CHAPTER 5 EIGENVALUES AND EIGENVECTORS

5.1 Eigenvalues and Eigenvectors

2. $A\mathbf{x} = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = 0\mathbf{x}$ therefore $\mathbf{x}$ is an eigenvector of A corresponding to the eigenvalue 0

4. (a) $\det(\lambda I - A) = \lambda^2 - 2\lambda - 3 = (\lambda - 3)(\lambda + 1)$; the eigenvalues are $\lambda = 3$ and $\lambda = -1$

(b) $\det(\lambda I - A) = \lambda^2 - 8\lambda + 16 = (\lambda - 4)^2$; the eigenvalue is $\lambda = 4$

(c) $\det(\lambda I - A) = \lambda^2 - 12$; the eigenvalues are $\lambda = \sqrt{12} = 2\sqrt{3}$ and $\lambda = -\sqrt{12} = -2\sqrt{3}$

(d) $\det(\lambda I - A) = \lambda^2 + 3$; no real eigenvalues

(e) $\det(\lambda I - A) = \lambda^2$; the eigenvalue is $\lambda = 0$

(f) $\det(\lambda I - A) = \lambda^2 - 2\lambda + 1 = (\lambda - 1)^2$; the eigenvalue is $\lambda = 1$

6. (a) Cofactor expansion along the second column yields $\det(\lambda I - A) = \begin{vmatrix} \lambda - 4 & 0 & -1 \\ 2 & \lambda - 1 & 0 \\ 2 & 0 & \lambda - 1 \end{vmatrix}$

$= (\lambda - 1)\begin{vmatrix} \lambda - 4 & -1 \\ 2 & \lambda - 1 \end{vmatrix} = (\lambda - 1)[(\lambda - 4)(\lambda - 1) - (-1)(2)] = (\lambda - 1)(\lambda^2 - 5\lambda + 6)$

$= (\lambda - 1)(\lambda - 2)(\lambda - 3)$; the characteristic equation is $(\lambda - 1)(\lambda - 2)(\lambda - 3) = 0$

(b) We use the arrow technique to evaluate the determinant: $\det(\lambda I - A) = \begin{vmatrix} \lambda - 3 & 0 & 5 \\ -\frac{1}{5} & \lambda + 1 & 0 \\ -1 & -1 & \lambda + 2 \end{vmatrix} =$

$[(\lambda - 3)(\lambda + 1)(\lambda + 2) + 0 + 1] - [-5(\lambda + 1) + 0 + 0] = \lambda^3 - 2\lambda$;

the characteristic equation is $\lambda^3 - 2\lambda = 0$

(c) We use the arrow technique to evaluate the determinant: $\det(\lambda I - A) = \begin{vmatrix} \lambda + 2 & 0 & -1 \\ 6 & \lambda + 2 & 0 \\ -19 & -5 & \lambda + 4 \end{vmatrix} =$

$[(\lambda + 2)^2(\lambda + 4) + 0 + 30] - [19(\lambda + 2) + 0 + 0] = \lambda^3 + 8\lambda^2 + \lambda + 8$;

the characteristic equation is $\lambda^3 + 8\lambda^2 + \lambda + 8 = 0$

(d) We use the arrow technique to evaluate the determinant: $\det(\lambda I - A) = \begin{vmatrix} \lambda+1 & 0 & -1 \\ 1 & \lambda-3 & 0 \\ 4 & -13 & \lambda+1 \end{vmatrix} =$

$[(\lambda+1)^2(\lambda-3) + 0 + 13] - [-4(\lambda-3) + 0 + 0] = \lambda^3 - \lambda^2 - \lambda - 2;$

the characteristic equation is $\lambda^3 - \lambda^2 - \lambda - 2$

(e) We use the arrow technique to evaluate the determinant: $\det(\lambda I - A) = \begin{vmatrix} \lambda-5 & 0 & -1 \\ -1 & \lambda-1 & 0 \\ 7 & -1 & \lambda \end{vmatrix} =$

