} 7 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 35 \end{bmatrix}$$

9. $$\begin{bmatrix} 3 & 1 & 2 \\ -1 & 2 & 4 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 9 \\ -10 \end{bmatrix}$$

10. $$\begin{bmatrix} 3 & 2 & -1 \\ 4 & -1 & 0 \\ -5 & 2 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 14 \\ -19 \end{bmatrix}$$

11. $$\begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 9 & 13 \end{bmatrix}$$

12. $$\begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 3 & 0 \\ 3 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 10 & 3 & 6 \\ 5 & -3 & 4 \end{bmatrix}$$

13. $\begin{bmatrix} 2 & 1 & 2 \\ 3 & 2 & 4 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 4 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 9 \\ 5 & 16 \end{bmatrix}$

14. $\begin{bmatrix} -1 & 2 \\ 4 & 3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 2 \\ 3 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 3 & 6 \\ 17 & 10 & 20 \\ 3 & 2 & 4 \end{bmatrix}$

15. $\begin{bmatrix} 0.1 & 0.9 \\ 0.2 & 0.8 \end{bmatrix}\begin{bmatrix} 1.2 & 0.4 \\ 0.5 & 2.1 \end{bmatrix} = \begin{bmatrix} 0.1(1.2)+0.9(0.5) & 0.1(0.4)+0.9(2.1) \\ 0.2(1.2)+0.8(0.5) & 0.2(0.4)+0.8(2.1) \end{bmatrix} = \begin{bmatrix} 0.57 & 1.93 \\ 0.64 & 1.76 \end{bmatrix}$

16. $\begin{bmatrix} 1.2 & 0.3 \\ 0.4 & 0.5 \end{bmatrix}\begin{bmatrix} 0.2 & 0.6 \\ 0.4 & -0.5 \end{bmatrix} = \begin{bmatrix} 0.36 & 0.57 \\ 0.28 & -0.01 \end{bmatrix}.$

17. $\begin{bmatrix} 6 & -3 & 0 \\ -2 & 1 & -8 \\ 4 & -4 & 9 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 6 & -3 & 0 \\ -2 & 1 & -8 \\ 4 & -4 & 9 \end{bmatrix}.$

18. $\begin{bmatrix} 2 & 4 \\ -1 & -5 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} 2 & -2 & 4 \\ 1 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 8 & 8 & 4 \\ -7 & -13 & 1 \\ 5 & -9 & 13 \end{bmatrix}.$

19. $\begin{bmatrix} 3 & 0 & -2 & 1 \\ 1 & 2 & 0 & -1 \end{bmatrix}\begin{bmatrix} 2 & 1 & -1 \\ -1 & 2 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 1 & -3 \\ 1 & 7 & -3 \end{bmatrix}.$

20. $\begin{bmatrix} 2 & 1 & -3 & 0 \\ 4 & -2 & -1 & 1 \\ -1 & 2 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 4 \\ 3 & -3 \\ 0 & -5 \end{bmatrix} = \begin{bmatrix} -4 & 11 \\ 3 & -14 \\ 0 & 4 \end{bmatrix}.$

21. $4\begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & 1 \\ 3 & 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 1 \\ 1 & 4 & 0 \\ 0 & 1 & -2 \end{bmatrix} = \begin{bmatrix} -4 & -20 & 4 \\ 4 & 12 & 0 \\ 12 & 32 & 20 \end{bmatrix}$

22. $$3\begin{bmatrix}2 & -1 & 0\\ 2 & 1 & 2\\ 1 & 0 & -1\end{bmatrix}\begin{bmatrix}2 & 3 & 1\\ 3 & -3 & 0\\ 0 & 1 & -1\end{bmatrix}=\begin{bmatrix}3 & 27 & 6\\ 21 & 15 & 0\\ 6 & 6 & 6\end{bmatrix}.$$

23. $$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}4 & -3 & 2\\ 7 & 1 & -5\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}4 & -3 & 2\\ 7 & 1 & -5\end{bmatrix}=\begin{bmatrix}4 & -3 & 2\\ 7 & 1 & -5\end{bmatrix}.$$

