

3.  $(-\frac{17}{7} + \frac{6}{7}t, 3 - t, -\frac{18}{7} + \frac{1}{7}t, t)$     4.  $(2t, 1 - t, 1 + t, t)$     5. No solution

6.  $(2.5810 - 0.0406t, 3.2462 - 3.8226t, 1.6619 - 2.1548t, 0.2942 + 0.3531t, t)$

## 2.4 CONCEPT QUESTIONS, page 107

- A matrix is an ordered rectangular array of real numbers.
  - A matrix has size (or dimension)  $m \times n$  if it has  $m$  rows and  $n$  columns.
  - A row matrix is one of size  $1 \times n$ .
  - A column matrix is one of size  $m \times 1$ .
  - A square matrix is one of size  $n \times n$ .
- Two matrices are equal if they have the same size and the corresponding entries are equal. For example,

$$\begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & (4-1) \\ (1-2) & (1+1) \end{bmatrix}$$

3.

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & -2 & 1 \\ 4 & 1 & 3 \end{bmatrix}$$

The entries satisfy  $a_{ij} = a_{ji}$ , that is  $A$  is symmetric with respect to the main diagonal.

## EXERCISES 2.4, page 107

- The size of  $A$  is  $4 \times 4$ ; the size of  $B$  is  $4 \times 3$ ; the size of  $C$  is  $1 \times 5$ , and the size of  $D$  is  $4 \times 1$ .
- $a_{14} = -4$ ;  $a_{21} = -11$ ;  $a_{31} = 6$ ;  $a_{43} = 5$ .
- These are entries of the matrix  $B$ . The entry  $b_{13}$  refers to the entry in the first row and third column and is equal to 2. Similarly,  $b_{31} = 3$ , and  $b_{43} = 8$ .
- The row matrix is the matrix  $C$ . The transpose of the matrix  $C$  is

$$C^T = \begin{bmatrix} 1 \\ 0 \\ 3 \\ 4 \\ 5 \end{bmatrix}.$$

5. The column matrix is the matrix  $D$ . The transpose of the matrix  $D$  is

$$D^T = [1 \ 3 \ -2 \ 0].$$

6. The square matrix is  $A$ . The transpose is

$$A^T = \begin{bmatrix} 2 & -11 & 6 & 5 \\ -3 & 2 & 0 & 1 \\ 9 & 6 & 2 & 5 \\ -4 & 7 & 9 & -8 \end{bmatrix}.$$

7.  $A$  is of size  $3 \times 2$ ;  $B$  is of size  $3 \times 2$ ;  $C$  and  $D$  are of size  $3 \times 3$ .

8.  $A$  is of size  $3 \times 2$  and  $C$  is of size  $3 \times 3$ ; therefore, their sum does not exist.

$$9. \quad A + B = \begin{bmatrix} -1 & 2 \\ 3 & -2 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 3 & 1 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ 6 & -1 \\ 2 & 2 \end{bmatrix}.$$

$$10. \quad 2A - 3B = \begin{bmatrix} -2 & 4 \\ 6 & -4 \\ 8 & 0 \end{bmatrix} - \begin{bmatrix} 6 & 12 \\ 9 & 3 \\ -6 & 6 \end{bmatrix} = \begin{bmatrix} -8 & -8 \\ -3 & -7 \\ 14 & -6 \end{bmatrix}.$$

$$11. \quad C - D = \begin{bmatrix} 3 & -1 & 0 \\ 2 & -2 & 3 \\ 4 & 6 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -2 & 4 \\ 3 & 6 & 2 \\ -2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -4 \\ -1 & -8 & 1 \\ 6 & 3 & 1 \end{bmatrix}.$$

$$12. \quad 4D - 2C = \begin{bmatrix} 8 & -8 & 16 \\ 12 & 24 & 8 \\ -8 & 12 & 4 \end{bmatrix} + \begin{bmatrix} -6 & 2 & 0 \\ -4 & 4 & -6 \\ -8 & -12 & -4 \end{bmatrix} = \begin{bmatrix} 2 & -6 & 16 \\ 8 & 28 & 2 \\ -16 & 0 & 0 \end{bmatrix}.$$

$$13. \begin{bmatrix} 6 & 3 & 8 \\ 4 & 5 & 6 \end{bmatrix} - \begin{bmatrix} 3 & -2 & -1 \\ 0 & -5 & -7 \end{bmatrix} = \begin{bmatrix} 3 & 5 & 9 \\ 4 & 10 & 13 \end{bmatrix}.$$

