

Investigation: Chords of a circle

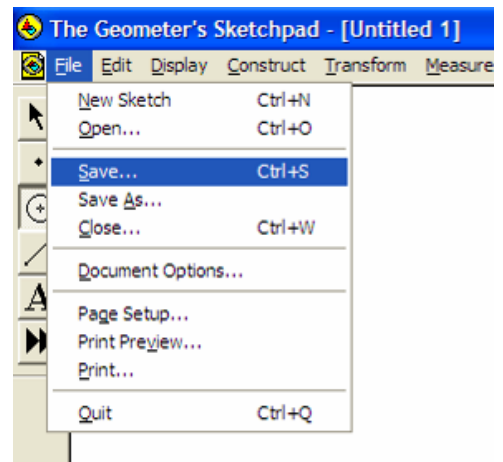
In this investigation, you will discover some properties of the chords of a circle and their corresponding arcs. Follow the instructions to create the sketch and complete the Investigation. Turn in just the Investigation part of the worksheet.

Getting started:

1. Start Sketchpad if it isn't already running or choose **New Sketch** from the **File** menu.
A new, blank sketch window appears.

This is the menubar →

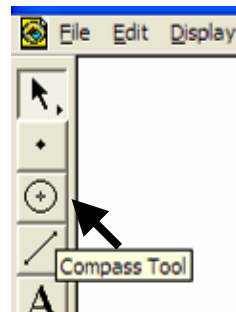
When these instructions tell you to select a command from the menu, for example select **Save** from the **File** menu, that means you should click on the word "File" on the menubar, then when the drop-down list appears, click on "Save".



At any time, if you want to undo your last operation, select **Undo** from the **Edit** menu.

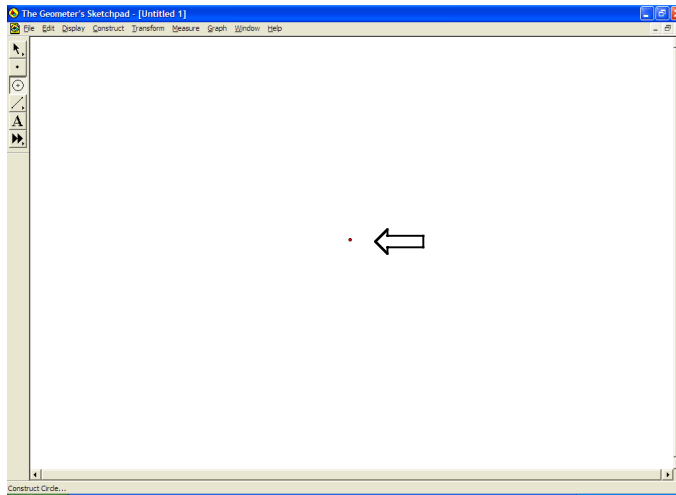
Sketch:

2. Construct a circle.
 - a. Click on the **Compass** tool in the **Toolbox**.

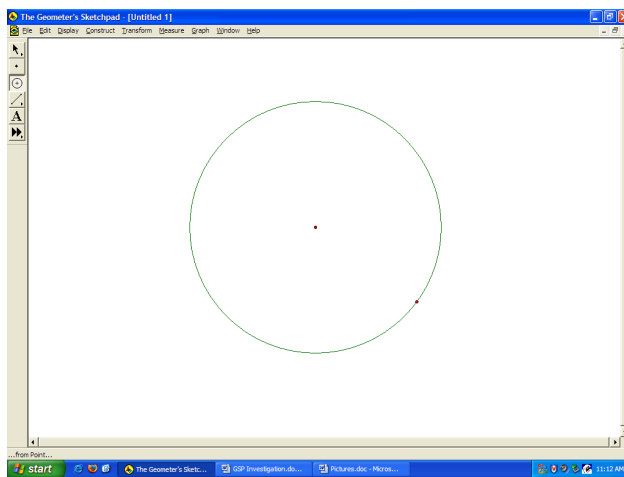


Geometer Sketchpad Worksheet

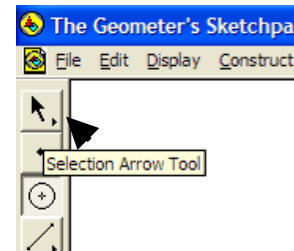
- b. Click once at approximately the center of the sketch.
This will be the center of the circle.



- c. Drag your mouse in any direction until the circle takes up about half of the sketch space.

