Exterior Angles Investigation

Name ______________________________________________      Date _____________

1) Start Geometer’s Sketchpad if it isn’t already running or choose New Sketch from the File menu.  
   A new, blank sketch window titled “Untitled N” appears.

2) If necessary, resize the Geometer’s Sketchpad window and the nested “Untitled N” window to allow room to draw.

3) Today we will investigate some properties of the exterior angles of triangles. We will begin by drawing an acute triangle.

4) Choose the Segment tool from the Toolbox.

5) Using the mouse, position the cursor in the lower central portion of the blank sketch window and left click once.  
   This will create a single point in the sketch.

6) Without clicking again, slide the mouse to the right.  
   You will see a second point appear (following your cursor’s movement) and a segment stretching between the two points.

7) When the second point is halfway between the first point and the right edge of the sketch window, click again to complete the segment.  
   At this point, the segment will appear highlighted (it will look like a triple stripe).

8) We will now draw the second segment of the triangle.
   a) Move the cursor over the last point drawn.  
      A colored halo will appear around the point.
   b) Click while the halo is visible to reuse this endpoint for the second segment, and then move the cursor up and to the left.  
      Again, you will see a second point appear and a segment stretching between the two points.
   c) Click once to complete the second segment.

9) We will use a different approach to construct the final segment.
   a) Click on the Selection Arrow tool to exit segment drawing mode.
   b) Click anywhere in blank portion of the sketch to deselect the second segment.  
      The segment will appear normal instead of triple-striped.
   c) Next click on the first point drawn (lower left) and last point drawn (highest point).  
      These two points and nothing else will appear highlighted.
   d) From the Construct menu, choose Segment.
10) Now we will label the vertices of our triangle.
   a) Click anywhere in blank portion of the sketch to deselect the third segment.
   b) Using the mouse, click once on the highest vertex, and then (moving clockwise) once on the vertex at the lower right, and then once on the vertex at the lower left. 
      *All three points will appear highlighted.*
   c) From the **Display** menu, choose **Show Labels**.
   d) Click anywhere in blank portion of the sketch to deselect the vertices of the triangle.

11) To explore the **exterior angles** of the triangle, we must extend the segments of the triangle to form rays.

12) We can draw an exterior angle at vertex B by drawing either ray $\overrightarrow{AB}$ or ray $\overrightarrow{CB}$.

13) Since we labeled the vertices in clockwise fashion, we will draw the exterior angles in a similar fashion.
   a) Click first on point A and then on point B.
   b) From the **Construct** menu, select **Ray**.
   c) Click anywhere in blank portion of the sketch to deselect ray $\overrightarrow{AB}$.
   d) Repeat steps a through c to draw rays $\overrightarrow{BC}$ and $\overrightarrow{CA}$.

14) Now we will place a named point on each ray so that we can refer to the exterior angles by name.
   a) Select the **Point** tool from the Toolbox.
      *The cursor will change to an arrow with a point at the tip.*
   b) Use the mouse to position the cursor over ray $\overrightarrow{AB}$ so that point B lies between the cursor and point A.
      *When the cursor is properly positioned, ray $\overrightarrow{AB}$ will be highlighted.*
   c) While ray $\overrightarrow{AB}$ is highlighted, click once to create a new point.
   d) With the Point tool still selected, add similar points to rays $\overrightarrow{BC}$ and $\overrightarrow{CA}$.
   e) Select the **Selection Arrow** tool to exit point drawing mode.
   f) At this point the only element highlighted should be the last point drawn (on ray $\overrightarrow{CA}$).
g) Moving clockwise, select the remaining unnamed points on rays $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

h) With all three unnamed points selected, choose **Show Labels** from the **Display** menu to name the points.

i) Examine all your labels. If a label is intersected by a ray, you can move the label away by pressing and dragging directly on the label. You will know the label is selected when the cursor changes to a small pointing hand.

j) Double clicking on a label will bring up a **Properties of Point** dialogue box. Use this feature to change the labels of the last three points drawn to P, Q and R as shown below.

k) Finally, click on white space to deselect all elements.

15) We have drawn the exterior angles $\angle PAB$, $\angle QBC$, and $\angle RCA$ of triangle $\triangle ABC$.