$$14. \begin{bmatrix} 2 & -3 & 4 & -1 \\ 3 & 1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 3 & -2 & -4 \\ 6 & 2 & 0 & -3 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 2 & -5 \\ 9 & 3 & 0 & -3 \end{bmatrix}.$$

$$15. \begin{bmatrix} 1 & 4 & -5 \\ 3 & -8 & 6 \end{bmatrix} + \begin{bmatrix} 4 & 0 & -2 \\ 3 & 6 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 8 & 9 \\ -11 & 2 & -5 \end{bmatrix} = \begin{bmatrix} 3 & -4 & -16 \\ 17 & -4 & 16 \end{bmatrix}.$$

$$16. 3 \begin{bmatrix} 1 & 1 & -3 \\ 3 & 2 & 3 \\ 7 & -1 & 6 \end{bmatrix} + 4 \begin{bmatrix} -2 & -1 & 8 \\ 4 & 2 & 2 \\ 3 & 6 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 3 & -9 \\ 9 & 6 & 9 \\ 21 & -3 & 18 \end{bmatrix} + \begin{bmatrix} -8 & -4 & 32 \\ 16 & 8 & 8 \\ 12 & 24 & 12 \end{bmatrix} \\ = \begin{bmatrix} -5 & -1 & 23 \\ 25 & 14 & 17 \\ 33 & 21 & 30 \end{bmatrix}.$$

$$17. \begin{bmatrix} 1.2 & 4.5 & -4.2 \\ 8.2 & 6.3 & -3.2 \end{bmatrix} - \begin{bmatrix} 3.1 & 1.5 & -3.6 \\ 2.2 & -3.3 & -4.4 \end{bmatrix} = \begin{bmatrix} -1.9 & 3.0 & -0.6 \\ 6.0 & 9.6 & 1.2 \end{bmatrix}.$$

$$18. \begin{bmatrix} 0.06 & 0.12 \\ 0.43 & 1.11 \\ 1.55 & -0.43 \end{bmatrix} - \begin{bmatrix} 0.77 & -0.75 \\ 0.22 & -0.65 \\ 1.09 & -0.57 \end{bmatrix} = \begin{bmatrix} -0.71 & 0.87 \\ 0.21 & 1.76 \\ 0.46 & 0.14 \end{bmatrix}.$$

$$19. \frac{1}{2} \begin{bmatrix} 1 & 0 & 0 & -4 \\ 3 & 0 & -1 & 6 \\ -2 & 1 & -4 & 2 \end{bmatrix} + \frac{4}{3} \begin{bmatrix} 3 & 0 & -1 & 4 \\ -2 & 1 & -6 & 2 \\ 8 & 2 & 0 & -2 \end{bmatrix} - \frac{1}{3} \begin{bmatrix} 3 & -9 & -1 & 0 \\ 6 & 2 & 0 & -6 \\ 0 & 1 & -3 & 1 \end{bmatrix} \\ = \begin{bmatrix} \frac{7}{2} & 3 & -1 & \frac{10}{3} \\ -\frac{19}{6} & \frac{2}{3} & -\frac{17}{2} & \frac{23}{3} \\ \frac{29}{3} & \frac{17}{6} & -1 & -2 \end{bmatrix}.$$

$$20. \quad 0.5 \begin{bmatrix} 1 & 3 & 5 \\ 5 & 2 & -1 \\ -2 & 0 & 1 \end{bmatrix} - 0.2 \begin{bmatrix} 2 & 3 & 4 \\ -1 & 1 & -4 \\ 3 & 5 & -5 \end{bmatrix} + 0.6 \begin{bmatrix} 3 & 4 & -1 \\ 4 & 5 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1.9 & 3.3 & 1.1 \\ 5.1 & 3.8 & 0.9 \\ -1 & -1 & 1.5 \end{bmatrix}.$$

$$21. \quad \begin{bmatrix} 2x-2 & 3 & 2 \\ 2 & 4 & y-2 \\ 2z & -3 & 2 \end{bmatrix} = \begin{bmatrix} 3 & u & 2 \\ 2 & 4 & 5 \\ 4 & -3 & 2 \end{bmatrix}.$$

