Section 3.4: The Chain Rule

The chain rule is a rule that lets us take the derivative of a composite function.

**Form 1 of the Chain Rule**

If $C$ is a function of $p$, and $p$ is a function of $t$, then
\[
\frac{dC}{dt} = \frac{dC}{dp} \frac{dp}{dt}
\]

Example 1:

$C(p) = 2p^2 - 1$ and $p(t) = \ln t$

Find $\frac{dC}{dt}$

Example 2:

$A(y) = 14 + \frac{100}{y}$ is the average cost in cents to produce $y$ Yo-Yo’s. $y(t) = 3t^2 + 100t + 5000$ represents the number of Yo-yo’s produced $t$ years after 2000.

Find $\frac{dA}{dt}$

At what rate is the average cost changing in 2002?
Form 2 of the Chain Rule
If \( y = h(g(x)) \), then \( \frac{dy}{dx} = h'(g(x))g'(x) \)

Example 3:
\( y = (x^4 + 3)^5 \)

\[
y = \frac{1}{2x^2+2}
\]

\( y = e^{2.5x} \)

Example 4:(Logistic Functions)
The percentage of households with TVs that subscribed to cable \( t \) years after 1970 can be modeled by the logistic equation:
\[
P(t) = \frac{68}{1 + 38e^{-0.25t}}
\]

Find \( P'(t) \) which represents the rate of change for the percentage of households with TVs who subscribe to cable.