$[(\lambda-5)(\lambda-1)\lambda + 0 - 1] - [-7(\lambda-1) + 0 + 0] = \lambda^3 - 6\lambda^2 + 12\lambda - 8;$

the characteristic equation is $\lambda^3 - 6\lambda^2 + 12\lambda - 8 = 0$

(f) We use the arrow technique to evaluate the determinant: $\det(\lambda I - A) = \begin{vmatrix} \lambda-5 & -6 & -2 \\ 0 & \lambda+1 & 8 \\ -1 & 0 & \lambda+2 \end{vmatrix} =$

$[(\lambda-5)(\lambda+1)(\lambda+2) + 48 + 0] - [2(\lambda+1) + 0 + 0] = \lambda^3 - 2\lambda^2 - 15\lambda + 36;$

the characteristic equation is $\lambda^3 - 2\lambda^2 - 15\lambda + 36 = 0$

8. (a) The reduced row echelon form of $I - A = \begin{bmatrix} -3 & 0 & -1 \\ 2 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$. The general solution of

$(I - A)\mathbf{x} = \mathbf{0}$ is $x_1 = 0, x_2 = t, x_3 = 0$. In vector form, $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ t \\ 0 \end{bmatrix} = t\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$. A basis for the eigenspace

corresponding to $\lambda = 1$ is $\{(0,1,0)\}$.

The reduced row echelon form of $2I - A = \begin{bmatrix} -2 & 0 & -1 \\ 2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$. The general solution of

$(2I - A)\mathbf{x} = \mathbf{0}$ is $x_1 = -\frac{1}{2}t, x_2 = t, x_3 = t$. In vector form, $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}t \\ t \\ t \end{bmatrix} = t\begin{bmatrix} -\frac{1}{2} \\ 1 \\ 1 \end{bmatrix}$. A basis for the

eigenspace corresponding to $\lambda = 2$ is $\{(-1,2,2)\}$ (scaled by a factor of 2 for convenience).

The reduced row echelon form of $3I - A = \begin{bmatrix} -1 & 0 & -1 \\ 2 & 2 & 0 \\ 2 & 0 & 2 \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$. The general solution of

$(3I - A)\mathbf{x} = \mathbf{0}$ is $x_1 = -t, x_2 = t, x_3 = t$. In vector form, $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -t \\ t \\ t \end{bmatrix} = t\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$. A basis for the eigenspace

corresponding to $\lambda = 3$ is $\{(-1,1,1)\}$.

(b) The reduced row echelon form of $0I - A = \begin{bmatrix} -3 & 0 & 5 \\ -\frac{1}{5} & 1 & 0 \\ -1 & -1 & 2 \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 & -\frac{5}{3} \\ 0 & 1 & -\frac{1}{3} \\ 0 & 0 & 0 \end{bmatrix}$. The general solution of

$(0I - A)\mathbf{x} = \mathbf{0}$ is $x_1 = \frac{5}{3}t, x_2 = \frac{1}{3}t, x_3 = t$. In vector form, $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{5}{3}t \\ \frac{1}{3}t \\ t \end{bmatrix} = t\begin{bmatrix} \frac{5}{3} \\ \frac{1}{3} \\ 1 \end{bmatrix}$. A basis for the eigenspace

corresponding to $\lambda = 0$ is $\{(5,1,3)\}$ (scaled by a factor 3 for convenience).

The reduced row echelon form of $\sqrt{2}I - A = \begin{bmatrix} \sqrt{2}-3 & 0 & 5 \\ -\frac{1}{5} & \sqrt{2}+1 & 0 \\ -1 & -1 & \sqrt{2}+2 \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 & \frac{-20-15\sqrt{2}}{6+5\sqrt{2}} \\ 0 & 1 & \frac{-2-\sqrt{2}}{6+5\sqrt{2}} \\ 0 & 0 & 0 \end{bmatrix}$.

