All prime statements P, Q, R, etc. are statements.

If P is a statement, then \( \neg P \) is a statement.
If P and Q are statements, then \( P \lor Q \) is a statement.
If P and Q are statements, then \( P \land Q \) is a statement.
If P and Q are statements, then \( P \Rightarrow Q \) is a statement.
If P and Q are statements, then \( P \iff Q \) is a statement.

Idempotent Law (1) \( P \lor P \equiv P \)
Idempotent Law (2) \( P \land P \equiv P \)

Law of double negation \( \neg (\neg P) \equiv P \) [ same as \( \neg (\neg P) \iff P \) ]

Or form of implication \( P \Rightarrow Q \iff \neg P \lor Q \)

(when changing from implication to or form
reference or form; but when changing from or form to implication
reference implication form)

Contrapositive form of implication \( P \Rightarrow Q \iff \neg Q \Rightarrow \neg P \)

De Morgan Law (1) \( \neg P \lor \neg Q \equiv \neg (P \land Q) \)
De Morgan Law (2) \( \neg P \land \neg Q \equiv \neg (P \lor Q) \)

Direct Proof Law \( P \land R \Rightarrow Q \equiv (P \Rightarrow (R \Rightarrow Q)) \)

Indirect Proof Law \( P \land \neg Q \Rightarrow \text{always false} \equiv P \Rightarrow Q \)

Law of the Excluded Middle (1) \( P \land \neg P \text{ always false} \)
Law of the Excluded Middle (2) \( P \lor \neg P \text{ always true}^{1} \)

Commutative Law of “or” (1) \( P \lor Q \equiv Q \lor P \)
Commutative Law of “and” (2) \( P \land Q \equiv Q \land P \)

Associative Law of “or” (1) \( P \lor (Q \lor R) \equiv (P \lor Q) \lor R \equiv P \lor Q \lor R \)
Associative Law of “and” (2) \( P \land (Q \land R) \equiv (P \land Q) \land R \equiv P \land Q \land R \)

\[^{1}\text{This is obvious, but let’s take a closer look. Note } P \lor \neg P \text{ is logically equivalent to } \neg P \lor P \text{ which by the or form into implication form is } P \Rightarrow P \text{ !!! Now, if anyone (usually in the Social Sciences) says the Law of the Excluded Middle is an antiquated, outdated, or invalid law ask them, “‘If - - - -, then - - - -’ [fill in the blank] is a fallacy?”} \]
Distributive Law of “and over or” (1) \[ P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R) \]

Distributive Law of “or over and” (2) \[ P \lor (Q \land R) \equiv (P \lor Q) \land (P \lor R) \]

Law of Addition \[ P \Rightarrow P \lor Q \]

Law of Simplification \[ P \land Q \Rightarrow P \]

Modus Ponens \[ [(P \Rightarrow Q) \land P] \Rightarrow Q \]

Modus Tollens \[ [(P \Rightarrow Q) \land \neg Q] \Rightarrow \neg P \]

Disjunctive Syllogism \[ [(P \lor Q) \land \neg Q] \Rightarrow P \]

Hypothetical Syllogism (Transitivity) \[ (P \Rightarrow Q) \land (Q \Rightarrow R) \Rightarrow [P \Rightarrow R] \]

Assume the hypothesis of the conclusion \[ (P \Rightarrow (R \Rightarrow Q)) \Rightarrow (P \land R) \Rightarrow Q \]

FALLACIES:

Asserting the conclusion \[ [(P \Rightarrow Q) \land Q] \Rightarrow P \]
(assuming the conclusion) (fallacy of the converse)
It is actually the case that \[ [(P \Rightarrow Q) \land Q] \Rightarrow P \] necessarily!

Asserting the premise \[ (P \Rightarrow Q) \Rightarrow P \]
(assuming the premise must always be true)
It is actually the case that \[ (P \Rightarrow Q) \Rightarrow P \] necessarily!

Fallacy of the inverse \[ [(P \Rightarrow Q) \land \neg P] \Rightarrow \neg Q \]
It is actually the case that \[ [(P \Rightarrow Q) \land \neg P] \Rightarrow \neg Q \] necessarily!

Fallacy (1) \[ P \lor Q \Rightarrow P \]
(they reversed the law of addition)

Fallacy (2) \[ P \Rightarrow P \land Q \]
(they reversed the law of simplification)

There are MANY more fallacies we could list; but, these are the most common. Avoid them!