

Welsh's problem on the number of bases of matroids

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Abstract

In this paper, we study a problem raised by Dominic Welsh on the existence of matroids with prescribed size, rank, and number of bases. In case of corank being at most 3, by an explicit construction of matrices, we are able to show the existence of such matroids, except the counterexample found by Mayhew and Royle. This idea also extends to give asymptotic results on the existence of matroids with any fixed corank.

1 Background

Let (n, r, b) be a triple of integers such that $0 < r \leq n$ and $1 \leq b \leq \binom{n}{r}$. Given a triple (n, r, b) , Welsh [?] asked if there exists a matroid with n elements, rank r , and exactly b bases. Such a matroid would be called an (n, r, b) -matroid. It was conjectured that an (n, r, b) -matroid exists for every such triple, until Mayhew and Royle [?] found the lone counterexample to date, namely $(n, r, b) = (6, 3, 11)$. However, they proposed the following conjecture.

Conjecture 1.1. *An (n, r, b) -matroid exists for all $0 < r \leq n$ and $1 \leq b \leq \binom{n}{r}$ except $(n, r, b) = (6, 3, 11)$.*

In this paper, we verify this conjecture for a large family of triples (n, r, b) .

2 Introduction

A *matroid* $\mathcal{M} = (X, \mathcal{I})$ is a combinatorial structure defined on a finite ground set X of n elements, together with a family \mathcal{I} of subsets of X called *independent sets*, satisfying the following three properties:

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1. $\emptyset \in \mathcal{I}$, or \mathcal{I} is nonempty.
2. If $I \in \mathcal{I}$ and $J \subseteq I$, then $J \in \mathcal{I}$. This is sometimes known as *hereditary property*.
3. If $I, J \in \mathcal{I}$ and $|J| < |I|$, then there exists $x \in I \setminus J$ such that $J \cup \{x\} \in \mathcal{I}$. This is known as *augmentation property* or *exchange property*.

A maximal independent set is called a *basis* of the matroid. A direct consequence of the augmentation property is that all bases have the same cardinality. This cardinality is defined as the *rank* of \mathcal{M} , and the difference between the size of X and the rank of \mathcal{M} is defined as the *corank* of \mathcal{M} .

One of the most important examples of matroids is a *linear matroid*. A linear matroid is defined from a matrix A over a field \mathbb{F} , where X is the set of columns of A , and an independent set $I \in \mathcal{I}$ is a collection of columns which is linearly independent over \mathbb{F} . For a linear matroid, the rank is exactly the rank of the matrix A , and the corank is the difference between the number of columns and the rank of A . Since the elements of a linear matroid are the columns of a matrix, a linear matroid is also called a *column matroid*.

In this paper, we are going to show that a linear (n, r, b) -matroid exists for a large family of triples (n, r, b) . Throughout this paper, A is an $r \times n$ matrix over \mathbb{Q} with rank r and corank $k = n - r$, and the number of $r \times r$ invertible submatrices of A is b . Note that b is an invariant if we perform row operations on A or permute the columns of A , so we can always assume that $A = (I_r | M)$, where M is an $r \times k$ matrix. We start with the following observation that there is a natural duality between the rank and corank of a linear matroid.

Proposition 2.1. *The number of invertible square submatrices of M is b , and the number of invertible $k \times k$ submatrices of $(I_k | M^\top)$ is also b . (Here, we adopt the convention that an empty matrix is invertible.)*

Proof. Let \mathcal{S} be the set of all invertible $r \times r$ submatrices of A , where $r \geq 0$, and let \mathcal{T} be the set of all invertible square submatrices of M .

Let S be a matrix in \mathcal{S} with columns i_1, i_2, \dots, i_r , where $i_1, \dots, i_j \leq r$ and $i_{j+1}, \dots, i_r > r$. Then there is a bijection between \mathcal{S} and \mathcal{T} which sends S to the square submatrix of M with rows $\{1, 2, \dots, r\} \setminus \{i_1, i_2, \dots, i_j\}$ and columns $i_{j+1} - r, i_{j+2} - r, \dots, i_r - r$. Hence, the number of invertible square submatrices of M is b .

It is then obvious that the number of invertible square submatrices of M^\top is b , and the above bijective argument in turn implies that the number of invertible $k \times k$ submatrices of $(I_k | M^\top)$ is b . \square

In view of this proposition, to determine the existence of a linear (n, r, b) -matroid, we would try to construct an $r \times k$ matrix M with exactly b invertible square submatrices, where $k = n - r$. Such a matrix M will be called an (r, k, b) -matrix. Note that we only need to consider the case when $k \leq r$ for our purpose of studying this Welsh's problem, since if $k > r$, we can instead consider the existence of an (k, r, b) -matrix. Also note that the existence of a linear (n, r, b) -matroid is equivalent to the existence of an (r, k, b) -matrix. Next, we observe that the naive "extension by zero" construction exhibits a useful relation between the existences of various (r, k, b) -matrices.

Lemma 2.2. *If there exists an (r_0, k_0, b) -matrix, then for all $r \geq r_0$, $k \geq k_0$, (r, k, b) -matrix exists.*

Proof. If M_0 is an (r_0, k_0, b) -matrix, by building an $r \times k$ matrix M such that M_0 is a submatrix and all the extra entries are 0's, M is an (r, k, b) -matrix. \square

Combining the results from Sections ?? and ??, we show that Conjecture ?? holds for all $1 \leq b \leq \binom{r+3}{r}$, except that $(n, r, b) = (6, 3, 11)$. In Section ??, we prove Conjecture ?? for all large r and large b . We strengthen this result in Section ?? such that Conjecture ?? holds for all large enough r , regardless the values of n and b . This gives the closest to complete answer to the Welsh's problem and provides a plausible reason of the existence of counterexample for small values of r .

3 Existence of linear matroids with corank at most 2

In this section, we will study the existence of (r, k, b) -matrices for $k \leq 2$. The cases $k = 0$ and $k = 1$ are very straight forward. For the first nontrivial case $k = 2$, the following lemma in number theory will help us to show the existence of matrices satisfying the parameters $(r, 2, b)$.

