The properties of addition of natural numbers can be derived from a short set of axioms. The axioms are called the Peano Axioms:

There exists a set, P, which is defined by the following four axioms.

Axiom 1: There exists a natural number, call it 1, that is not the successor of any other natural number.

Axiom 2: Every natural number has a unique successor. If $k \in P$, then let $k'$ denote the successor of $k$.

Axiom 3: Every natural number except one is the successor of exactly one natural number.

Axiom 4: If $M$ is a set of natural numbers such that

(i) $1 \in M$

(ii) for each $k \in P$, if $k \in M$, then $k' \in P$,

then $P = M$.

So, the Peano axioms assert the uniqueness of the naturals that this successor property along with the element 1 creates the entirety of the natural numbers. No matter how you name the set (you can call it Ray, or you can call it Jay, . . .) if it has these properties then it really is the naturals.

From these axioms arise the natural numbers by defining what addition by one means.

Definition 3.6.1: For every $k \in \mathbb{N}$, define $k + 1 = k'$.

Then, note inductively, the entire understanding of addition flows from this definition (likewise multiplication, etc.).