Now, by the definition of equality of matrices,

$$u = 3$$

$$2x - 2 = 3 \text{ and } 2x = 5, \text{ or } x = 5/2,$$

$$y - 2 = 5, \text{ and } y = 7,$$

$$2z = 4, \text{ and } z = 2.$$

$$22. \quad \begin{bmatrix} x & -2 \\ 3 & y \end{bmatrix} + \begin{bmatrix} -2 & z \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 2u & 4 \end{bmatrix}; \quad \begin{bmatrix} x-2 & -2+z \\ 2 & y+2 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 2u & 4 \end{bmatrix}.$$

Now, by the definition of equality of matrices,

$$x - 2 = 4, \text{ so } x = 6$$

$$-2 + z = -2, \text{ so } z = 0$$

$$2 = 2u, \text{ so } u = 1$$

$$y + 2 = 4, \text{ so } y = 2.$$

$$23. \quad \begin{bmatrix} 1 & x \\ 2y & -3 \end{bmatrix} - 4 \begin{bmatrix} 2 & -2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 3z & 10 \\ 4 & -u \end{bmatrix}; \quad \begin{bmatrix} -7 & x+8 \\ 2y & -15 \end{bmatrix} = \begin{bmatrix} 3z & 10 \\ 4 & -u \end{bmatrix}.$$

Now, by the definition of equality of matrices,

$$-u = -15, \text{ so } u = 15$$

$$x + 8 = 10, \text{ so } x = 2$$

$$2y = 4, \text{ so } y = 2$$

$$3z = -7, \text{ so } z = -7/3.$$

$$24. \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ x & -1 \end{bmatrix} - 3 \begin{bmatrix} y-1 & 2 \\ 1 & 2 \\ 4 & 2z+1 \end{bmatrix} = 2 \begin{bmatrix} -4 & -u \\ 0 & -1 \\ 4 & 4 \end{bmatrix}; \begin{bmatrix} -3y+4 & -4 \\ 0 & -2 \\ x-12 & -6z-4 \end{bmatrix} = \begin{bmatrix} -8 & -2u \\ 0 & -2 \\ 8 & 8 \end{bmatrix}.$$

Now, by the definition of equality of matrices,

$$-2u = -4, \text{ so } u = 2$$

$$x - 12 = 8, \text{ so } x = 20$$

$$-3y + 4 = -8, \text{ so } -3y = -12, y = 4$$

$$-6z - 4 = 8, \text{ so } -6z = 12, \text{ and } z = -2.$$

25. To verify the Commutative Law for matrix addition, let us show that  $A + B = B + A$ .

$$\text{Now, } A + B = \begin{bmatrix} 2 & -4 & 3 \\ 4 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 4 & -3 & 2 \\ 1 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 6 & -7 & 5 \\ 5 & 2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -3 & 2 \\ 1 & 0 & 4 \end{bmatrix} + \begin{bmatrix} 2 & -4 & 3 \\ 4 & 2 & 1 \end{bmatrix} = B + A.$$

26. To verify the Associative Law for matrix addition, let us show that

$A + (B + C) = (A + B) + C$ . Now,

$$B + C = \begin{bmatrix} 4 & -3 & 2 \\ 1 & 0 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 2 \\ 3 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & -3 & 4 \\ 4 & -2 & 5 \end{bmatrix}$$

$$\text{and } A + (B + C) = \begin{bmatrix} 2 & -4 & 3 \\ 4 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 5 & -3 & 4 \\ 4 & -2 & 5 \end{bmatrix} = \begin{bmatrix} 7 & -7 & 7 \\ 8 & 0 & 6 \end{bmatrix}.$$

$$\text{Next, } A + B = \begin{bmatrix} 2 & -4 & 3 \\ 4 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 4 & -3 & 2 \\ 1 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 6 & -7 & 5 \\ 5 & 2 & 5 \end{bmatrix}$$

$$\text{and } (A + B) + C = \begin{bmatrix} 6 & -7 & 5 \\ 5 & 2 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 2 \\ 3 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -7 & 7 \\ 8 & 0 & 6 \end{bmatrix}.$$