The general solution of $(\sqrt{2}I - A)\mathbf{x} = \mathbf{0}$ is $x_1 = \frac{20+15\sqrt{2}}{6+5\sqrt{2}}t, x_2 = \frac{2+\sqrt{2}}{6+5\sqrt{2}}t, x_3 = t$.

In vector form, $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{20+15\sqrt{2}}{6+5\sqrt{2}}t \\ \frac{2+\sqrt{2}}{6+5\sqrt{2}}t \\ t \end{bmatrix} = t\begin{bmatrix} \frac{20+15\sqrt{2}}{6+5\sqrt{2}} \\ \frac{2+\sqrt{2}}{6+5\sqrt{2}} \\ 1 \end{bmatrix}$.

A basis for the eigenspace corresponding to $\lambda = \sqrt{2}$ is $\{(20+15\sqrt{2}, 2+\sqrt{2}, 6+5\sqrt{2})\}$ (scaled by a factor $6+5\sqrt{2}$ for convenience).

The reduced row echelon form of $-\sqrt{2}I - A = \begin{bmatrix} -\sqrt{2}-3 & 0 & 5 \\ -\frac{1}{5} & -\sqrt{2}+1 & 0 \\ -1 & -1 & -\sqrt{2}+2 \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 & \frac{-20+15\sqrt{2}}{6-5\sqrt{2}} \\ 0 & 1 & \frac{-2+\sqrt{2}}{6-5\sqrt{2}} \\ 0 & 0 & 0 \end{bmatrix}$.

The general solution of $(-\sqrt{2}I - A)\mathbf{x} = \mathbf{0}$ is $x_1 = \frac{20-15\sqrt{2}}{6-5\sqrt{2}}t, x_2 = \frac{2-\sqrt{2}}{6-5\sqrt{2}}t, x_3 = t$.

In vector form, $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{20-15\sqrt{2}}{6-5\sqrt{2}}t \\ \frac{2-\sqrt{2}}{6-5\sqrt{2}}t \\ t \end{bmatrix} = t\begin{bmatrix} \frac{20-15\sqrt{2}}{6-5\sqrt{2}} \\ \frac{2-\sqrt{2}}{6-5\sqrt{2}} \\ 1 \end{bmatrix}$.

A basis for the eigenspace corresponding to $\lambda = -\sqrt{2}$ is $\{(20-15\sqrt{2}, 2-\sqrt{2}, 6-5\sqrt{2})\}$ (scaled by a factor $6-5\sqrt{2}$ for convenience).

(c) The reduced row echelon form of $-8I - A = \begin{bmatrix} -6 & 0 & -1 \\ 6 & -6 & 0 \\ -19 & -5 & -4 \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 & \frac{1}{6} \\ 0 & 1 & \frac{1}{6} \\ 0 & 0 & 0 \end{bmatrix}$.

The general solution of $(-8I - A)\mathbf{x} = \mathbf{0}$ is $x_1 = -\frac{1}{6}t, x_2 = -\frac{1}{6}t, x_3 = t$.

In vector form, $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -\frac{1}{6}t \\ -\frac{1}{6}t \\ t \end{bmatrix} = t\begin{bmatrix} -\frac{1}{6} \\ -\frac{1}{6} \\ 1 \end{bmatrix}$.

A basis for the eigenspace corresponding to $\lambda = -8$ is $\{(-1,-1,6)\}$ (scaled by a factor 6 for convenience).

(d) The reduced row echelon form of $2I - A = \begin{bmatrix} 3 & 0 & -1 \\ 1 & -1 & 0 \\ 4 & 13 & 3 \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 1 & -\frac{1}{3} \\ 0 & 0 & 0 \end{bmatrix}$.

The general solution of $(2I - A)\mathbf{x} = \mathbf{0}$ is $x_1 = \frac{1}{3}t, x_2 = \frac{1}{3}t, x_3 = t$.

In vector form, $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{3}t \\ \frac{1}{3}t \\ t \end{bmatrix} = t\begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ 1 \end{bmatrix}$.

A basis for the eigenspace corresponding to $\lambda = 2$ is $\{(1,1,3)\}$ (scaled by a factor 3 for convenience).