24. $$2\begin{bmatrix}3 & 2 & -1\\ 0 & 1 & 3\\ 2 & 0 & 3\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 0\\ 0 & -1 & -2\\ 1 & 3 & 1\end{bmatrix}=2\begin{bmatrix}3 & 2 & -1\\ 0 & 1 & 3\\ 2 & 0 & 3\end{bmatrix}\begin{bmatrix}1 & 2 & 0\\ 0 & -1 & -2\\ 1 & 3 & 1\end{bmatrix}$$
$$=2\begin{bmatrix}2 & 1 & -5\\ 3 & 8 & 1\\ 5 & 13 & 3\end{bmatrix}=\begin{bmatrix}4 & 2 & -10\\ 6 & 16 & 2\\ 10 & 26 & 6\end{bmatrix}.$$

25. To verify the associative law for matrix multiplication, we will show that $(AB)C = A(BC)$.

$$AB=\begin{bmatrix}1 & 0 & -2\\ 1 & -3 & 2\\ -2 & 1 & 1\end{bmatrix}\begin{bmatrix}3 & 1 & 0\\ 2 & 2 & 0\\ 1 & -3 & -1\end{bmatrix}=\begin{bmatrix}1 & 7 & 2\\ -1 & -11 & -2\\ -3 & -3 & -1\end{bmatrix}$$
$$(AB)C=\begin{bmatrix}1 & 7 & 2\\ -1 & -11 & -2\\ -3 & -3 & -1\end{bmatrix}\begin{bmatrix}2 & -1 & 0\\ 1 & -1 & 2\\ 3 & -2 & 1\end{bmatrix}=\begin{bmatrix}15 & -12 & 16\\ -19 & 16 & -24\\ -12 & 8 & -7\end{bmatrix}$$
$$BC=\begin{bmatrix}3 & 1 & 0\\ 2 & 2 & 0\\ 1 & -3 & -1\end{bmatrix}\begin{bmatrix}2 & -1 & 0\\ 1 & -1 & 2\\ 3 & -2 & 1\end{bmatrix}=\begin{bmatrix}7 & -4 & 2\\ 6 & -4 & 4\\ -4 & 4 & -7\end{bmatrix}$$
$$A(BC)=\begin{bmatrix}1 & 0 & -2\\ 1 & -3 & 2\\ -2 & 1 & 1\end{bmatrix}\begin{bmatrix}7 & -4 & 2\\ 6 & -4 & 4\\ -4 & 4 & -7\end{bmatrix}=\begin{bmatrix}15 & -12 & 16\\ -19 & 16 & -24\\ -12 & 8 & -7\end{bmatrix}.$$

26. To verify the distributive law for matrix multiplication , we will show that $A(B + C) = AB + AC$.

$$A(B+C)=\begin{bmatrix}1&0&-2\\1&-3&2\\-2&1&1\end{bmatrix}\begin{bmatrix}5&0&0\\3&1&2\\4&-5&0\end{bmatrix}=\begin{bmatrix}-3&10&0\\4&-13&-6\\-3&-4&2\end{bmatrix}$$

$$AB+AC=\begin{bmatrix}1&7&2\\-1&-11&-2\\-3&-3&-1\end{bmatrix}+\begin{bmatrix}-4&3&-2\\5&-2&-4\\0&-1&3\end{bmatrix}=\begin{bmatrix}-3&10&0\\4&-13&-6\\-3&-4&2\end{bmatrix}$$

27. $$AB=\begin{bmatrix}1&2\\3&4\end{bmatrix}\begin{bmatrix}2&1\\4&3\end{bmatrix}=\begin{bmatrix}10&7\\22&15\end{bmatrix}$$

$$BA=\begin{bmatrix}2&1\\4&3\end{bmatrix}\begin{bmatrix}1&2\\3&4\end{bmatrix}=\begin{bmatrix}5&8\\13&20\end{bmatrix}$$

Therefore, $AB \neq BA$ and matrix multiplication is not commutative.

28. a. $$AB=\begin{bmatrix}0&3&0\\1&0&1\\0&2&0\end{bmatrix}\begin{bmatrix}2&4&5\\3&-1&-6\\4&3&4\end{bmatrix}=\begin{bmatrix}9&-3&-18\\6&7&9\\6&-2&-12\end{bmatrix}$$

b. $$AC=\begin{bmatrix}0&3&0\\1&0&1\\0&2&0\end{bmatrix}\begin{bmatrix}4&5&6\\3&-1&-6\\2&2&3\end{bmatrix}=\begin{bmatrix}9&-3&-18\\6&7&9\\6&-2&-12\end{bmatrix}.$$

c. From the results of (a) and (b), we see that $AB = AC$ does not imply that $B = C$.

29. $$AB=\begin{bmatrix}3&0\\8&0\end{bmatrix}\begin{bmatrix}0&0\\4&5\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}$$

$AB = 0$, but neither A nor B is the zero matrix. Therefore, $AB = 0$, does not imply that A or B is the zero matrix.