$$27. (3+5)A = 8A = 8 \begin{bmatrix} 3 & 1 \\ 2 & 4 \\ -4 & 0 \end{bmatrix} = \begin{bmatrix} 24 & 8 \\ 16 & 32 \\ -32 & 0 \end{bmatrix} = 3 \begin{bmatrix} 3 & 1 \\ 2 & 4 \\ -4 & 0 \end{bmatrix} + 5 \begin{bmatrix} 3 & 1 \\ 2 & 4 \\ -4 & 0 \end{bmatrix} \\ = 3A + 5A.$$

$$28. \quad 2(4A) = 2(4) \begin{bmatrix} 3 & 1 \\ 2 & 4 \\ -4 & 0 \end{bmatrix} = 2 \begin{bmatrix} 12 & 4 \\ 8 & 16 \\ -16 & 0 \end{bmatrix} = \begin{bmatrix} 24 & 8 \\ 16 & 32 \\ -32 & 0 \end{bmatrix} = (2 \cdot 4) \begin{bmatrix} 3 & 1 \\ 2 & 4 \\ -4 & 0 \end{bmatrix} = 8 \begin{bmatrix} 3 & 1 \\ 2 & 4 \\ -4 & 0 \end{bmatrix}.$$

$$29. \quad 4(A+B) = 4 \left[ \begin{bmatrix} 3 & 1 \\ 2 & 4 \\ -4 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 3 & 2 \end{bmatrix} \right] = 4 \begin{bmatrix} 4 & 3 \\ 1 & 4 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 16 & 12 \\ 4 & 16 \\ -4 & 8 \end{bmatrix}$$

$$4A+4B = 4 \begin{bmatrix} 3 & 1 \\ 2 & 4 \\ -4 & 0 \end{bmatrix} + 4 \begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 16 & 12 \\ 4 & 16 \\ -4 & 8 \end{bmatrix}.$$

$$30. \quad 2(A-3B) = 2 \left[ \begin{bmatrix} 3 & 1 \\ 2 & 4 \\ -4 & 0 \end{bmatrix} - 3 \begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 3 & 2 \end{bmatrix} \right] = 2 \begin{bmatrix} 0 & -5 \\ 5 & 4 \\ -13 & -6 \end{bmatrix} = \begin{bmatrix} 0 & -10 \\ 10 & 8 \\ -26 & -12 \end{bmatrix}.$$

$$2A-6B = 2 \begin{bmatrix} 3 & 1 \\ 2 & 4 \\ -4 & 0 \end{bmatrix} - 6 \begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 0 & -10 \\ 10 & 8 \\ -26 & -12 \end{bmatrix}.$$

$$31. \quad [3 \quad 2 \quad -1 \quad 5]^T = \begin{bmatrix} 3 \\ 2 \\ -1 \\ 5 \end{bmatrix}.$$

$$32. \quad \begin{bmatrix} 4 & 2 & 0 & -1 \\ 3 & 4 & -1 & 5 \end{bmatrix}^T = \begin{bmatrix} 4 & 3 \\ 2 & 4 \\ 0 & -1 \\ -1 & 5 \end{bmatrix}.$$

$$33. \quad \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & 2 \\ 0 & 1 & 0 \end{bmatrix}^T = \begin{bmatrix} 1 & 3 & 0 \\ -1 & 4 & 1 \\ 2 & 2 & 0 \end{bmatrix}.$$

$$34. \quad \begin{bmatrix} 1 & 2 & 6 & 4 \\ 2 & 3 & 2 & 5 \\ 6 & 2 & 3 & 0 \\ 4 & 5 & 0 & 2 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 & 6 & 4 \\ 2 & 3 & 2 & 5 \\ 6 & 2 & 3 & 0 \\ 4 & 5 & 0 & 2 \end{bmatrix}.$$

$$35. \begin{array}{r} \\ \\ \\ \end{array} \begin{array}{cccc} & 1 & 2 & 3 & 4 \\ \begin{array}{l} \text{Mr. Cross} \\ \text{Mr. Jones} \\ \text{Mr. Smith} \end{array} & \begin{bmatrix} 220 & 215 & 210 & 205 \\ 220 & 210 & 200 & 195 \\ 215 & 205 & 195 & 190 \end{bmatrix} \end{array}$$