(e) The reduced row echelon form of $2I - A = \begin{bmatrix} -3 & 0 & -1 \\ -1 & 1 & 0 \\ 7 & -1 & 2 \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 & \frac{1}{3} \\ 0 & 1 & \frac{1}{3} \\ 0 & 0 & 0 \end{bmatrix}$.

The general solution of $(2I - A)\mathbf{x} = \mathbf{0}$ is $x_1 = -\frac{1}{3}t, x_2 = -\frac{1}{3}t, x_3 = t$.

In vector form, $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -\frac{1}{3}t \\ -\frac{1}{3}t \\ t \end{bmatrix} = t\begin{bmatrix} -\frac{1}{3} \\ -\frac{1}{3} \\ 1 \end{bmatrix}$.

A basis for the eigenspace corresponding to $\lambda = 2$ is $\{(-1,-1,3)\}$ (scaled by a factor 3 for convenience).

(f) The reduced row echelon form of $3I - A = \begin{bmatrix} -2 & -6 & -2 \\ 0 & 4 & 8 \\ -1 & 0 & 5 \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$. The general solution of $(3I - A)\mathbf{x} = \mathbf{0}$ is $x_1 = 5t, x_2 = -2t, x_3 = t$. In vector form, $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5t \\ -2t \\ t \end{bmatrix} = t\begin{bmatrix} -5 \\ -2 \\ 1 \end{bmatrix}$.

A basis for the eigenspace corresponding to $\lambda = 3$ is $\{(5,-2,1)\}$.

The reduced row echelon form of $-4I - A = \begin{bmatrix} -9 & -6 & -2 \\ 0 & -3 & 8 \\ -1 & 0 & -2 \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -\frac{8}{3} \\ 0 & 0 & 0 \end{bmatrix}$. The general solution of $(-4I - A)\mathbf{x} = \mathbf{0}$ is $x_1 = -2t, x_2 = \frac{8}{3}t, x_3 = t$. In vector form, $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2t \\ \frac{8}{3}t \\ t \end{bmatrix} = t\begin{bmatrix} -2 \\ \frac{8}{3} \\ 1 \end{bmatrix}$.

A basis for the eigenspace corresponding to $\lambda = -4$ is $\{(-6,8,3)\}$ (scaled by a factor 3 for convenience).

10. In each part we employ the procedure described in Example 3 in Section 5.1.

(a) $\det(\lambda I - A) = \lambda^4 + \lambda^3 - 3\lambda^2 - \lambda + 2$.

The only integer solutions of the characteristic equation are ± 1 and ± 2.

Since $\det(1I - A) = 0$, $\lambda - 1$ must be a factor of the characteristic polynomial.

Dividing $\lambda - 1$ into $\lambda^4 + \lambda^3 - 3\lambda^2 - \lambda + 2$ leads to $\det(\lambda I - A) = (\lambda - 1)(\lambda^3 + 2\lambda^2 - \lambda - 2)$.

The cubic polynomial can be found to be zero at $\lambda = 1$ as well.

Dividing $\lambda - 1$ into $\lambda^3 + 2\lambda^2 - \lambda - 2$ yields

$\det(\lambda I - A) = (\lambda - 1)^2(\lambda^2 + 3\lambda + 2) = (\lambda - 1)^2(\lambda + 1)(\lambda + 2)$.

We conclude that the eigenvalues are $1, -1$, and -2.

(b) $\det(\lambda I - A) = \lambda^4 - 8\lambda^3 + 19\lambda^2 - 24\lambda + 48$.