Lemma 3.1. *Let $s \geq 5$ be a positive integer, and let k be a nonnegative integer such that $k \leq \frac{s^2-5s}{4}$. Then there exist nonnegative integers a_1, a_2, \dots, a_s such that $a_1 + a_2 + \dots + a_s = s$ and $a_1^2 + a_2^2 + \dots + a_s^2 = s + 2k$.*

Proof. For $5 \leq s \leq 32$, we verified the lemma by Mathematica; for $s \geq 33$, we will use strong induction on s .

Suppose the statement is true for all integers u such that $5 \leq u < s$ for some $s \geq 33$, i.e., for all nonnegative integers $k' \leq \frac{u^2-5u}{4}$, there exist nonnegative integers a_1, \dots, a_u such that $a_1 + \dots + a_u = u$ and $a_1^2 + \dots + a_u^2 = u + 2k'$.

Let t and k be integers such that $0 < t \leq s - 5$ and $0 \leq k - \frac{t^2-t}{2} \leq \frac{(s-t)^2-5(s-t)}{4}$. Then $u := s - t$ falls in the range $5 \leq u < s$, and $k' := k - \frac{t^2-t}{2} \leq \frac{u^2-5u}{4}$. By the induction hypothesis, there are nonnegative integers a_1, \dots, a_{s-t} such that $a_1 + \dots + a_{s-t} = s - t$ and $a_1^2 + \dots + a_{s-t}^2 = s - t + 2(k - \frac{t^2-t}{2}) = s + 2k - t^2$. If we set $a_{s-t+1} = t$ and $a_{s-t+2} = \dots = a_s = 0$, then $a_1 + \dots + a_s = s$ and $a_1^2 + \dots + a_s^2 = s + 2k$, implying that the statement holds true for k satisfying $0 \leq k - \frac{t^2-t}{2} \leq \frac{(s-t)^2-5(s-t)}{4}$, or equivalently, $\frac{t^2-t}{2} \leq k \leq \frac{3t^2-2st+3t+s^2-5s}{4}$.

It now suffices to show that the union of the intervals $I(t) := \left[\frac{t^2-t}{2}, \frac{3t^2-2st+3t+s^2-5s}{4} \right]$ for $0 < t \leq s - 5$ covers $\left[0, \frac{s^2-5s}{4} \right]$ when $s \geq 33$. Let $\alpha(t) = \frac{t^2-t}{2}$ and $\beta(t) = \frac{3t^2-2st+3t+s^2-5s}{4}$.

Claim 1. $\frac{s^2-5s}{4} \leq \beta(t)$ if and only if $t \geq \frac{2}{3}s - 1$, which is attainable for some t in the range $0 < t \leq s - 5$ if $s \geq 12$.

Proof of claim 1. This inequality holds if and only if $3t^2 - 2st + 3t \geq 0$, which is equivalent to $t \geq \frac{2}{3}s - 1$ since t is positive. We finish by noticing that when $s \geq 12$, $s - 5 \geq \frac{2}{3}s - 1$. \square

Claim 2. $\alpha(t-1) \leq \alpha(t) \leq \beta(t)$.

Proof of claim 2. The first inequality holds since $\alpha(t)$ is an increasing function for $t \geq 1$, since $\alpha'(t) = \frac{2t-1}{2} > 0$ when $t \geq 1$, considering α as a continuous function on \mathbb{R} . The second inequality holds if and only if $5(s-t) \leq (s-t)^2$, which is always true since $t \leq s-5$. \square

Claim 3. $\alpha(t) \leq \beta(t-1)$ if and only if $t \leq \frac{2s+1-\sqrt{16s+1}}{2}$.

Proof of claim 3. This inequality holds if and only if $t^2 - (2s+1)t + s^2 - 3s \geq 0$, which occurs if and only if $t \leq \frac{2s+1-\sqrt{16s+1}}{2}$ or $t \geq \frac{2s+1+\sqrt{16s+1}}{2}$. However, $t \geq \frac{2s+1+\sqrt{16s+1}}{2}$ is rejected since $t < s$. \square

By claims 2 and 3, if $t \leq \frac{2s+1-\sqrt{16s+1}}{2}$, then $I(1) \cup \dots \cup I(t-1) \cup I(t)$ forms one closed interval. If $\lceil \frac{2}{3}s - 1 \rceil \leq \frac{2s+1-\sqrt{16s+1}}{2}$, then claim 1 implies that $\left[0, \frac{s^2-5s}{4}\right] \subseteq \bigcup_{t=1}^{\lceil \frac{2}{3}s-1 \rceil} I(t)$.

To obtain $\lceil \frac{2}{3}s - 1 \rceil \leq \frac{2s+1-\sqrt{16s+1}}{2}$, we look for integers s satisfying $\frac{2}{3}s \leq \frac{2s+1-\sqrt{16s+1}}{2}$, or equivalently, $3\sqrt{16s+1} \leq 2s+3$. This inequality holds if $33s \leq s^2$, or $s \geq 33$. \square

Theorem 3.2. *If $k \leq 2$, then for all integers b such that $1 \leq b \leq \binom{r+k}{r}$, there exists an (r, k, b) -matrix M .*

Proof. It is trivial for $k = 0$. If $k = 1$, then M is a column vector with the first $b-1$ entries 1's and the rest 0's.

If $k = 2$, let the first column of M have the first s entries 1's, the second column have the first s entries nonzero, and the rest be all 0's. Furthermore, assume that there are a_i i 's in the second column, $1 \leq i \leq s$, where $a_1 + a_2 + \dots + a_s = s$. Then the number of invertible square submatrices of M is

$$\begin{aligned} 1 + 2s + \sum_{i < j} a_i a_j &= 1 + 2s + \frac{1}{2}(\sum_i a_i)^2 - \frac{1}{2}(\sum_i a_i^2) \\ &= 1 + 2s + \frac{1}{2}s^2 - \frac{1}{2}(\sum_i a_i^2), \end{aligned}$$

and we would like to set it to be b , which gives $s + 2\binom{s+2}{2} - b = \sum_i a_i^2$.

By Lemma ??, if $0 \leq \binom{s+2}{2} - b \leq \frac{s^2-5s}{4}$, or equivalently $\frac{s^2+11s+4}{4} \leq b \leq \frac{s^2+3s+2}{2}$, there is a solution for a_i 's. It is easy to check that the intervals $[\frac{s^2+11s+4}{4}, \frac{s^2+3s+2}{2}]$ cover all integers $b \geq 39$, and the only missing integers are in $[1, 20] \cup [22, 26] \cup [29, 32] \cup [37, 38]$. Here, we finish the proof by constructing M explicitly for each of these b 's.

