Play Ball - Equally: Math Programming Lends a Hand to Little League Baseball

Francis J. Vasko
Mathematics & CIS Department, Kutztown UniversityKutztown, PA 19530

Recently a friend asked me if mathematics could be used to solve a problem that happens all too often with Little League baseball teams. With the current Little League season ending, he was frustrated with the problem of trying to get each team member to play about the same amount of time. He felt strongly that Little League baseball should be about developing the skills of all the team members, and that not just the best players should be on the field most of the time with the players who really need the practice and experience sitting on the bench.

He explained to me that each of twelve members on the team had several positions he or she wanted to or could play. The specific positions for each player are given in Table 1. The number after each position is a measure of how desirable it is to have that team member play that particular position. The higher the number the more desirable it is to have that team member play that particular position.

TABLE 1

TEAM MEMBER	PLAYABLE POSITIONS
#1	PITCHER (8), SECOND BASE (4), EXTRA HITTER
#2	PITCHER (5), SHORT STOP (2), THIRD BASE (8), EXTRA HITTER
#3	PITCHER (2), RIGHT FIELD (3), CENTER FIELD (8), LEFT FIELD (4), EXTRA HITTER
#4	FIRST BASE (9), RIGHT FIELD (2), CENTER FIELD (3), LEFT FIELD (4), EXTRA HITTER
#5	SECOND BASE (7), RIGHT FIELD (2), CENTER FIELD (2), LEFT FIELD (2), EXTRA HITTER
#6	FIRST BASE (4), SHORT STOP (7), THIRD BASE (3), EXTRA HITTER
#7	PITCHER (4), FIRST BASE (7), EXTRA HITTER
#8	CATCHER (8), SHORT STOP (5), THIRD BASE (2), EXTRA HITTER
#9	CATCHER (6), RIGHT FIELD (3), CENTER FIELD (3), LEFT FIELD (3), EXTRA HITTER
#10	PITCHER (9), THIRD BASE (2), EXTRA HITTER
#11	PITCHER (3), SHORT STOP (7), THIRD BASE (3), EXTRA HITTER
#12	CATCHER (4), SECOND BASE (4), RIGHT FIELD (4), CENTER FIELD (3), LEFT FIELD (2), EXTRA HITTER

He further explained that the season consisted of 17 games with each game split into two half games. Players typically played for a complete half game. Even if, for example, a pitcher was having difficulty he would be switched with another player on the field as opposed to being taken out of the game. If necessary, players could be replaced half way through the game. Also, in addition to the traditional nine field positions there was a tenth non-fielding position referred to as the extra hitter (EH) position. The EH got to bat but did not play the field.

With 17 games per season (34 half-games), and since there are 10 positions and 12 team members, each team member should ideally play (10/12)34 = 28 1/3 half-games. Since ball players typically play complete half-games, the best that could be achieved is to make sure that each of the 12 players plays at least 28 half-games and no more than 29 half-games. Also, each player had to be assigned only to positions he or she could or wanted to play (Table 1). In addition to these constraints, each of the ten positions must be assigned to a ball player for each of the 34 half-games. Finally, it would be nice to be able to make the assignments such that certain player-position pairings could be favored. For example, if a particular player was a great pitcher, within the limitations outlined above, we might want to favor that player getting to pitch as much as possible.

In order to express these limitations or constraints algebraically, we will need to introduce some variables. Recall that we are trying to assign team members to positions in an equitable manner over the duration of the baseball season. Therefore, we define one variable for each feasible team member-position pair. The value of the variable indicates how many times a team member plays a particular position in a season. Since there are a total of 51 feasible player-position pairings given in Table 1, there are a total of 51 variables required.

Essentially, we have outlined an integer linear program. The mathematical formulation based on the above discussion is now given.

LL BASEBALL FORMULATION 1:

Maximize $\sum_i \sum_j p_{ij} x_{ij}$

Subject to:

$\sum_j a_{ij}x_{ij} \leq \text{MAXGAMES} \quad i = 1,\ldots,12$ (1)

$\sum_j a_{ij}x_{ij} \geq \text{MINGAMES} \quad i = 1,\ldots,12$ (2)

$\sum_i a_{ij}x_{ij} = \text{NUMGAMES} \quad j = 1,\ldots,10$ (3)

x_{ij} nonnegative integer (4)

where x_{ij} is the number of times team member i plays position j. The a_{ij} coefficient is 1, if team member i is capable of playing position j and 0 otherwise. The p_{ij} coefficients represent the benefit of having team member i play position j. MINGAMES and MAXGAMES are the minimum and maximum number of half-games, respectively, that a player can play in a season. NUMGAMES is the number of half-games in a season. Constraint set (1) ensures that no team member plays more than the maximum number of games allowed per player. Constraint set (2) ensures that each team member plays at least the minimum number of games required for each team member. Constraint set (3) ensures that each position is assigned to a team member for each half-game of the season. Thus, problem one has 51 integer variables and 34 constraints.

With 17 games per season (34 half-games), the value of NUMGAMES was set at 34. As stated previously, since there are 10 positions and 12 team members, each team member should play (10/12)34 = 28 1/3 half-games. Hence, MINGAMES = 28 and MAXGAMES = 29.