The only integer solutions of the characteristic equation are $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 16, \pm 24$, and ± 48. Successively substituting these into the characteristic polynomial, we find $\det(4I - A) = 0$ so that $\lambda - 4$ must be a factor of the polynomial. Dividing $\lambda - 4$ into $\lambda^4 - 8\lambda^3 + 19\lambda^2 - 24\lambda + 48$ we obtain $\det(\lambda I - A) = (\lambda - 4)(\lambda^3 - 4\lambda^2 + 3\lambda - 12)$. The cubic polynomial can have integer zeros $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6,$ and ± 12. Substituting $\lambda = 3$ results in a zero value, therefore dividing $\lambda - 4$ into $\lambda^3 - 4\lambda^2 + 3\lambda - 12$ leads to $\det(\lambda I - A) = (\lambda - 4)^2(\lambda^2 + 3)$.

The only real eigenvalue is 4.

12. (a) The matrix is upper triangular, therefore by Theorem 5.1.2 its eigenvalues are -1 and 5, the entries on the main diagonal.

(b) The matrix is lower triangular, therefore by Theorem 5.1.2 its eigenvalues are 3, 7, and 1, the entries on the main diagonal.

(c) The matrix is diagonal, therefore by Theorem 5.1.2 its eigenvalues are $-\frac{1}{3}$, 1, and $\frac{1}{2}$, the entries on the main diagonal.

14. We begin by finding eigenvalues and eigenvectors of the matrix A.

$$\det(\lambda I - A) = \begin{vmatrix} \lambda + 1 & 2 & 2 \\ -1 & \lambda - 2 & -1 \\ 1 & 1 & \lambda \end{vmatrix} = [(\lambda + 1)(\lambda - 2)\lambda - 2 - 2] - [2(\lambda - 2) - (\lambda + 1) - 2\lambda]$$

$$= \lambda^3 - \lambda^2 - \lambda + 1 = \lambda^2(\lambda - 1) - 1(\lambda - 1) = (\lambda^2 - 1)(\lambda - 1) = (\lambda + 1)(\lambda - 1)^2$$

From the characteristic equation $(\lambda + 1)(\lambda - 1)^2 = 0$ it follows that A has eigenvalues 1 and -1.

The reduced row echelon form of $I - A = \begin{bmatrix} 2 & 2 & 2 \\ -1 & -1 & -1 \\ 1 & 1 & 1 \end{bmatrix}$ is $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

The general solution of $(I - A)\mathbf{x} = \mathbf{0}$ is $x_1 = -s - t$, $x_2 = s$, $x_3 = t$.

A basis for the eigenspace of the matrix A corresponding to $\lambda = 1$ is $\{(-1,1,0), (-1,0,1)\}$.

The reduced row echelon form of $-1I - A = \begin{bmatrix} 0 & 2 & 2 \\ -1 & -3 & -1 \\ 1 & 1 & -1 \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$.

The general solution of $(I - A)\mathbf{x} = \mathbf{0}$ is $x_1 = 2t$, $x_2 = -t$, $x_3 = t$.

A basis for the eigenspace of the matrix A corresponding to $\lambda = 1$ is $\{(2,-1,1)\}$.

Using Theorem 5.1.4, we conclude that the eigenvalues of the matrix A^{25} are $1^{25} = 1$ and $(-1)^{25} = -1$. Furthermore, the bases for eigenspaces corresponding to these eigenvalues are $\{(-1,1,0), (-1,0,1)\}$ and $\{(2,-1,1)\}$, respectively.

16. Since $p(\lambda) = \det(\lambda I - A)$, it follows that $p(0) = \det(-A) = (-1)^n \det(A)$.

(a) $n = 3$ and $p(0) = 5$ therefore $\det(A) = -5$

(b) $n = 4$ and $p(0) = 7$ therefore $\det(A) = 7$

18. Denoting $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, we have $\det(\lambda I - A) = \begin{vmatrix} \lambda - a_{11} & -a_{12} \\ -a_{21} & \lambda - a_{22} \end{vmatrix} = (\lambda - a_{11})(\lambda - a_{22}) - a_{12}a_{21} =$

$\lambda^2 - \underbrace{(a_{11} + a_{22})}_{\text{tr}(A)}\lambda + \underbrace{a_{11}a_{22} - a_{12}a_{22}}_{\det(A)}$

26. $\det(\lambda I - A) = \begin{vmatrix} \lambda + 2 & -2 & -3 \\ 2 & \lambda - 3 & -2 \\ 4 & -2 & \lambda - 5 \end{vmatrix} = \lambda^3 - 6\lambda^2 + 11\lambda - 6;$

The only integer solutions of the characteristic equation are $\pm 1, \pm 2, \pm 3, \pm 6$.