In the following table, $\mathbf{0}$ represents a column vector of all 0's (possibly of length 0), which fills up the column so that M has r rows.

$b =$	1	2	3	4	5	6	7	8
$M =$	$\begin{bmatrix} \mathbf{0} & \mathbf{0} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ 1 & 0 \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 1 & 0 \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 0 \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$

$b =$	9	10	11	12	13	14	15	16																																																																																																									
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Theorem 3.3. *Conjecture ?? holds for all $1 \leq b \leq \binom{r+2}{r}$.*

Proof. By Theorem ?? and Lemma ??, for all $1 \leq b \leq \binom{r+2}{r}$, for all r and k such that $\binom{r+k}{r} \geq b$, (r, k, b) -matrices and hence linear (n, r, b) -matroids exist. □

4 Existence of linear matroids with ample amount of bases

In Section ??, we used an induction argument together with computational exhaustion to show the existence of linear (n, r, b) -matroids with $1 \leq b \leq \binom{r+2}{r}$. In this section, we will establish an asymptotic existence result by studying a special class of matrices obtained from a generic hyperplane arrangement. A *generic hyperplane arrangement* is a finite collection $\mathcal{H} = \{\Gamma_1, \Gamma_2, \dots, \Gamma_\ell\}$ of $(k - 1)$ -dimensional linear subspaces of \mathbb{Q}^k satisfying

1. The hyperplanes Γ_j 's do not contain any coordinate directions, i.e. $\mathbf{e}_i \notin \Gamma_j$ for all $i = 1, 2, \dots, k$ and $j = 1, 2, \dots, \ell$, where $\mathbf{e}_i = (0, \dots, 1, \dots, 0)$ is the i -th coordinate vector.
2. No hyperplanes are of the form $\{(x_1, x_2, \dots, x_k) : x_i = x_j\}$ for some $i \neq j$.
3. For all $j = 1, 2, \dots, \min(k, \ell)$, the intersection of any j hyperplanes from \mathcal{H} is a $(k - j)$ -dimensional subspace.

Given a generic hyperplane arrangement \mathcal{H} together with integers $n_0, n_1, \dots, n_\ell \geq k$, a set of generic $(n_0, n_1, \dots, n_\ell)$ -vectors from \mathcal{H} is a collection of vectors, where n_j of them come from Γ_j and n_0 of them from the complement of the union of the hyperplanes, such that

1. No vector lies in more than one hyperplane.
2. Any $k - 1$ vectors are linearly independent.
3. Any k vectors are linearly dependent only when they are all chosen from the same hyperplane.

It is well known that such generic hyperplane arrangements and sets of generic vectors of any size always exist. The former can be shown by choosing normal vectors in general positions and the latter follows easily from an inductive construction.

Now, given nonnegative integers a_k, a_{k+1}, \dots, a_r with $\sum_{s=k}^r a_s s \leq r$, we first pick a generic hyperplane arrangement \mathcal{H} of size $\ell = \sum_{s=k}^r a_s$. Then we choose a set of generic $(n_0, n_1, \dots, n_\ell)$ -vectors from \mathcal{H} , where $n_0 = r - \sum_{s=k}^r a_s s$, and among n_1, n_2, \dots, n_ℓ , there are exactly a_s copies of s for each $s = k, k + 1, \dots, r$. If we put these vectors together as row vectors of a matrix, we get an $r \times k$ matrix M .

This construction builds an (r, k, b) -matrix M with $b = \binom{r}{k} - \bar{b}$, where $\bar{b} = \sum_{s=k}^r a_s \binom{r}{s}$ denotes the number of singular $r \times r$ submatrices in A , or the number of singular square submatrices in M . In the following proposition, we determine the range of \bar{b} that is achievable by our construction.

Proposition 4.1. *For each $k \geq 3$, there exists $r_0 \in \mathbb{N}$ such that for all integers $r \geq r_0$, and all $0 \leq \bar{b} \leq \binom{r+k-1}{k-1}$, there exist nonnegative integers a_k, a_{k+1}, \dots, a_r such that*

- (i) $\sum_{s=k}^r a_s \binom{r}{s} = \bar{b}$, and
- (ii) $\sum_{s=k}^r a_s s \leq r$.

Proof. Let κ be a real number greater than $(k^{\frac{1}{k}} + 1)^k$. Then $\kappa > (k^{\frac{1}{k}})^k + k \cdot k^{\frac{1}{k}} > 2k$.

Let $\bar{b}_0 = \bar{b}$. For each integer $i \geq 0$, if $\bar{b}_i \geq \binom{\kappa}{k}$, choose $s_i \in \mathbb{N}$ such that $\binom{s_i}{k} \leq \bar{b}_i < \binom{s_i+1}{k}$. Note that $1 \leq \bar{b}_i / \binom{s_i}{k} < \binom{s_i+1}{k} / \binom{s_i}{k}$ which is less than 2 since $\binom{s_i+1}{k} \geq 2\binom{s_i}{k}$ if and only if $s_i \leq 2k - 1$. Let $a_{s_i} = 1$, and $\bar{b}_{i+1} = \bar{b}_i - \binom{s_i}{k}$. If $\bar{b}_i < \binom{\kappa}{k}$ for some $i \geq 0$, then let $a_k = \bar{b}_i$ and all other undetermined a_s be 0.

From this definition, condition (i) is clearly satisfied. For condition (ii), $\sum_{s=k}^r a_s s = s_0 + s_1 + \cdots + s_m + \bar{b}_{m+1}$ for some $m \geq 0$. To give an upper bound to this sum, let $f_k(x) = \binom{x}{k}$ and $f_{k-1}(x) = \binom{x}{k-1}$. Note that they are both strictly increasing functions when $x \geq k$, so $f_k^{-1} f_{k-1}$ is also a strictly increasing function.