16) Note that one interior angle is adjacent to each exterior angle. The two interior angles which are not adjacent to an exterior angle are called the **remote interior angles** of that exterior angle.

17) Now we will use Sketchpad’s measuring capabilities to find the measures of all the interior and exterior angles in our sketch.

   a) Click on points P, A and B in that order. *The three points are highlighted.*

   b) From the **Measure** menu, select **Angle**.
The highlighted measure of angle $\angle PAB$ appears in the upper left corner of the sketch.

c) Click in white space to deselect all elements.

d) Repeat steps a through c to determine the measure of the other two exterior angles ($\angle QBC$ and $\angle RCA$).

e) Repeat steps a through c to determine the measure of the three interior angles ($\angle ABC$, $\angle BCA$, $\angle CAB$).

18) We also need to get the three pair-wise sums of the interior angles.

   a) From the Measure menu, select Calculate.

      *A New Calculation dialogue box which looks like a calculator appears.*

   b) Click on $m\angle CAB = NN. N^\circ$ in the sketch to transfer $m\angle CAB$ to the calculator.

   c) Click on the + key of the calculator keypad.

   d) Click on $m\angle ABC = NN. N^\circ$ in the sketch to transfer $m\angle ABC$ to the calculator.

      *The calculator will display the expression $m\angle CAB + m\angle ABC$.*

   e) Click on the OK button to dismiss the calculator and transfer the expression to the sketch.

   f) Click in white space to deselect the expression.

   g) Repeat steps a through f to compute the sums of the other two pairs of interior angles.

19) Study the angle measures displayed on your sketch. Look for relationships.

20) With the Selection Arrow tool still selected, select and drag any vertex of the triangle and watch the angle measures. Make sure you form right and obtuse triangles.

21) If desired, you can also select and drag the angle measures around in order to place certain measures in close proximity to one another.

22) Referring to your sketch for help, answer the following questions:

   a) Which interior angle is adjacent to exterior angle $\angle PAB$? $\angle CAB$ or $\angle BAC$

   b) What is the sum of $m\angle PAB$ and the measure of its adjacent interior angle? $180^\circ$

   c) What is the relationship between angle $\angle PAB$ and its adjacent interior angle and why?

      *They are supplementary angles because they form a linear pair.*

   d) What is the relationship between any exterior angle and its adjacent interior angle?

      *They are supplementary angles.*
e) Back in step 13, when we chose to draw the exterior angles of triangle \( \triangle ABC \) using rays which emanated from the figure in a clockwise fashion, suppose we had made the opposite choice. In other words, suppose we created our exterior angles by drawing rays \( \overrightarrow{AC}, \overrightarrow{CB}, \) and \( \overrightarrow{BA} \). And suppose we had placed point \( Q \) on ray \( \overrightarrow{CB} \). (You can add a ray to the drawing and then delete it with undo if you have trouble visualizing this.) How would the measure of this other exterior angle at vertex \( B \) \( (\angle QBA) \) compare to \( m\angle QBC \)? Why?

They would be equal

because either angle is a supplement of angle \( \angle ABC \).

f) Write an equation for \( m\angle PAB \) in terms of \( m\angle CAB \).

\[ m\angle PAB = 180 - m\angle CAB \]

g) Which angles are remote interior angles of exterior angle \( \angle PAB \)?

\( \angle ABC \) and \( \angle BCA \)

h) Complete the following equation and state which theorem you used.

\[ m\angle ABC + m\angle BCA + m\angle CAB = 180; \]

Angle Sum Theorem for Triangles

i) Rewrite the equation from h by isolating the remote interior angles of exterior angle \( \angle PAB \) on the left hand side of the equation.

\[ m\angle ABC + m\angle BCA = 180 - m\angle CAB \]

j) Combine the equations from f and i into a single equation relating exterior angle \( \angle PAB \) to the measures of its remote interior angles.

\[ m\angle PAB = m\angle ABC + m\angle BCA \]

k) Write a general conjecture about the measure of an exterior angle of a triangle and the measures of its two remote interior angles.

The measure of an exterior angle of a triangle is equal to the sum of the measures of its two remote interior angles.