

Math 181 Section 050
Section 2.3: Limit Laws
September 3, 2013

The following limit laws are presented in section 2.3. Every time you use one of these laws in homework problems from this section, you should use the labels below to indicate which law you are using.

Suppose that c is a constant, m and n are positive integers, and both

$$\lim_{x \rightarrow a} f(x) \text{ and } \lim_{x \rightarrow a} g(x)$$

exist. Then the following properties hold (and the analogous laws hold for one-sided limits):

1. Sum Law

$$\lim_{x \rightarrow a} [f(x) + g(x)] = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$$

2. Difference Law

$$\lim_{x \rightarrow a} [f(x) - g(x)] = \lim_{x \rightarrow a} f(x) - \lim_{x \rightarrow a} g(x)$$

3. Constant Multiple Law

$$\lim_{x \rightarrow a} [cf(x)] = c \lim_{x \rightarrow a} f(x)$$

4. Product Law

$$\lim_{x \rightarrow a} [f(x)g(x)] = \lim_{x \rightarrow a} f(x) \lim_{x \rightarrow a} g(x)$$

5. Quotient Law

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)} \quad \left(\text{for } \lim_{x \rightarrow a} g(x) \neq 0 \right)$$

6. Power Law

$$\lim_{x \rightarrow a} [f(x)]^n = \left[\lim_{x \rightarrow a} f(x) \right]^n$$

7. Fractional Power Law

$$\lim_{x \rightarrow a} [f(x)^{n/m}] = \left[\lim_{x \rightarrow a} f(x) \right]^{n/m}$$

- Two-sided limits: If m is even and n/m is in lowest terms, the Fractional Power Law requires $f(x) \geq 0$ for x near a .
- Right-sided limits: If m is even and n/m is in lowest terms, the Fractional Power Law requires $f(x) \geq 0$ for x near a with $x > a$.
- Left-sided limits: If m is even and n/m is in lowest terms, the Fractional Power Law requires $f(x) \geq 0$ for x near a with $x < a$.