For all $i \geq 0$, $\bar{b}_{i+1} = \bar{b}_i - \binom{s_i}{k} < \binom{s_i+1}{k} - \binom{s_i}{k} = \binom{s_i}{k-1}$. Hence, $s_0 \leq (f_k^{-1} f_{k-1})(r + k - 1)$ and $s_{i+1} \leq f_k^{-1}(\bar{b}_{i+1}) < (f_k^{-1} f_{k-1})(s_i)$. When $x \geq \kappa > 2k$,

$$f_k^{-1} f_{k-1}(x) < f_k^{-1}\left(\frac{x^{k-1}}{(k-1)!}\right) = f_k^{-1}\left(\frac{(k^{1/k} x^{(k-1)/k})^k}{k!}\right) < k^{\frac{1}{k}} x^{\frac{k-1}{k}} + k - 1,$$

which is less than $(k^{\frac{1}{k}} + 1)x^{\frac{k-1}{k}}$ when $x > (k-1)^{\frac{k}{k-1}}$. Since $2k-1 = (1+1)^{k-1} > 1+k-1 > k-1$, we have $(k-1)^{\frac{1}{k-1}} < 2$, implying $2k > (k-1)^{\frac{k}{k-1}}$. Therefore,

$$f_k^{-1} f_{k-1}(x) < (k^{\frac{1}{k}} + 1)x^{\frac{k-1}{k}}$$

for all $x \geq 2k$. In particular,

$$s_0 < (k^{\frac{1}{k}} + 1)(r + k - 1)^{\frac{k-1}{k}}, \quad (1)$$

and for $i \geq 1$,

$$\begin{aligned} s_i &\leq (f_k^{-1} f_{k-1})^{i+1}(r + k - 1) < (k^{\frac{1}{k}} + 1)^{1 + \frac{k-1}{k} + (\frac{k-1}{k})^2 + \cdots + (\frac{k-1}{k})^i} (r + k - 1)^{(\frac{k-1}{k})^{i+1}} \\ &\leq (k^{\frac{1}{k}} + 1)^k (r + k - 1)^{(\frac{k-1}{k})^{i+1}}. \end{aligned}$$

As $(f_k^{-1} f_{k-1})(s_m) < \kappa$, we would like to find the minimum integer m' such that

$$(k^{\frac{1}{k}} + 1)^k (r + k - 1)^{(\frac{k-1}{k})^{m'+2}} \leq \kappa.$$

This is equivalent to

$$(m' + 2) \log\left(\frac{k-1}{k}\right) \leq \log \log\left(\frac{\kappa}{(k^{1/k} + 1)^k}\right) - \log \log(r + k - 1),$$

or

$$m' \geq \left(\log \log(r + k - 1) - \log \log\left(\frac{\kappa}{(k^{1/k} + 1)^k}\right) \right) / \log\left(\frac{k}{k-1}\right) - 2. \quad (2)$$

Combining (??) and (??), we have $s_0 + s_1 + \cdots + s_m + \bar{b}_{m+1} \leq m s_0 + \binom{\kappa}{k} \leq m' s_0 + \binom{\kappa}{k} = O(r^{\frac{k-1}{k}} \log \log r)$ for fixed k , which is less than r when r is large. \square

This gives the following immediate asymptotic result on the existence of linear (n, r, b) -matroids.

Theorem 4.2. For each fixed $k \geq 3$, let $r_0 \in \mathbb{N}$ be as defined in Proposition ???. Then for all integers $r \geq r_0$, for all integers b such that $b \geq \binom{r_0+k-1}{k}$, linear (n, r, b) -matroids always exist, where $n = r + k$.

Proof. By Proposition ??, for every integer $r \geq r_0$, for all integers b such that $\binom{r+k-1}{k} = \binom{r+k}{k} - \binom{r+k-1}{k-1} \leq b \leq \binom{r+k}{k}$, we can construct an (r, k, b) -matrix following the procedures introduced at the beginning of Section ??. Finally, we are done by noticing that the intervals $[\binom{r_0+k-1+i}{k}, \binom{r_0+k+i}{k}]$, $i = 0, 1, \dots, r - r_0$, cover all integers $\binom{r_0+k-1}{k} \leq b \leq \binom{n}{r}$. \square

In fact, we can remove the lower bound on the values of b in Theorem ??.

Theorem 4.3. For each fixed $k \geq 3$, there exists $R \in \mathbb{N}$ such that for all $r \geq R$, linear (n, r, b) -matroids always exist, where $n = r + k$.

The proof of this theorem will be shown at the end of Section ??.

5 Existence of linear matroids with corank at most 3

In this section, we will show that when $k = 3$, there always exists a linear $(r+3, r, b)$ -matroid except $(n, r, b) = (6, 3, 11)$. For $4 \leq r \leq 10$, the existence of an $(r, 3, b)$ -matrix will be verified through explicit constructions, which is described in Appendix ??. Then we will base on the method employed in Proposition ?? and modify it into an inductive argument.

Proposition 5.1. For each $r \geq 49$ and for all $0 \leq \bar{b} \leq \binom{r+2}{2}$, there exist nonnegative integers a_3, a_4, \dots, a_r such that

$$(i) \sum_{s=3}^r a_s \binom{s}{3} = \bar{b}, \text{ and}$$

$$(ii) \sum_{s=3}^r a_s s \leq r.$$

Proof. Let the statement of the proposition be denoted by $\mathcal{P}(r)$ for $r \geq 49$. In Appendix ??, we show that $\mathcal{P}(r)$ holds for $49 \leq r \leq 203$. Suppose that $\mathcal{P}(r)$ is true for all $49 \leq r \leq r_0$ for some $r_0 \geq 203$. We would like to check the validity of $\mathcal{P}(r_0 + 1)$.

Let \bar{b} be an integer such that $0 \leq \bar{b} \leq \binom{r_0+3}{2}$. If $\bar{b} \leq \binom{r_0+2}{2}$, we can apply $\mathcal{P}(r_0)$ to \bar{b} to obtain nonnegative integers a_3, a_4, \dots, a_{r_0} and $a_{r_0+1} = 0$ such that

$$(i) \sum_{s=3}^{r_0+1} a_s \binom{s}{3} = \bar{b}, \text{ and}$$

$$(ii) \sum_{s=3}^{r_0+1} a_s s \leq r_0 \leq r_0 + 1,$$

so it suffices to consider $\binom{r_0+2}{2} < \bar{b} \leq \binom{r_0+3}{2}$. Note that there exists a unique integer s_0 such that $\binom{s_0}{3} \leq \bar{b} < \binom{s_0+1}{3}$.