Since $\det(1I - A) = 0$, we divide $\lambda - 1$ into $\lambda^3 - 6\lambda^2 + 11\lambda - 6$ to obtain

$\det(\lambda I - A) = (\lambda - 1)(\lambda^2 - 5\lambda + 6) = (\lambda - 1)(\lambda - 2)(\lambda - 3)$.

The reduced row echelon form of $I - A = \begin{bmatrix} 3 & -2 & -3 \\ 2 & -2 & -2 \\ 4 & -2 & -4 \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

The general solution of $(I - A)\mathbf{x} = \mathbf{0}$ is $x_1 = t$, $x_2 = 0$, $x_3 = t$.

A basis for the eigenspace of the matrix A corresponding to $\lambda = 1$ is $\{(1,0,1)\}$.

The reduced row echelon form of $2I - A = \begin{bmatrix} 4 & -2 & -3 \\ 2 & -1 & -2 \\ 4 & -2 & -3 \end{bmatrix}$ is $\begin{bmatrix} 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$.

The general solution of $(2I - A)\mathbf{x} = \mathbf{0}$ is $x_1 = \frac{1}{2}t$, $x_2 = t$, $x_3 = 0$.

A basis for the eigenspace of the matrix A corresponding to $\lambda = 2$ is $\{(1,2,0)\}$.

The reduced row echelon form of $3I - A = \begin{bmatrix} 5 & -2 & -3 \\ 2 & 0 & -2 \\ 4 & -2 & -2 \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$.

The general solution of $(3I - A)\mathbf{x} = \mathbf{0}$ is $x_1 = t$, $x_2 = t$, $x_3 = t$.

A basis for the eigenspace of the matrix A corresponding to $\lambda = 3$ is $\{(1,1,1)\}$.

(a) From the result of Exercise 23, the matrix A^{-1} has

- eigenvalue 1 with a basis for the corresponding eigenspace $\{(1,0,1)\}$,
- eigenvalue $\frac{1}{2}$ with a basis for the corresponding eigenspace $\{(1,2,0)\}$,
- eigenvalue $\frac{1}{3}$ with a basis for the corresponding eigenspace $\{(1,1,1)\}$.

(b) From the result of Exercise 24, the matrix $A - 3I$ has

- eigenvalue -2 with a basis for the corresponding eigenspace $\{(1,0,1)\}$,
- eigenvalue -1 with a basis for the corresponding eigenspace $\{(1,2,0)\}$,
- eigenvalue 0 with a basis for the corresponding eigenspace $\{(1,1,1)\}$.

(c) From the result of Exercise 24, the matrix $A + 2I$ has

- eigenvalue 3 with a basis for the corresponding eigenspace $\{(1,0,1)\}$,
- eigenvalue 4 with a basis for the corresponding eigenspace $\{(1,2,0)\}$,
- eigenvalue 5 with a basis for the corresponding eigenspace $\{(1,1,1)\}$.

28. (a) Since the degree of $p(\lambda)$ is 6, A is a 6×6 matrix (see Exercise 17)

(b) $p(0) \neq 0$, therefore 0 is not an eigenvalue of A. From parts (a) and (t) of Theorem 5.1.6, A is invertible.

(c) A has three eigenspaces since it has three distinct eigenvalues, each corresponding to an eigenspace.

5.2 Diagonalization

2. $\begin{vmatrix} 4 & -1 \\ 2 & 4 \end{vmatrix} = 18$ does not equal $\begin{vmatrix} 4 & 1 \\ 2 & 4 \end{vmatrix} = 14$ therefore, by Table 1 in Section 5.2, A and B are not similar matrices.