30. $$A^2=\begin{bmatrix}2&2\\-2&-2\end{bmatrix}\begin{bmatrix}2&2\\-2&-2\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}=0$$

Thus, for matrices, $A^2 = 0$ does not imply $A = 0$. If $a^2 = 0$, where a is a real number, then $a = 0$.

31. $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} a-b & 3b \\ c-d & 3d \end{bmatrix} = \begin{bmatrix} -1 & -3 \\ 3 & 6 \end{bmatrix}$

Then $3b = -3$, and $b = -1$

$3d = 6$, and $d = 2$

$a - b = -1$, and $a = b - 1 = -2$.

$c - d = 3$, and $c = d + 3 = 5$

Therefore, $A = \begin{bmatrix} -2 & -1 \\ 5 & 2 \end{bmatrix}$.

32. a. $(A+B)^2 = \begin{bmatrix} 7 & -1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 7 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 47 & -10 \\ 20 & 7 \end{bmatrix}$.

b. $A^2 + 2AB + B^2 = \begin{bmatrix} 9 & 5 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} 28 & -10 \\ 8 & 4 \end{bmatrix} + \begin{bmatrix} 12 & -10 \\ 10 & -3 \end{bmatrix} = \begin{bmatrix} 49 & -15 \\ 18 & 5 \end{bmatrix}$.

c. From the results of (a) and (b), we see that, in general, $(A+B)^2 \neq A^2 + 2AB + B^2$.

33. a.

$$A^T = \begin{bmatrix} 2 & 5 \\ 4 & -6 \end{bmatrix} \text{ and } (A^T)^T = \begin{bmatrix} 2 & 4 \\ 5 & -6 \end{bmatrix} = A$$

b.

$$(A+B)^T = \begin{bmatrix} 6 & 12 \\ -2 & -3 \end{bmatrix}^T = \begin{bmatrix} 6 & -2 \\ 12 & -3 \end{bmatrix}$$

$$A^T + B^T = \begin{bmatrix} 2 & 5 \\ 4 & -6 \end{bmatrix} + \begin{bmatrix} 4 & -7 \\ 8 & 3 \end{bmatrix} = \begin{bmatrix} 6 & -2 \\ 12 & -3 \end{bmatrix}$$

c.

$$AB = \begin{bmatrix} 2 & 4 \\ 5 & -6 \end{bmatrix}\begin{bmatrix} 4 & 8 \\ -7 & 3 \end{bmatrix} = \begin{bmatrix} -20 & 28 \\ 62 & 22 \end{bmatrix}, \quad \text{so} \quad (AB)^T = \begin{bmatrix} -20 & 62 \\ 28 & 22 \end{bmatrix}.$$

$$B^T A^T = \begin{bmatrix} 4 & -7 \\ 8 & 3 \end{bmatrix}\begin{bmatrix} 2 & 5 \\ 4 & -6 \end{bmatrix} = \begin{bmatrix} -20 & 62 \\ 28 & 22 \end{bmatrix} = (AB)^T$$

34. a. $A^T = \begin{bmatrix} 1 & -2 \\ 3 & -1 \end{bmatrix}$ and $(A^T)^T = \begin{bmatrix} 1 & 3 \\ -2 & -1 \end{bmatrix} = A.$

b. $(A+B)^T = \begin{bmatrix} 4 & -1 \\ 0 & -3 \end{bmatrix}^T = \begin{bmatrix} 4 & 0 \\ -1 & -3 \end{bmatrix}.$

$$A^T + B^T = \begin{bmatrix} 1 & -2 \\ 3 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 2 \\ -4 & -2 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ -1 & -3 \end{bmatrix} = (A+B)^T.$$

c. $AB = \begin{bmatrix} 1 & 3 \\ -2 & -1 \end{bmatrix}\begin{bmatrix} 3 & -4 \\ 2 & -2 \end{bmatrix} = \begin{bmatrix} 9 & -10 \\ -8 & 10 \end{bmatrix}$

so $(AB)^T = \begin{bmatrix} 9 & -8 \\ -10 & 10 \end{bmatrix}.$

$$B^T A^T = \begin{bmatrix} 3 & 2 \\ -4 & -2 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 9 & -8 \\ -10 & 10 \end{bmatrix} = (AB)^T.$$