Set $\bar{b}' := \bar{b} - \binom{s_0}{3}$. If $\bar{b}' = 0$, we are done. If $\bar{b}' \geq 1$, we have $1 \leq \bar{b}' < \binom{s_0+1}{3} - \binom{s_0}{3} = \binom{s_0}{2}$. Clearly, $s_0 < r_0 + 2$, so we can apply the induction hypothesis $\mathcal{P}(s_0 - 2)$ to \bar{b}' as long as $s_0 \geq 51$. From this, we can find nonnegative integers $a_3, a_4, \dots, a_{s_0-2}$ with

$$(i) \sum_{s=3}^{s_0-2} a_s \binom{s}{3} = \bar{b}', \text{ and}$$

$$(ii) \sum_{s=3}^{s_0-2} a_s s \leq s_0 - 2.$$

By setting $a_{s_0-1} = 0$, $a_{s_0} = 1$, and $a_{s_0+1} = \dots = a_{r_0+1} = 0$, we have

$$(i) \sum_{s=3}^{r_0+1} a_s \binom{s}{3} = \bar{b}, \text{ and}$$

$$(ii) \sum_{s=3}^{r_0+1} a_s s \leq 2s_0 - 2.$$

It suffices to show that $s_0 \geq 51$ and $2s_0 - 2 \leq r_0 + 1$. First, observe that for $r_0 \geq 203$, we have

$$\binom{s_0+1}{3} > \bar{b} > \binom{r_0+2}{2} \geq \binom{205}{2} = 20910,$$

or equivalently, $(s_0 + 1)s_0(s_0 - 1) > 125460$. A straight forward calculation shows that it holds if and only if $s_0 \geq 51$.

Next, for any integer $x \geq 1$,

$$\begin{aligned} \binom{x+3}{3} - \binom{x+3}{2} &= \frac{(x+3)(x+1)(x-1)}{48} - \frac{(x+3)(x+2)}{2} \\ &= \frac{1}{48}(x+3)(x^2 - 24x - 49) \geq 0 \end{aligned}$$

if and only if $x^2 - 24x - 49 \geq 0$. By solving the quadratic inequality, it is easy to see that it holds for all integers $x \geq 26$. Since $r_0 \geq 203$ and $\binom{x}{3}$ is a strictly increasing function for $x \geq 3$, we have

$$\binom{s_0}{3} \leq \bar{b} \leq \binom{r_0+3}{2} \leq \binom{\frac{r_0+3}{2}}{3},$$

which implies $\frac{r_0+3}{2} \geq s_0$ or $2s_0 - 2 \leq r_0 + 1$. □

To complete the case for $k = 3$, we still need to consider $r = 3$ and $11 \leq r \leq 48$. When $r = 3$, Theorem ?? implies that linear $(6, 3, b)$ -matroid exists for $1 \leq b \leq 10$, and the following constructions produce the $(3, 3, b)$ -matrices M , where $12 \leq b \leq 20$.

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As for $11 \leq r \leq 48$, we have to slightly modify the generic hyperplane arrangement construction as described in Section ?? to reduce the number of rows r needed to produce a matrix M with a prescribed number of singular submatrices \bar{b} . For simplicity, we will only show the construction for $k = 3$, but the corresponding one for general k is similar. There are three types of modification:

1. Allow some of the 2-planes to contain exactly one coordinate direction, i.e. 2-planes with equations $ax + by = 0$, $by + cz = 0$, or $ax + cz = 0$, where $a, b, c \neq 0$, and pick vectors from these 2-planes on the same line parallel but distinct to the axis. Each of these modified 2-planes increases \bar{b} by changing a summand of $\binom{s}{3}$ to $\binom{s+1}{3}$. In other words, we save one row every time we modify a 2-plane.
2. Take one to three 2-planes to be those orthogonal to a coordinate axis, i.e. $x = 0$, $y = 0$, or $z = 0$. The resulting \bar{b} will increase by changing a summand of $\binom{s}{3}$ to $\binom{s+2}{3}$. In other words, we can save two rows for up to three times.
3. For each vector chosen from the complement of the union of hyperplanes, we duplicate them t times in the matrix M . Each such duplication will increase the resulting \bar{b} by $\binom{t}{3} + \binom{t}{2}(r - t + 3)$.

Proposition 5.2. *For each r such that $11 \leq r \leq 48$, and for each $0 \leq \bar{b} \leq \binom{r+2}{2}$, there exists an $(r, 3, \binom{r+3}{r} - \bar{b})$ -matrix M .*

Proof. For each r and \bar{b} such that $11 \leq r \leq 48$ and $0 \leq \bar{b} \leq \binom{r+2}{2}$, let $\bar{b}_0 = \bar{b}$. For each integer $i \geq 0$, let t_i be largest integer such that $\binom{t_i}{3} + \binom{t_i}{2}(r - t_i + 3) \leq \bar{b}_i$, let $c_i = \lfloor \bar{b}_i / (\binom{t_i}{3} + \binom{t_i}{2}(r - t_i + 3)) \rfloor$, and let $\bar{b}_{i+1} = \bar{b}_i - c_i(\binom{t_i}{3} + \binom{t_i}{2}(r - t_i + 3))$. We repeat the process until $\bar{b}_{i+1} < \binom{2}{3} + \binom{2}{2}(r - 2 + 3) = r + 1$. In this process, we are building an $r \times 3$ matrix M using only rows from the Type 3 modification.

After using only rows from Type 3 modification, we still need to pick appropriate rows to give an additional \bar{b}_{i+1} singular submatrices if $\bar{b}_{i+1} > 0$. Note that $\bar{b}_{i+1} \leq r \leq 48$. The following is a list of partitions of integers from 1 to 48. Each summand $\binom{s+2}{3}$ corresponds to s rows from Type 2 modification, and summand $\binom{s+1}{3}$ corresponds to s rows from Type 1 modification.

