

Name: \_\_\_\_\_

Date: \_\_\_\_\_

### **Investigation: The Angle Sums of Convex Polygons (Using Version 3)**

In this investigation you will discover how to calculate the sum of the interior angles of any convex polygon without actually having to measure each individual angle. A convex polygon is one in which its interior angles do not exceed  $180^\circ$ .

#### **Sketch (Triangle)**

**Step 1:** *Plot 3 points about 1- 2 inches apart from one another.* Do this by selecting the “Point” button (It looks like a dot) on the toolbar and then clicking on the sketch to plot the points.

**Step 2:** *Connect the points with segments.* Do this by clicking on the “Arrow” button (It looks like the mouse icon) and dragging a box over the three points, which will highlight them. Then go to the “Construct” menu and then select “Segment.” After this action is completed, segments will appear forming a triangle. On the PC you can use CTRL + I after highlighting the points instead of going into the menu.

**Step 3:** *Label the vertices of the triangle.* Do this by holding down the Shift key and clicking on each of the points. Make sure that the segments are not also selected. Then go to the “Display” menu and select the option to “Show Labels.” After this action is completed, letters will appear, labeling the vertices. They should be the letters A, B, and C. They may be on top of a point or segment click on them and drag to move them out of the way if necessary. After satisfied with the position of the letters, deselect all objects by clicking in an empty place.

**Step 4:** *Use Geometers Sketchpad to measure each of the angles of the triangle.* Do this by holding down the Shift key and clicking A, B, and C in order. Then go to the “Measure” menu and select “Angles.” Then  $\angle ABC$  will be displayed in the top left corner of the worksheet. Click on the pad to reset the highlighting. Do this procedure starting with B and then with C to get the other two angle measurements. Place your data in the table below.

$\angle ABC$	$\angle BCA$	$\angle CAB$

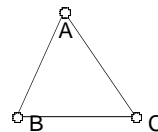
**Step 5:** *Calculate the angle sum for the triangle.* Do this by selecting “Measure” followed by “Calculate.” A calculator visual will pop up. Click on the measure for  $\angle ABC$ ; then follow by clicking “+”; then click the measure for  $\angle BCA$ ; then “+” again; and finally click on the measure for  $\angle CAB$ . After all this, click “Ok” and an angle sum equation will pop up. See the diagram on the next page for an example up to this point.

$$m\angle ABC = 66^\circ$$

$$m\angle BCA = 56^\circ$$

$$m\angle CAB = 59^\circ$$

$$m\angle ABC + m\angle BCA + m\angle CAB = 180^\circ$$



## Investigate

According to Geometer's Sketchpad, what is the angle sum for your triangle? \_\_\_\_\_

Click somewhere away from the figure to deselect objects. Now click and drag a vertex and move the point around. Do the individual angles change while you do this? \_\_\_\_\_

Does the angle sum change while you move any of the vertices? \_\_\_\_\_

What can you conclude about the angle sum of any triangle? \_\_\_\_\_

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## Sketch (Quadrilateral)

Now we are going to investigate the interior angle sum of a convex quadrilateral. First, go to the "File" menu and select "New Sketch."

**Step 1:** *Place four points on the sketchpad* that roughly would form the corners of a rectangle. As before, select the "Point" button prior to placing any points.

**Step 2:** *Connect the points with segments* as was done for the triangle. Recall, you must select the "Arrow" button and then make a box over the four points. Finally, go to the "Construct" menu and select "Segment." The appearing segments should form a quadrilateral.

**Step 3:** *Label the vertices.* Deselect the segments. Hold down the Shift key and highlight all the points. Go to the "Display" menu and select "Show Labels." The letters should appear. The letters should be A, B, C, and D. Click and drag them in order to place them in a clearly viewed position. Then deselect all objects.

**Step 4:** *Measure each of the interior angles at the vertices* of the quadrilateral using Geometer's Sketchpad. Click on the "Arrow" button and then hold down the shift key. Select the points A, B, and then C in that particular order. Then go to the "Measure" menu and select angle. The angle measurement should appear in the upper left corner of the worksheet. Do this with the other three angles, paying attention to the order in which you select the points. The order always places the vertex second.

**Step 5:** Use the "Calculate" button in the "Measure" menu to *find the sum of the interior angles in the quadrilateral.* When the calculator comes up, select each of the angle measurements with a plus in between. The syntax should resemble what is below.  
$$= \angle ABC + \angle BCD + \angle CDA + \angle DAB$$

\*Check: Are you sure your quadrilateral is convex. Make sure no angles exceed  $180^\circ$ . \*

## Investigate

Fill in the table below for the angles of your quadrilateral.

$\angle ABC$	$\angle BCD$	$\angle CDA$	$\angle DAB$

According to Geometer's Sketchpad, what is the angle sum for the quadrilateral? \_\_\_\_\_

Construct a diagonal from a vertex to a nonadjacent vertex. Do this by selecting two nonadjacent points (while holding down the Shift key). Then construct a segment by going into the "Construct" menu and selecting "Segment."

How many shapes are formed? \_\_\_\_\_

What kind of shapes are they? \_\_\_\_\_

What would the angle sum for each of these shapes? \_\_\_\_\_

What is the sum of both of their total angles? \_\_\_\_\_

What can you conclude about the resulting picture? \_\_\_\_\_

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## Additional Sketches & Investigations (Polygons with Additional Sides)

Select "New Sketch" from the "File" menu. Construct a convex pentagon and perform the same analysis. Fill in the table below.

$\angle ABC$	$\angle BCD$	$\angle CDE$	$\angle DEA$	$\angle EAB$

Calculate the sum of the five interior angles. What is the interior angle sum? \_\_\_\_\_

Construct segments from a single vertex to nonadjacent vertices.

How many shapes are formed? \_\_\_\_\_

What would be the angle sum for each of these shapes? \_\_\_\_\_

What logically would be the total angle sums for these shapes? \_\_\_\_\_

Select "New Sketch" from the "File" menu. Now construct a convex hexagon and perform the same analysis. Fill in the table below.

$\angle ABC$	$\angle BCD$	$\angle CDE$	$\angle DEF$	$\angle EFA$	$\angle FAB$

Calculate the sum of the six interior angles. What is the angle sum? \_\_\_\_\_

Construct segments from a single vertex to nonadjacent vertices.

How many shapes are formed? \_\_\_\_\_

What would be the angle sum for each of these shapes? \_\_\_\_\_

What logically would be the total of all the angle sums for these shapes? \_\_\_\_\_

### **Identify the Pattern**

Now, considering all the constructed shapes, what is the pattern of association between the number of sides and the total interior angle sum? \_\_\_\_\_

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Using the variable  $n$  as the number of sides, construct a formula that finds the interior angle sum given  $n$ . (Hint: it is a factor of  $180^\circ$ ).