35. The given system of linear equations can be represented by the matrix equation $AX = B$, where

$$A = \begin{bmatrix} 2 & -3 \\ 3 & -4 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \text{ and } B = \begin{bmatrix} 7 \\ 8 \end{bmatrix}.$$

36. The given system of linear equations can be represented by the matrix equation $AX = B$, where

$$A = \begin{bmatrix} 2 & 0 \\ 3 & -2 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \text{ and } B = \begin{bmatrix} 7 \\ 12 \end{bmatrix}.$$

37. The given system of linear equations can be represented by the matrix equation $AX = B$, where

$$A = \begin{bmatrix} 2 & -3 & 4 \\ 0 & 2 & -3 \\ 1 & -1 & 2 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 6 \\ 7 \\ 4 \end{bmatrix}.$$

38. The given system of linear equations can be represented by the matrix equation $AX = B$, where

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 3 & 4 & -2 \\ 2 & -3 & 7 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} -1 \\ 1 \\ 6 \end{bmatrix}.$$

39. The given system of linear equations can be represented by the matrix equation $AX = B$, where

$$A = \begin{bmatrix} -1 & 1 & 1 \\ 2 & -1 & -1 \\ -3 & 2 & 4 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix}.$$

40. The given system of linear equations can be represented by the matrix equation $AX = B$, where

$$A = \begin{bmatrix} 3 & -5 & 4 \\ 4 & 2 & -3 \\ -1 & 0 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad B = \begin{bmatrix} 10 \\ -12 \\ -2 \end{bmatrix}.$$

41. a. $$AB = \begin{bmatrix} 200 & 300 & 100 & 200 \\ 100 & 200 & 400 & 0 \end{bmatrix} \begin{bmatrix} 54 \\ 48 \\ 98 \\ 82 \end{bmatrix} = \begin{bmatrix} 51{,}400 \\ 54{,}200 \end{bmatrix}$$

b. The first entry shows that William's total stockholdings are \$51,400, while the second entry shows that Michael's stockholdings are \$54,200.

42. a.

$$A = \begin{matrix} \text{baht} & \text{rupiahs} & \text{ringgits} & \text{S. dollars} \\ [1200 & 80000 & 42 & 36] \end{matrix}$$

b. $$B = \begin{bmatrix} 0.03 \\ 0.00011 \\ 0.294 \\ 0.656 \end{bmatrix} \begin{matrix} \text{baht} \\ \text{rupiahs} \\ \text{ringgits} \\ \text{S. dollars} \end{matrix}$$

c. He will have

$$AB = \begin{bmatrix} 1200 & 80000 & 42 & 36 \end{bmatrix} \begin{bmatrix} 0.03 \\ 0.00011 \\ 0.294 \\ 0.656 \end{bmatrix} = 80.764, \text{ or } \$80.76.$$

43. a.

$$\begin{matrix} \text{N} & \text{S} & \text{D} & \text{R} \\ \text{Krones} & \text{Krones} & \text{Krones} & \text{Rubles} \end{matrix}$$

$$A = \begin{bmatrix} 82 & 68 & 62 & 1200 \\ 64 & 74 & 44 & 1600 \end{bmatrix} \begin{matrix} \text{Kaitlin} \\ \text{Emma} \end{matrix}$$

$$B = \begin{bmatrix} 0.1651 \\ 0.1462 \\ 0.1811 \\ 0.0387 \end{bmatrix} \begin{matrix} N \\ S \\ D \\ R \end{matrix}$$

$$AB = \begin{bmatrix} 82 & 68 & 62 & 1200 \\ 64 & 74 & 44 & 1600 \end{bmatrix} \begin{bmatrix} 0.1651 \\ 0.1462 \\ 0.1811 \\ 0.0387 \end{bmatrix} \begin{matrix} N \\ S \\ D \\ R \end{matrix}$$

$$= \begin{bmatrix} 81.148 \\ 91.2736 \end{bmatrix} \begin{matrix} \text{Kaitlin} \\ \text{Emma} \end{matrix}$$

So Kaitlin will have \$81.15 and Emma will have \$91.27.

44. The column vector that represents the profit for each type of house is

$$B = \begin{bmatrix} 20{,}000 \\ 22{,}000 \\ 25{,}000 \\ 30{,}000 \end{bmatrix}.$$

The column vector that gives the total profit for Bond Brothers is

$$AB = \begin{bmatrix} 60 & 80 & 120 & 40 \\ 20 & 30 & 60 & 10 \\ 10 & 15 & 30 & 5 \end{bmatrix} \begin{bmatrix} 20{,}000 \\ 22{,}000 \\ 25{,}000 \\ 30{,}000 \end{bmatrix}$$

$$= \begin{bmatrix} 7{,}160{,}000 \\ 2{,}860{,}000 \\ 1{,}430{,}000 \end{bmatrix}.$$

Therefore, Bond Brothers expects to make \$7,160,000 in New York, \$2,860,000 in Connecticut, and \$1,430,000 in Massachusetts, and the total profit is \$11,450,000.