Upon solving problem one using standard spreadsheet software on a PC, the solution was feasible with all players playing in either 28 or 29 half-games. However, as can be seen from Table 2, there were some obvious problems with this solution. First of all, team members #1 and #11 were assigned to the extra hitter position for 17 half-games. This meant they would only get to play fielding positions for 11 half-games which was unacceptable. To remedy this, it seemed appropriate that every team member should be assigned to the extra hitter position about the

TABLE 2

TEAM MEMBER	POSITIONS ASSIGNED
#1	PITCHER-5, SECOND BASE-6, EXTRA HITTER-17
#2	PITCHER-0, SHORT STOP-0, THIRD BASE-29, EXTRA HITTER-0
#3	PITCHER-0, RIGHT FIELD-0, CENTER FIELD-29, LEFT FIELD-0, EXTRA HITTER-0
#4	FIRST BASE-6, RIGHT FIELD-0, CENTER FIELD-0, LEFT FIELD-22, EXTRA HITTER-0
#5	SECOND BASE-29, RIGHT FIELD-0, CENTER FIELD-0, LEFT FIELD-0, EXTRA HITTER-0
#6	FIRST BASE-0, SHORT STOP-28, THIRD BASE-0, EXTRA HITTER-0
#7	PITCHER-0, FIRST BASE-28, EXTRA HITTER-0
#8	CATCHER-29, SHORT STOP-0, THIRD BASE-0, EXTRA HITTER-0
#9	CATCHER-5, RIGHT FIELD-6, CENTER FIELD-5, LEFT FIELD-12, EXTRA HITTER-0
#10	PITCHER-28, THIRD BASE-0, EXTRA HITTER-0
#11	PITCHER-0, SHORT STOP-6, THIRD BASE-5, EXTRA HITTER-17
#12	CATCHER-0, SECOND BASE-0, RIGHT FIELD-28, CENTER FIELD-0, LEFT FIELD-0, EXTRA HITTER-0

same number of times. Since that position needed to be occupied 34 times a season, each player should be an EH about 34/12 = 2 5/6 times per season. Constraints were added to ensure that no player was assigned the EH position (position 10) more than 3 times per season. Furthermore, team member #8 was assigned to catch (position 2) for 29 half-games and team member #10 was assigned to pitch (position 1) for 28 half-games. Both of these positions are very strenuous to play and the number of times that a team member plays as a pitcher or as a catcher had to be limited. Determining exactly what the limits should be was not so obvious, but was easy to model by adding simple bounding constraints. The revised integer linear programming formulation now had a total of 55 constraints and is given below.

LL BASEBALL FORMULATION 2:

Maximize SiSj $p_{ij}x_{ij}$

Subject to:

$\sum_j a_{ij}x_{ij} \le$ MAXGAMES i = 1,…,12 (1)

$\sum_j a_{ij}x_{ij} \ge$ MINGAMES i = 1,…,12 (2)

$\sum_i a_{ij}x_{ij} =$ NUMGAMES j = 1,…,10 (3)

x_{ij} nonnegative integer (4)

$x_{i10} \le 3$ i = 1,…,12 (5)

$x_{i1} \le 14$ i = 1,2,3,7,10,11 (6)

$x_{i2} \le 14$ i = 8,9,12 (7)

The solution obtained from solving formulation two is summarized in Table 3. These results provide a template for assigning team members to positions such that every player gets the same amount of time on the baseball field. Although the output from this revised formulation was now what my friend was looking for, we both agreed that a certain amount of courage would be needed to implement this plan (See Figure 1)

TABLE 3

TEAM MEMBER	POSITIONS ASSIGNED
#1	PITCHER-0, SECOND BASE-25, EXTRA HITTER-3
#2	PITCHER-2, SHORT STOP-0, THIRD BASE-23, EXTRA HITTER-3
#3	PITCHER-0, RIGHT FIELD-0, CENTER FIELD-28, LEFT FIELD-0, EXTRA HITTER-1
#4	FIRST BASE-0, RIGHT FIELD-0, CENTER FIELD-0, LEFT FIELD-26, EXTRA HITTER-3
#5	SECOND BASE-9, RIGHT FIELD-14, CENTER FIELD-0, LEFT FIELD-2, EXTRA HITTER-3
#6	FIRST BASE-13, SHORT STOP-12, THIRD BASE-0, EXTRA HITTER-3
#7	PITCHER-4, FIRST BASE-21, EXTRA HITTER-3
#8	CATCHER-14, SHORT STOP-11, THIRD BASE-0, EXTRA HITTER-3
#9	CATCHER-14, RIGHT FIELD-0, CENTER FIELD-9, LEFT FIELD-6, EXTRA HITTER-3
#10	PITCHER-14, THIRD BASE-11, EXTRA HITTER-3
#11	PITCHER-14, SHORT STOP-11, THIRD BASE-0, EXTRA HITTER-3
#12	CATCHER-6, SECOND BASE-0, RIGHT FIELD-20, CENTER FIELD-0, LEFT FIELD-0, EXTRA HITTER-3

Figure 1: "Courage Needed to Implement the Plan"

ACKNOWLEDGEMENTS

I would like to thank Ned and Sue Parkes for bringing this problem to my attention and for their confidence that it could be solved mathematically.

BIOGRAPHY

Francis J. Vasko is a Professor of Mathematics and Computer and Information Science at Kutztown University of Pennsylvania. Before coming to Kutztown University in September 1986, he worked for more than eight years as an employee in the Research Department at Bethlehem Steel solving a variety of real-world applications in operations research (he continues to serve as a consultant to Bethlehem Steel Corporation). His current research focuses on using a variety and mixture of combinatorial optimization techniques in order to more accurately model and solve important real-world applications in production planning, strategic planning, and resource allocation.