University of Florida
Dept. of Computer & Information Science & Engineering

COT 3100
Applications of Discrete Structures

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Slides for a Course Based on the Text
Discrete Mathematics & Its Applications
(5th Edition)
by Kenneth H. Rosen

Module #11: **Sequences**

Rosen 5th ed., §3.2

~9 slides, ~½ lecture

§3.2: Sequences & Strings

- A *sequence* or *series* is just like an ordered *n*-tuple, except:
  - Each element in the series has an associated *index* number.
  - A sequence or series may be infinite.
- A *summation* is a compact notation for the sum of all terms in a (possibly infinite) series.

Sequences

- Formally: A *sequence* or *series* $\{a_n\}$ is identified with a *generating function* $f:S\rightarrow A$ for some subset $S\subseteq\mathbf{N}$ (often $S=\mathbf{N}$ or $S=\mathbf{N}-\{0\}$) and for some set A.
- If f is a generating function for a series $\{a_n\}$, then for $n\in S$, the symbol a_n denotes $f(n)$, also called *term n* of the sequence.
- The *index* of a_n is n. (Or, often i is used.)

Sequence Examples

- Many sources just write “the sequence a_1, a_2, …” instead of $\{a_n\}$, to ensure that the set of indices is clear.
  - Our book leaves it ambiguous.
- An example of an infinite series:
  - Consider the series $\{a_n\} = a_1, a_2, \ldots$, where $(\forall n \geq 1)\ a_n = f(n) = 1/n$.
  - Then $\{a_n\} = 1, 1/2, 1/3, \ldots$

Example with Repetitions

- Consider the sequence $\{b_n\} = b_0, b_1, \ldots$ (note 0 is an index) where $b_n = (-1)^n$.
- $\{b_n\} = 1, -1, 1, -1, \ldots$
- Note repetitions! $\{b_n\}$ denotes an infinite sequence of 1's and –1's, *not* the 2-element set $\{1, -1\}$.

Recognizing Sequences

- Sometimes, you're given the first few terms of a sequence, and you are asked to find the sequence's generating function, or a procedure to enumerate the sequence.
- Examples: What's the next number?
  - 1,2,3,4,… 5 (the 5th smallest number >0)
  - 1,3,5,7,9,… 11 (the 6th smallest odd number >0)
  - 2,3,5,7,11,... 13 (the 6th smallest prime number)

The Trouble with Recognition

- The problem of finding "the" generating function given just an initial subsequence is *not well defined*.
- This is because there are *infinitely* many computable functions that will generate *any* given initial subsequence.
- We implicitly are supposed to find the *simplest* such function (because this one is assumed to be most likely), but, how should we define the *simplicity* of a function?
  - We might define simplicity as the reciprocal of complexity, but…
  - There are *many* plausible, competing definitions of complexity, and this is an active research area.
- So, these questions really have *no* objective right answer!

What are Strings, Really?

- This book says "finite sequences of the form $a_1, a_2, \ldots, a_n$ are called *strings*", but *infinite* strings are also used sometimes.
- Strings are often restricted to sequences composed of *symbols* drawn from a finite *alphabet*, and may be indexed from 0 or 1.
- Either way, the length of a (finite) string is its number of terms (or of distinct indexes).

Strings, more formally

- Let Σ be a finite set of *symbols*, *i.e.* an *alphabet.*
- A *string s* over alphabet Σ is any sequence $\{s_i\}$ of symbols, $s_i \in \Sigma$, indexed by **N** or **N**−{0}.
- If *a*, *b*, *c*, … are symbols, the string *s* = *a*, *b*, *c*, … can also be written *abc* …(*i.e.*, without commas).
- If *s* is a finite string and *t* is a string, the *concatenation of s with* t, written *st*, is the string consisting of the symbols in *s*, in sequence, followed by the symbols in *t*, in sequence.

More String Notation

- The length $|s|$ of a finite string s is its number of *positions* (*i.e.*, its number of index values i).
- If s is a finite string and $n \in \mathbf{N}$, s^n denotes the concatenation of n copies of s.
- ε denotes the empty string, the string of length 0.
- If Σ is an alphabet and $n \in \mathbf{N}$,
  $\Sigma^n \equiv \{s \mid s \text{ is a string over } \Sigma \text{ of length } n\}$, and
  $\Sigma^* \equiv \{s \mid s \text{ is a finite string over } \Sigma\}$.