45.

$$B = \begin{bmatrix} 0.78 \\ 0.88 \\ 0.80 \end{bmatrix}$$

$$AB = \begin{bmatrix} 18.2 & 28.2 & 40.5 \\ 19.6 & 28.6 & 42.6 \\ 20.8 & 30.4 & 46.4 \end{bmatrix} \begin{bmatrix} 0.78 \\ 0.88 \\ 0.80 \end{bmatrix} = \begin{bmatrix} 71.412 \\ 74.536 \\ 80.096 \end{bmatrix}$$

So in 2006, \$71.412 million was put towards program cost, in 2007, \$74.536 million was put towards program cost, and in 2008, \$80.096 million was put towards program cost.

46. The column vector that represents the admission prices is

$$B = \begin{bmatrix} 4 \\ 6 \\ 8 \end{bmatrix}.$$

The column vector that gives the gross receipts for each theater is

$$AB = \begin{bmatrix} 225 & 110 & 50 \\ 75 & 180 & 225 \\ 280 & 85 & 110 \\ 0 & 250 & 225 \end{bmatrix} \begin{bmatrix} 4 \\ 6 \\ 8 \end{bmatrix} = \begin{bmatrix} 1960 \\ 3180 \\ 2510 \\ 3300 \end{bmatrix}.$$

The total revenue collected is given by

1960 + 3180 + 2510 + 3300, or $10950.

47.
$$BA = \begin{bmatrix} 30{,}000 & 40{,}000 & 20{,}000 \end{bmatrix} \begin{matrix} \begin{matrix} D & R & I \end{matrix} \\ \begin{bmatrix} 0.50 & 0.30 & 0.20 \\ 0.45 & 0.40 & 0.15 \\ 0.40 & 0.50 & 0.10 \end{bmatrix} \end{matrix}$$

$$= \begin{matrix} \begin{matrix} D & R & I \end{matrix} \\ \begin{bmatrix} 41{,}000 & 35{,}000 & 14{,}000 \end{bmatrix} \end{matrix}$$

48. $\begin{bmatrix} 4000 & 3000 & 3000 \\ 2000 & 5000 & 3000 \\ 2000 & 3000 & 5000 \end{bmatrix} \begin{bmatrix} 0.18 \\ 0.24 \\ 0.12 \end{bmatrix} = \begin{bmatrix} 1800 & 1920 & 1680 \end{bmatrix}$; Maria; Steven

49.
$$AB = \begin{bmatrix} 2700 & 3000 \\ 800 & 700 \\ 500 & 300 \end{bmatrix} \begin{bmatrix} 0.25 & 0.20 & 0.30 & 0.25 \\ 0.30 & 0.35 & 0.25 & 0.10 \end{bmatrix} = \begin{bmatrix} 1575 & 1590 & 1560 & 975 \\ 410 & 405 & 415 & 270 \\ 215 & 205 & 225 & 155 \end{bmatrix}$$

50. a.
$$PA = \begin{bmatrix} 700 & 1000 & 800 \\ 500 & 800 & 600 \end{bmatrix} \begin{bmatrix} 1.3 & 20 & 12 \\ 1.5 & 30 & 5 \\ 2.5 & 25 & 15 \end{bmatrix} = \begin{bmatrix} 4410 & 64000 & 25400 \\ 3350 & 49000 & 19000 \end{bmatrix}$$

b. $PAC = \begin{bmatrix} 4410 & 64000 & 25400 \\ 3350 & 49000 & 19000 \end{bmatrix} \begin{bmatrix} 4.50 \\ 0.10 \\ 0.25 \end{bmatrix} = \begin{bmatrix} 32{,}595 \\ 24{,}725 \end{bmatrix}$

c. The total cost of materials incurred by Ace Novelty in filling the order is 32,595 + 24,725 = 57,320, or $57,320.