36. a. The required matrices are

$$A = \begin{array}{r} \text{Lesley} \\ \text{Tom} \end{array} \begin{bmatrix} 500 & 350 & 200 & 400 \\ 400 & 450 & 300 & 200 \end{bmatrix}, \quad B = \begin{array}{r} \text{Lesley} \\ \text{Tom} \end{array} \begin{bmatrix} 50 & 50 & 0 & 100 \\ 0 & 80 & 100 & 50 \end{bmatrix}$$

b. Their holdings at the end of the year are

$$C = A + B = \begin{array}{r} \text{Lesley} \\ \text{Tom} \end{array} \begin{bmatrix} 550 & 400 & 200 & 500 \\ 400 & 530 & 400 & 250 \end{bmatrix}$$

$$37. B = (1.03)A = 1.03 \begin{bmatrix} 340 & 360 & 380 \\ 410 & 430 & 440 \\ 620 & 660 & 700 \end{bmatrix} = \begin{array}{l} I \\ II \\ III \end{array} \begin{array}{ccc} M_1 & M_2 & M_3 \\ \begin{bmatrix} 350.2 & 370.8 & 391.4 \\ 422.3 & 442.9 & 453.2 \\ 638.6 & 679.8 & 721 \end{bmatrix} \end{array}$$

38. We want to find  $r$  so that

$$(1.01r)A = B$$

By the definition of scalar multiplication, we just need to consider the corresponding

entries in the first row and first column. Thus,

$$(1 + 0.01r)(340) = 357$$

$$0.01r = \frac{357}{340} - 1$$

and  $r = 5$ . So the percentage increase is 5%.

39. a.  $D = A + B - C$

$$= \begin{bmatrix} 2820 & 1470 & 1120 \\ 1030 & 520 & 480 \\ 1170 & 540 & 460 \end{bmatrix} + \begin{bmatrix} 260 & 120 & 110 \\ 140 & 60 & 50 \\ 120 & 70 & 50 \end{bmatrix} - \begin{bmatrix} 120 & 80 & 80 \\ 70 & 30 & 40 \\ 60 & 20 & 40 \end{bmatrix}$$

$$= \begin{bmatrix} 2960 & 1510 & 1150 \\ 1100 & 550 & 490 \\ 1230 & 590 & 470 \end{bmatrix}.$$

$$\text{b. } E = 1.1D = 1.1 \begin{bmatrix} 2960 & 1510 & 1150 \\ 1100 & 550 & 490 \\ 1230 & 590 & 470 \end{bmatrix} = \begin{bmatrix} 3256 & 1661 & 1265 \\ 1210 & 605 & 539 \\ 1353 & 649 & 517 \end{bmatrix}.$$

$$40. \quad \text{a.} \quad \begin{array}{cccc} & \textit{Text.} & \textit{Fict.} & \textit{Non-} \\ & & & \textit{Fict.} & \textit{Ref.} \\ \textit{A} = \begin{array}{l} \textit{Hard} \\ \textit{Paper} \end{array} & \begin{bmatrix} 5280 & 1680 & 2320 & 1890 \\ 1940 & 2810 & 1490 & 2070 \end{bmatrix} \end{array}$$

$$\text{b.} \quad \begin{array}{cccc} & \textit{Text.} & \textit{Fict.} & \textit{Non-} \\ & & & \textit{Fict.} & \textit{Ref.} \\ \textit{B} = \begin{array}{l} \textit{Hard} \\ \textit{Paper} \end{array} & \begin{bmatrix} 6340 & 2220 & 1790 & 1980 \\ 2050 & 3100 & 1720 & 2710 \end{bmatrix} \end{array}$$

$$\text{c.} \quad \begin{array}{cccc} & \textit{Text.} & \textit{Fict.} & \textit{Non-} \\ & & & \textit{Fict.} & \textit{Ref.} \\ \textit{C} = \begin{array}{l} \textit{Hard} \\ \textit{Paper} \end{array} & \begin{bmatrix} 11620 & 3900 & 4110 & 3870 \\ 3990 & 5910 & 3210 & 4780 \end{bmatrix} \end{array}$$

$$41. \quad \textit{A} = \begin{array}{l} \textit{MA} \\ \textit{U.S.} \end{array} \begin{bmatrix} 6.88 & 7.05 & 7.18 \\ 4.13 & 4.09 & 4.06 \end{bmatrix}$$

$$42. \quad \textit{A} = \begin{array}{l} \textit{M} \\ \textit{W} \end{array} \begin{bmatrix} 1 & 2.6 & 7 & 18.8 & 36.3 \\ 0.8 & 1.8 & 4.4 & 12.2 & 27.6 \end{bmatrix}$$