$$\begin{aligned}
1 &= \binom{1+2}{3} \\
2 &= \binom{1+2}{3} + \binom{1+2}{3} \\
3 &= \binom{1+2}{3} + \binom{1+2}{3} + \binom{1+2}{3} \\
4 &= \binom{2+2}{3} + \binom{1+2}{3} \\
5 &= \binom{2+2}{3} + \binom{1+2}{3} \\
6 &= \binom{2+2}{3} + \binom{1+2}{3} + \binom{1+2}{3} \\
7 &= \binom{2+2}{3} + \binom{1+2}{3} + \binom{1+2}{3} + \binom{2+1}{3} \\
8 &= \binom{2+2}{3} + \binom{2+2}{3} \\
9 &= \binom{2+2}{3} + \binom{2+2}{3} + \binom{1+2}{3} \\
10 &= \binom{3+2}{3} \\
11 &= \binom{3+2}{3} + \binom{1+2}{3} \\
12 &= \binom{3+2}{3} + \binom{1+2}{3} + \binom{1+2}{3} \\
13 &= \binom{3+2}{3} + \binom{1+2}{3} + \binom{1+2}{3} + \binom{2+1}{3} \\
14 &= \binom{3+2}{3} + \binom{2+2}{3} \\
15 &= \binom{3+2}{3} + \binom{2+2}{3} + \binom{1+2}{3} \\
16 &= \binom{3+2}{3} + \binom{2+2}{3} + \binom{1+2}{3} + \binom{2+1}{3} \\
17 &= \binom{3+2}{3} + \binom{2+2}{3} + \binom{1+2}{3} + \binom{2+1}{3} + \binom{2+1}{3} \\
18 &= \binom{3+2}{3} + \binom{2+2}{3} + \binom{2+2}{3} \\
19 &= \binom{3+2}{3} + \binom{2+2}{3} + \binom{2+2}{3} + \binom{2+1}{3} \\
20 &= \binom{4+2}{3} \\
21 &= \binom{4+2}{3} + \binom{1+2}{3} \\
22 &= \binom{4+2}{3} + \binom{1+2}{3} + \binom{1+2}{3} \\
23 &= \binom{4+2}{3} + \binom{1+2}{3} + \binom{1+2}{3} + \binom{2+1}{3} \\
24 &= \binom{4+2}{3} + \binom{2+2}{3} \\
25 &= \binom{4+2}{3} + \binom{2+2}{3} + \binom{1+2}{3} \\
26 &= \binom{4+2}{3} + \binom{2+2}{3} + \binom{1+2}{3} + \binom{2+1}{3} \\
27 &= \binom{4+2}{3} + \binom{2+2}{3} + \binom{1+2}{3} + \binom{2+1}{3} + \binom{2+1}{3} \\
28 &= \binom{4+2}{3} + \binom{2+2}{3} + \binom{2+2}{3} \\
29 &= \binom{4+2}{3} + \binom{2+2}{3} + \binom{2+2}{3} + \binom{2+1}{3} \\
30 &= \binom{4+2}{3} + \binom{3+2}{3} \\
31 &= \binom{4+2}{3} + \binom{3+2}{3} + \binom{1+2}{3} \\
32 &= \binom{4+2}{3} + \binom{3+2}{3} + \binom{1+2}{3} + \binom{2+1}{3} \\
33 &= \binom{4+2}{3} + \binom{3+2}{3} + \binom{1+2}{3} + \binom{2+1}{3} + \binom{2+1}{3} \\
34 &= \binom{4+2}{3} + \binom{3+2}{3} + \binom{2+2}{3} \\
35 &= \binom{5+2}{3} \\
36 &= \binom{5+2}{3} + \binom{1+2}{3} \\
37 &= \binom{5+2}{3} + \binom{1+2}{3} + \binom{1+2}{3} \\
38 &= \binom{5+2}{3} + \binom{1+2}{3} + \binom{1+2}{3} + \binom{2+1}{3} \\
39 &= \binom{5+2}{3} + \binom{2+2}{3} \\
40 &= \binom{5+2}{3} + \binom{2+2}{3} + \binom{1+2}{3} \\
41 &= \binom{5+2}{3} + \binom{2+2}{3} + \binom{1+2}{3} + \binom{2+1}{3} \\
42 &= \binom{5+2}{3} + \binom{2+2}{3} + \binom{1+2}{3} + \binom{2+1}{3} + \binom{2+1}{3} \\
43 &= \binom{5+2}{3} + \binom{2+2}{3} + \binom{2+2}{3} \\
44 &= \binom{5+2}{3} + \binom{2+2}{3} + \binom{2+2}{3} + \binom{2+1}{3} \\
45 &= \binom{5+2}{3} + \binom{3+2}{3} \\
46 &= \binom{5+2}{3} + \binom{3+2}{3} + \binom{1+2}{3} \\
47 &= \binom{5+2}{3} + \binom{3+2}{3} + \binom{1+2}{3} + \binom{2+1}{3} \\
48 &= \binom{5+2}{3} + \binom{3+2}{3} + \binom{1+2}{3} + \binom{2+1}{3} + \binom{2+1}{3}
\end{aligned}$$

From this table, we can deduce directly the number of additional rows from Type 1 and Type 2 modifications we need to obtain \bar{b} singular square submatrices in M . Together with the rows from Type 3 modification, it remains to verify that the total number of rows we have used is at most r . Once again, we employ Mathematica to finish the verification, and the program codes are provided in Appendix ?? for reference. \square

Theorem 5.3. *Linear (n, r, b) -matroids exist for all $1 \leq b \leq \binom{r+3}{r}$, as long as $\binom{n}{r} \geq b$, except $(n, r, b) = (6, 3, 11)$.*

Proof. Propositions ?? and ?? imply that for all $r \geq 11$, linear (n, r, b) -matroids exist for all $\binom{r+2}{3} = \binom{r+3}{3} - \binom{r+2}{2} \leq b \leq \binom{r+3}{3}$. Hence, we are done by connecting with the explicit constructions for $3 \leq r \leq 10$. \square

Now, we have all the tools for proving Theorem ??.

Proof of Theorem ??. For each $i = 0, 1, 2, \dots, k-3$, let $r_0(k-i) \in \mathbb{N}$ be the constants obtained by applying Proposition ?? on $k-i$. Note that $r_0(k) \geq r_0(k-1) \geq \dots \geq r_0(3)$. Let $R_0 = r_0(k)$. Then there exist integers R_1, R_2, \dots, R_{k-3} such that

1. $R_0 \leq R_1 \leq R_2 \leq \dots \leq R_{k-3}$, and

2. $\binom{R_i+k-i-1}{k-i} \leq \binom{R_{i+1}+k-(i+1)}{k-(i+1)}$ for all $i = 0, 1, 2, \dots, k-4$.