51. $AC = \begin{bmatrix} 80 & 60 & 40 \end{bmatrix} \begin{bmatrix} 0.34 \\ 0.42 \\ 0.48 \end{bmatrix} = 71.6 \qquad BD = \begin{bmatrix} 300 & 150 & 250 \end{bmatrix} \begin{bmatrix} 0.24 \\ 0.31 \\ 0.35 \end{bmatrix} = 206$

$AC + BD = \begin{bmatrix} 277.60 \end{bmatrix}$, or $277.60. It represents Cindy's long distance bill for phone calls to those 3 cities.

52. a. $$AC = \begin{bmatrix} 320 & 280 & 460 & 280 \\ 480 & 360 & 580 & 0 \\ 540 & 420 & 200 & 880 \end{bmatrix} \begin{bmatrix} 120 \\ 180 \\ 260 \\ 500 \end{bmatrix} = \begin{bmatrix} 348{,}400 \\ 273{,}200 \\ 632{,}400 \end{bmatrix}.$$

The entries give the total production costs at locations I, II, and III for the month of May as $348,400, $273,200, and $632,400, respectively.

b. $$AD = \begin{bmatrix} 320 & 280 & 460 & 280 \\ 480 & 360 & 580 & 0 \\ 540 & 420 & 200 & 880 \end{bmatrix} \begin{bmatrix} 160 \\ 250 \\ 350 \\ 700 \end{bmatrix} = \begin{bmatrix} 478{,}200 \\ 369{,}800 \\ 877{,}400 \end{bmatrix}$$

The total revenue realized at locations I, II, and III for the month of May are $478,200, $369,800, and $877,400, respectively.

c. $$BC = \begin{bmatrix} 210 & 180 & 330 & 180 \\ 400 & 300 & 450 & 40 \\ 420 & 280 & 180 & 740 \end{bmatrix} \begin{bmatrix} 120 \\ 180 \\ 260 \\ 500 \end{bmatrix} = \begin{bmatrix} 233{,}400 \\ 239{,}000 \\ 517{,}600 \end{bmatrix}.$$

The total production costs at locations I, II, and III for the month of June are $233,400, $239,000, and $517,600, respectively.

d. $$BD = \begin{bmatrix} 210 & 180 & 330 & 180 \\ 400 & 300 & 450 & 40 \\ 420 & 280 & 180 & 740 \end{bmatrix} \begin{bmatrix} 160 \\ 250 \\ 350 \\ 700 \end{bmatrix} = \begin{bmatrix} 320{,}100 \\ 324{,}500 \\ 718{,}200 \end{bmatrix}.$$

The total revenue realized at locations I, II, and III for the month of June are $320,100, $324,500, and $718,200, respectively.

e. $(A+B)C = \begin{bmatrix} 530 & 460 & 790 & 460 \\ 880 & 660 & 1030 & 40 \\ 960 & 700 & 380 & 1620 \end{bmatrix} \begin{bmatrix} 120 \\ 180 \\ 260 \\ 500 \end{bmatrix} = \begin{bmatrix} 581{,}800 \\ 512{,}200 \\ 1{,}150{,}000 \end{bmatrix}.$

The total production costs in May and June at Locations I, II, and III are \$581,800, \$512,200, and \$1,150,000, respectively.

f. $(A+B)D = \begin{bmatrix} 530 & 460 & 790 & 460 \\ 880 & 660 & 1030 & 40 \\ 960 & 700 & 380 & 1620 \end{bmatrix} \begin{bmatrix} 160 \\ 250 \\ 350 \\ 700 \end{bmatrix} = \begin{bmatrix} 798{,}300 \\ 694{,}300 \\ 1{,}595{,}600 \end{bmatrix}.$

The total revenue realized in May and June in Locations I, II, and III are \$798,300, \$694,300, and \$1,595,600, respectively.

g. $A(D-C) = \begin{bmatrix} 320 & 280 & 460 & 280 \\ 480 & 360 & 580 & 0 \\ 540 & 420 & 200 & 880 \end{bmatrix} \begin{bmatrix} 40 \\ 70 \\ 90 \\ 200 \end{bmatrix} = \begin{bmatrix} 129{,}800 \\ 96{,}600 \\ 245{,}000 \end{bmatrix}.$

The profits in Locations I, II, and III in May are \$129,800, \$96,600, and \$245,000, respectively.

h. $B(D-C) = \begin{bmatrix} 210 & 180 & 330 & 180 \\ 400 & 300 & 450 & 40 \\ 420 & 280 & 180 & 740 \end{bmatrix} \begin{bmatrix} 40 \\ 70 \\ 90 \\ 200 \end{bmatrix} = \begin{bmatrix} 86{,}700 \\ 85{,}500 \\ 200{,}600 \end{bmatrix}$