43.

$$A = \begin{matrix} & \text{W} & \text{B} & \text{H} \\ \text{W} & 81 & 76.1 & 82.2 \\ \text{M} & 76 & 69.9 & 75.9 \end{matrix} ; B = \begin{matrix} & \text{W} & \text{M} \\ \text{W} & 81 & 76 \\ \text{B} & 76.1 & 69.9 \\ \text{H} & 82.2 & 75.9 \end{matrix}$$

44. 
$$A = \begin{matrix} & \text{H} & \text{H-D} & \text{Y} & \text{S} & \text{K} & \text{O} \\ 2001 & 27.9 & 21.9 & 19.2 & 11 & 9.1 & 10.9 \\ 2001 & 27.6 & 23.3 & 18.2 & 10.5 & 8.8 & 11.6 \end{matrix}$$

The sum of all elements in the first row is 100%. The sum of all elements in the second row is 100%. Harley-Davidson gained the most: 23.3-21.9 or 1.4%.

45. True. Each element in  $A + B$  is obtained by adding together the corresponding elements in  $A$  and  $B$ . Therefore, the matrix  $c(A + B)$  is obtained by multiplying each element in  $A + B$  by  $c$ . On the other hand,  $cA$  is obtained by multiplying each element in  $A$  by  $c$  and  $cB$  is obtained by multiplying each element in  $B$  by  $c$  and  $cA + cB$  is obtained by adding the corresponding elements in  $cA$  and  $cB$ . Thus  $c(A + B) = cA + cB$ .

46. True.  $(-1)B$  is the matrix whose elements are the negatives of the respective elements of  $B$ . The result now follows by the definition of matrix addition.

47. False. Take  $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$  and  $c = 2$ . Then

$$cA = 2 \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} \text{ and } (cA)^T = \begin{bmatrix} 2 & 6 \\ 4 & 8 \end{bmatrix}.$$

On the other hand,  $\frac{1}{c}A^T = \frac{1}{2} \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{3}{2} \\ 1 & 2 \end{bmatrix} \neq (cA)^T$

48. True.  $A^T$  is obtained from  $A$  by interchanging the rows and columns of  $A$ .  $(A^T)^T$  is obtained by interchanging the rows and columns of  $A^T$ . This leads to the original matrix  $A$ . Thus,  $(A^T)^T = A$ .

**USING TECHNOLOGY EXERCISES 2.4, page 113**

1.  $\begin{bmatrix} 15 & 38.75 & -67.5 & 33.75 \\ 51.25 & 40 & 52.5 & -38.75 \\ 21.25 & 35 & -65 & 105 \end{bmatrix}$
2.  $\begin{bmatrix} -52.08 & 26.88 & -11.76 & 10.08 \\ -26.04 & -22.68 & 10.08 & -14.28 \\ -10.08 & 11.76 & 14.28 & -23.52 \end{bmatrix}$
3.  $\begin{bmatrix} -5 & 6.3 & -6.8 & 3.9 \\ 1 & 0.5 & 5.4 & -4.8 \\ 0.5 & 4.2 & -3.5 & 5.6 \end{bmatrix}$
4.  $\begin{bmatrix} 5 & -6.3 & 6.8 & -3.9 \\ -1 & -0.5 & -5.4 & 4.8 \\ -0.5 & -4.2 & 3.5 & -5.6 \end{bmatrix}$
5.  $\begin{bmatrix} 16.44 & -3.65 & -3.66 & 0.63 \\ 12.77 & 10.64 & 2.58 & 0.05 \\ 5.09 & 0.28 & -10.84 & 17.64 \end{bmatrix}$
6.  $\begin{bmatrix} -8.02 & 11.95 & -13.72 & 7.71 \\ 3.34 & 2.13 & 10.86 & -9.4 \\ 1.53 & 8.26 & -8.03 & 12.88 \end{bmatrix}$
7.  $\begin{bmatrix} 22.2 & -0.3 & -12 & 4.5 \\ 21.6 & 17.7 & 9 & -4.2 \\ 8.7 & 4.2 & -20.7 & 33.6 \end{bmatrix}$
8.  $\begin{bmatrix} -12.142 & 26.091 & -32.968 & 17.979 \\ 12.584 & 8.983 & 25.974 & -21.606 \\ 5.473 & 19.11 & -22.633 & 36.4 \end{bmatrix}$