Let $R = R_{k-3}$, and fix $r \geq R$. By Theorem ??, an (r, k, b) -matrix exists for all integers b such that $\binom{R_0+k-1}{k} \leq b \leq \binom{r+k}{r}$. By Proposition ??, for each $i = 1, 2, \dots, k-3$, since $R_i \geq r_0(k-i)$, an $(R_i, k-i, b)$ -matrix exists for all integers b such that $\binom{R_i+k-i-1}{k-i} \leq b \leq \binom{R_i+k-i}{k-i}$. By Lemma ??, for each $i = 1, 2, \dots, k-3$, an (r, k, b) -matrix exists for all b such that $\binom{R_i+k-i-1}{k-i} \leq b \leq \binom{R_i+k-i}{k-i}$. Our definition of R_i 's implies that $[(\binom{R_0+k-1}{k}, \binom{r+k}{r})] \cup \bigcup_{i=1}^{k-3} [(\binom{R_i+k-i-1}{k-i}, \binom{R_i+k-i}{k-i})]$ covers all integers $\binom{R+2}{3} \leq b \leq \binom{r+k}{r}$. Therefore, an (r, k, b) -matrix exists for all $b \geq \binom{R+2}{3}$.

Finally, we are done by Theorem ??, which says an (r, k, b) -matrix exists for all $b \leq \binom{R+3}{3}$. \square

6 Conclusion and remarks

In our treatment, we always assume the base field is \mathbb{Q} . In fact, the same argument works for any infinite field. In particular, if we consider the algebraic closure $\overline{\mathbb{F}_p}$ for some prime p , the constructed (r, k, b) -matrix naturally descends to some finite extension of \mathbb{F}_p . However, we cannot ensure there is a fixed finite extension which captures all of them. In any case, the matroid structure arising from matrices will not be affected.

All our results focus on the situation when n and r are very close together. Recall from Lemma ??, we only need to consider $r \leq n \leq 2r$. Hence, our next goal is to investigate the case when n is close to $2r$. In view of the counterexample of the non-existence of $(6, 3, 11)$ -matroids, this latter goal should be much harder. But the general direction towards a complete solution to Conjecture ?? will be another asymptotic result for the existence of (r, k, b) -matrices for large enough k which should isolate a finite number of cases for direct checking.

A Construction of $(r+3, 3, b)$ -matroids for $4 \leq r \leq 10$

If we can construct a matrix $A' = (I_{r'}|M')$, where M' is an $r' \times 3$ matrix, such that A' has exactly b invertible $r' \times r'$ submatrices, then for all $r \geq r'$, the matrix $A = (I_r|M)$, where M is obtained by appending $r - r'$ rows of all zeros to M' , has exactly b invertible $r \times r$ submatrices. Hence, for $r \geq 5$, we only need to construct matrices with b invertible $r \times r$ submatrices, where $\binom{r+2}{3} < b \leq \binom{r+3}{3}$.

Theorem ?? implies that when $r = 4$ and $1 \leq b \leq \binom{r+2}{2} = 15$, there exists a matrix A of size $r \times (r+3)$ such that the number of invertible $r \times r$ submatrices is b . When $r = 4$ and $15 < b \leq \binom{r+3}{3} = 35$, we give the construction of $A = (I_r|M)$ as follows.

$b =$	16	17	18	19	20	21
$M =$	$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

$b =$	22	23	24	25	26	27	28
$M =$	$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$

$b =$	29	30	31	32	33	34	35
$M =$	$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 0 \\ 0 & 1 & 2 \\ 2 & 1 & 2 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 2 \\ 2 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 2 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \\ 2 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 4 \\ 2 & 1 & 2 \\ 3 & 3 & 2 \end{bmatrix}$

When $r = 5$ and $35 < b \leq \binom{r+3}{3} = 56$, we give the construction of $A = (I_r | M)$ as follows.

$b =$	36	37	38	39	40	41	42
$M =$	$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 0 & 0 \\ 1 & 1 & 1 \\ 2 & 1 & 3 \\ 3 & 1 & 3 \end{bmatrix}$	$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 1 & 2 \\ 2 & 3 & 0 \\ 2 & 3 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 1 & 3 \\ 2 & 1 & 3 \\ 2 & 2 & 2 \\ 4 & 4 & 4 \end{bmatrix}$	$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 4 \\ 0 & 2 & 2 \\ 1 & 2 & 0 \\ 3 & 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 3 \\ 0 & 3 & 3 \\ 1 & 1 & 3 \\ 2 & 1 & 3 \end{bmatrix}$	$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 0 \\ 3 & 3 & 2 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 2 & 2 \\ 2 & 3 & 0 \\ 2 & 3 & 3 \end{bmatrix}$

$b =$	43	44	45	46	47	48	49
$M =$	$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 3 & 2 \\ 1 & 4 & 0 \\ 2 & 1 & 0 \\ 2 & 2 & 2 \end{bmatrix}$	$\begin{bmatrix} 0 & 2 & 3 \\ 0 & 2 & 3 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \\ 4 & 0 & 2 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 3 \\ 2 & 0 & 1 \\ 2 & 0 & 2 \\ 2 & 4 & 3 \\ 3 & 2 & 3 \end{bmatrix}$	$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 2 \\ 1 & 2 & 1 \\ 1 & 2 & 3 \\ 3 & 2 & 4 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 & 3 \\ 0 & 2 & 2 \\ 1 & 1 & 3 \\ 2 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 & 3 \\ 1 & 2 & 3 \\ 2 & 1 & 3 \\ 2 & 3 & 1 \\ 2 & 3 & 1 \end{bmatrix}$	$\begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & 3 \\ 1 & 2 & 2 \\ 1 & 3 & 4 \\ 3 & 3 & 1 \end{bmatrix}$

$b =$	50	51	52	53	54	55	56
$M =$	$\begin{bmatrix} 0 & 1 & 3 \\ 0 & 2 & 2 \\ 1 & 0 & 3 \\ 1 & 1 & 0 \\ 3 & 1 & 4 \end{bmatrix}$	$\begin{bmatrix} 1 & 2 & 2 \\ 1 & 3 & 3 \\ 2 & 3 & 3 \\ 3 & 1 & 2 \\ 4 & 4 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 3 & 3 \\ 1 & 3 & 4 \\ 2 & 3 & 3 \\ 3 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 2 & 2 \\ 1 & 0 & 3 \\ 1 & 1 & 2 \\ 1 & 4 & 2 \\ 2 & 3 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 2 & 1 \\ 1 & 1 & 2 \\ 1 & 1 & 4 \\ 1 & 2 & 3 \\ 1 & 3 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 2 & 2 \\ 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 1 & 2 \\ 4 & 3 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 2 & 4 \\ 2 & 3 & 4 \\ 3 & 2 & 2 \\ 3 & 4 & 3 \\ 4 & 2 & 3 \end{bmatrix}$

When $r = 6$ and $56 < b \leq \binom{r+3}{3} = 84$, we give the construction of $A = (I_r|M)$ as follows.