The profits in Locations I, II, and III in June are \$86,700, \$85,500, and \$200,600, respectively.

i. $(A+B)(D-C) = \begin{bmatrix} 530 & 460 & 790 & 460 \\ 880 & 660 & 1030 & 40 \\ 960 & 700 & 380 & 1620 \end{bmatrix} \begin{bmatrix} 40 \\ 70 \\ 90 \\ 200 \end{bmatrix} = \begin{bmatrix} 216{,}500 \\ 182{,}100 \\ 445{,}600 \end{bmatrix}.$

The profits in Locations I, II, and III in May and June are \$216,500, \$182,100, \$445,600, respectively.

53. a. $MA^T = \begin{bmatrix} 400 & 1200 & 800 \\ 110 & 570 & 340 \\ 90 & 30 & 60 \end{bmatrix} \begin{bmatrix} 7 \\ 1 \\ 6 \end{bmatrix} = \begin{bmatrix} 8800 \\ 3380 \\ 1020 \end{bmatrix}.$

The amounts of vitamin A, vitamin C, and calcium taken by a girl in the first meal are 8800, 3380, and 1020 units respectively.

b.

$$MB^T = \begin{bmatrix} 400 & 1200 & 800 \\ 110 & 570 & 340 \\ 90 & 30 & 60 \end{bmatrix} \begin{bmatrix} 9 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 8800 \\ 3380 \\ 1020 \end{bmatrix}$$

The amounts of vitamin A, vitamin C, and calcium taken by a girl in the second meal are 8,800, 3380, and 1020 units, respectively.

c. $$M(A+B)^T = \begin{bmatrix} 400 & 1200 & 800 \\ 110 & 570 & 340 \\ 90 & 30 & 60 \end{bmatrix} \begin{bmatrix} 16 \\ 4 \\ 8 \end{bmatrix} = \begin{bmatrix} 17,600 \\ 6,760 \\ 2,040 \end{bmatrix}.$$

The amounts of vitamin A, vitamin C, and calcium taken by a girl in the two meals are 17,600, 6,760, and 2,040 units respectively.

54. a. To find the sales of each product in Branch A for the current year, we multiply the corresponding sales for the previous year by the factor 1.1. Similarly, we use the factor 1.15 for the sales in Branch B. These computations are carried out by computing

$$AB = \begin{bmatrix} 1.1 & 0 \\ 0 & 1.15 \end{bmatrix} \begin{bmatrix} 5 & 2 & 8 & 10 \\ 3 & 4 & 6 & 8 \end{bmatrix} = \begin{bmatrix} 5.5 & 2.2 & 8.8 & 11 \\ 3.45 & 4.6 & 6.9 & 9.2 \end{bmatrix}$$

b. $$\begin{bmatrix} 1.01r_1 & 0 & 0 & \cdots & 0 \\ 0 & 1.01r_2 & 0 & \cdots & 0 \\ 0 & 0 & 1.01r_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1.01r_m \end{bmatrix}$$

55. False. Let A be a matrix of order 2×3 and let B be a matrix of order 3×2. Then AB and BA are both defined. But, evidently, neither A nor B is a square matrix.

56. True. Let us show that $(cA)B = cAB$. The matrix cA is obtained from A by multiplying each element in A by c. Next, a typical element in $(cA)B$ is obtained by "multiplying" a row in cA into a column in B. Now, the element occupying the same

position in AB is obtained by multiplying the same row in A into the same column in B as in the previous step. Since cAB produces the matrix with each element in AB multiplied by c, the equality $(cA)B = cAB$ results. The result $A(cB) = cAB$ follows by a similar argument.

57. True. In order for the sum $B + C$ to be defined, B and C must have the same size, and in order for the product of A and $(B + C)$ to be defined, the number of columns of A must be equal to the number of rows of $(B + C)$.

58. True. From the schematic relating to the sizes of the matrices, we see

$$\underset{A}{(2\times 4)}\ \underset{B}{(4\times n)}\ \underset{A}{(2\times 4)}$$

we see that n must be 2. So B has size 4×2.