$b =$	57	58	59	60	61	62	63																																																																																																																														
$M =$	<table border="1"> <tr><td>3</td><td>0</td><td>0</td></tr> <tr><td>3</td><td>0</td><td>0</td></tr> <tr><td>1</td><td>1</td><td>2</td></tr> <tr><td>2</td><td>1</td><td>0</td></tr> <tr><td>2</td><td>4</td><td>4</td></tr> <tr><td>3</td><td>2</td><td>2</td></tr> </table>	3	0	0	3	0	0	1	1	2	2	1	0	2	4	4	3	2	2	<table border="1"> <tr><td>1</td><td>0</td><td>0</td></tr> <tr><td>2</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>2</td><td>2</td></tr> <tr><td>1</td><td>3</td><td>0</td></tr> <tr><td>3</td><td>1</td><td>2</td></tr> <tr><td>4</td><td>0</td><td>2</td></tr> </table>	1	0	0	2	0	0	0	2	2	1	3	0	3	1	2	4	0	2	<table border="1"> <tr><td>1</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>1</td><td>1</td></tr> <tr><td>2</td><td>2</td><td>0</td></tr> <tr><td>3</td><td>2</td><td>0</td></tr> <tr><td>3</td><td>3</td><td>2</td></tr> <tr><td>4</td><td>3</td><td>0</td></tr> </table>	1	0	0	0	1	1	2	2	0	3	2	0	3	3	2	4	3	0	<table border="1"> <tr><td>1</td><td>0</td><td>0</td></tr> <tr><td>3</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>2</td><td>4</td></tr> <tr><td>3</td><td>1</td><td>2</td></tr> <tr><td>4</td><td>1</td><td>3</td></tr> <tr><td>4</td><td>2</td><td>2</td></tr> </table>	1	0	0	3	0	0	0	2	4	3	1	2	4	1	3	4	2	2	<table border="1"> <tr><td>1</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>3</td><td>2</td></tr> <tr><td>0</td><td>3</td><td>2</td></tr> <tr><td>1</td><td>1</td><td>4</td></tr> <tr><td>1</td><td>2</td><td>0</td></tr> <tr><td>3</td><td>2</td><td>0</td></tr> </table>	1	0	0	0	3	2	0	3	2	1	1	4	1	2	0	3	2	0	<table border="1"> <tr><td>1</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>1</td><td>2</td></tr> <tr><td>0</td><td>2</td><td>2</td></tr> <tr><td>0</td><td>3</td><td>4</td></tr> <tr><td>1</td><td>0</td><td>1</td></tr> <tr><td>1</td><td>1</td><td>1</td></tr> </table>	1	0	0	0	1	2	0	2	2	0	3	4	1	0	1	1	1	1	<table border="1"> <tr><td>2</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>1</td><td>1</td></tr> <tr><td>0</td><td>3</td><td>3</td></tr> <tr><td>1</td><td>2</td><td>4</td></tr> <tr><td>1</td><td>4</td><td>4</td></tr> <tr><td>3</td><td>1</td><td>3</td></tr> </table>	2	0	0	0	1	1	0	3	3	1	2	4	1	4	4	3	1	3
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When $r = 7$ and $84 < b \leq \binom{r+3}{3} = 120$, we give the construction of $A = (I_r|M)$ as follows.

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When $r = 8$ and $120 < b \leq \binom{r+3}{3} = 165$, we give the construction of $A = (I_r|M)$ as follows.

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B Proof of Proposition ?? for $49 \leq r \leq 203$

We use Mathematica to check the validity of Proposition ?? for $49 \leq r \leq 203$.

```

IntDiv[n_, d_] := Block[{}, (n - Mod[n, d])/d];
Do[bin = Table[Binomial[s, 3], {s, r}]; a = Table[0, {s, r}];
  Do[btemp = bbar;
    Do[a[[s]] = IntDiv[btemp, bin[[s]]];
      btemp = Mod[btemp, bin[[s]]], {s, r, 3, -1}];
    If[(Sum[a[[s]]*s, {s, 3, r}]) > r,
      Print[r, " ", bbar, " ", Sum[a[[s]]*s, {s, 3, r}]],
      {bbar, 0, Binomial[r + 2, 2]}],
  {r, 49, 203}]

```

As there is no output after these lines finish running, we finish our verification.

C Proof of Proposition ?? for $11 \leq r \leq 48$

From the list in Proposition ??, we use `leng` below to record the number of rows from Type 1 and Type 2 modifications to obtain additional singular submatrices.

```
leng = {0, 1, 2, 3, 2, 3, 4, 6, 4, 5, 3, 4, 5, 7, 5, 6, 8, 10, 7, 9,
4, 5, 6, 8, 6, 7, 9, 11, 8, 10, 7, 8, 10, 12, 9, 5, 6, 7, 9, 7, 8,
10, 12, 9, 11, 8, 9, 11, 13};
Do[tbin = Table[Binomial[t, 3] + Binomial[t, 2] (r - t + 3), {t, r}];
  a = Table[0, {s, r}]; max = 0;
  Do[btemp = bbar;
    Do[a[[s]] = IntDiv[btemp, tbin[[s]]];
      btemp = Mod[btemp, tbin[[s]], {s, r, 2, -1}];
      If[Sum[a[[s]]*s, {s, 2, r}] + leng[[btemp + 1]] > max,
        max = Sum[a[[s]]*s, {s, 2, r}] + leng[[btemp + 1]],
        {bbar, Binomial[r + 2, 2]}];
      If[r < max, Print[r, " ", max]], {r, 11, 48}]
```

As there is no output after these lines finish running, we finish our verification.

References

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- [3] D. Welsh, Combinatorial problems in matroid theory, *Combinatorial Math. and its Applications (Proc. Conf. Oxford 1969)*, Academic Press, London, 1971.