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1. $\begin{bmatrix} 18.66 & 15.2 & -12 \\ 24.48 & 41.88 & 89.82 \\ 15.39 & 7.16 & -1.25 \end{bmatrix}$

2. $\begin{bmatrix} -11.38 & 4.87 & 51.85 & 49.02 \\ -14.97 & -6.81 & 19.92 & 7.68 \\ 9.22 & 6.95 & 5.06 & 5.92 \\ 33.03 & 16.15 & -21.56 & -15.12 \end{bmatrix}$

3. $\begin{bmatrix} 20.09 & 20.61 & -1.3 \\ 44.42 & 71.6 & 64.89 \\ 20.97 & 7.17 & -60.65 \end{bmatrix}$

4. $\begin{bmatrix} 41.61 & 46.63 & 8.1 \\ 108.78 & 172.92 & 104.85 \\ 47.52 & 14.35 & -180.7 \end{bmatrix}$

5. $\begin{bmatrix} 32.89 & 13.63 & -57.17 \\ -12.85 & -8.37 & 256.92 \\ 13.48 & 14.29 & 181.64 \end{bmatrix}$

6. $\begin{bmatrix} 83.37 & 156.39 & 173.82 & 169.23 \\ -49.67 & 16.07 & 37.79 & 87.89 \\ 72.60 & 57.67 & 27.60 & 9.52 \\ 145.26 & 51.24 & -39.26 & -92.93 \end{bmatrix}$

7. $\begin{bmatrix} 128.59 & 123.08 & -32.50 \\ 246.73 & 403.12 & 481.52 \\ 125.06 & 47.01 & -264.81 \end{bmatrix}$

8. $\begin{bmatrix} 379.02 & 249.37 & -540.47 & -500.46 \\ 157.15 & 154.34 & -269.76 & 53.12 \\ -6.78 & 1.20 & -29.96 & -83.85 \\ -270.29 & -174.56 & 295.49 & 67.10 \end{bmatrix}$

9. $\begin{bmatrix} 87 & 68 & 110 & 82 \\ 119 & 176 & 221 & 143 \\ 51 & 128 & 142 & 94 \\ 28 & 174 & 174 & 112 \end{bmatrix}$ $\begin{bmatrix} 113 & 117 & 72 & 101 & 90 \\ 72 & 85 & 36 & 72 & 76 \\ 81 & 69 & 76 & 87 & 30 \\ 133 & 157 & 56 & 121 & 146 \\ 154 & 157 & 94 & 127 & 122 \end{bmatrix}$

10. $\begin{bmatrix} 107.81 & -58.81 & 158.45 & 98.36 & 175.89 \\ 135.54 & -20.23 & 143.96 & 44.58 & 121.12 \\ 88.31 & 125.79 & 147.64 & 199.69 & 126.11 \\ 184.91 & 27.29 & 227.45 & 142.01 & 224.24 \\ 211.68 & -102.14 & 201.81 & -39.29 & 228.33 \end{bmatrix}$

$\begin{bmatrix} 243.56 & 196.98 & 153.97 & 20.36 & 311.72 \\ 148.25 & 101.54 & 71.07 & 152.12 & 113.96 \\ 213.16 & 203.26 & 147.46 & 33.67 & 268.89 \\ 212.75 & 185.59 & 155.87 & 11.05 & 278.86 \\ 65.57 & 38.57 & -46.63 & 21.51 & 101.95 \end{bmatrix}$; No

11. $\begin{bmatrix} 170 & 18.1 & 133.1 & -106.3 & 341.3 \\ 349 & 226.5 & 324.1 & 164 & 506.4 \\ 245.2 & 157.7 & 231.5 & 125.5 & 312.9 \\ 310 & 245.2 & 291 & 274.3 & 354.2 \end{bmatrix}$

12. a. $\begin{bmatrix} 113.6 & 103.3 & 60.2 & -118.3 & 254.6 \\ 217.2 & 202.2 & 134.7 & -1 & 319.6 \\ 144.4 & 130.9 & 96.4 & 13.3 & 188.8 \\ 173 & 165.8 & 93.9 & 93 & 181.9 \end{bmatrix}$

b. $\begin{bmatrix} 56.4 & -85.2 & 72.9 & 12 & 86.7 \\ 131.8 & 24.3 & 189.4 & 165 & 186.8 \\ 100.8 & 26.8 & 135.1 & 112.2 & 124.1 \\ 137 & 79.4 & 197.1 & 181.3 & 172.3 \end{bmatrix}$

c. $\begin{bmatrix} 170 & 18.1 & 133.1 & -106.3 & 341.3 \\ 349 & 226.5 & 324.1 & 164 & 506.4 \\ 245.2 & 157.7 & 231.5 & 125.5 & 312.9 \\ 310 & 245.2 & 291 & 274.3 & 354.2 \end{bmatrix}$ d. Yes