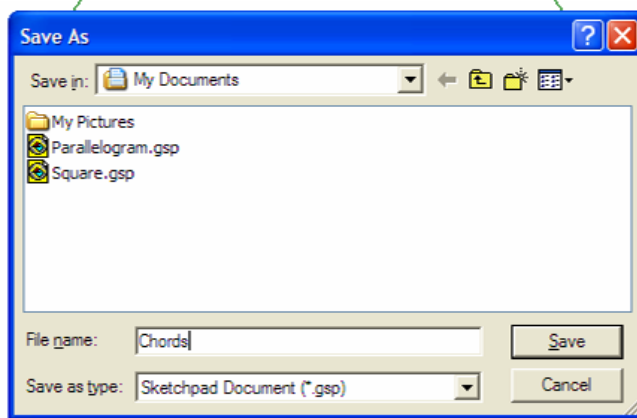


- d. Click once on the sketch to complete the circle.
e. Click on the **Selection Arrow** tool from the Toolbox.

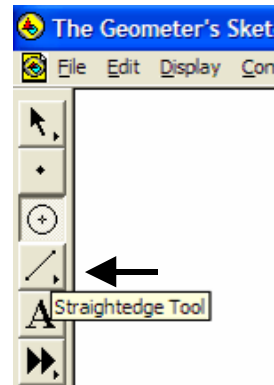


Geometer Sketchpad Worksheet

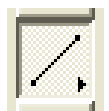
3. Save your work.
 - a. Select **Save** from the **File** menu.
A pop-up window will appear.
 - b. Select your student directory from the **Save in** drop down list.
 - c. Type in the name “Chords”.



- d. Click the **Save** button on the pop-up window.
4. Construct a chord anywhere on the circle.
 - a. Click on the **Straightedge** tool on the Toolbar.



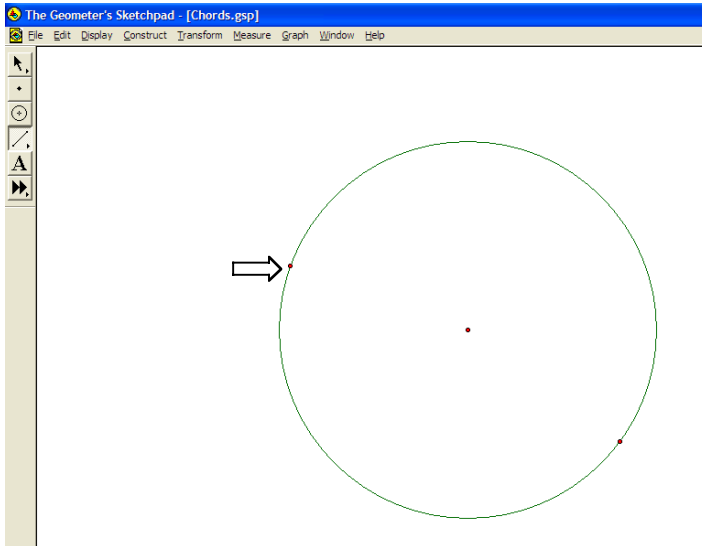
Note: If the **Straightedge** tool looks different from the one pictured, press and hold the mouse button over the current **Straightedge** tool and choose **Segment** tool from the palette that appears.



Geometer Sketchpad Worksheet

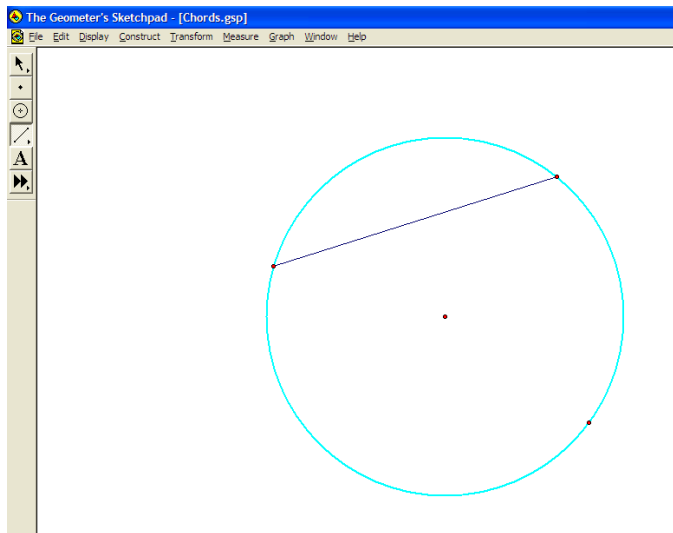
- b. Click anywhere on the circumference of the circle to create the first endpoint of the segment.

The circle will change colors (e.g., the circle will be green on a PC).



- c. Drag your mouse over to another point on the circle.

As your mouse reaches the circumference of the circle, the circle's color becomes turquoise.



Geometer Sketchpad Worksheet

- d. Click on the circle to create the second endpoint for the segment.

The circle's color becomes green again. The segment is highlighted in pink.

- e. Click the **Selection Arrow** tool to deselect all tools.

- f. Click anywhere in the blank space of the sketch to deselect the objects.

- g. Check your work so far:

- i. Click on one of the segment's endpoints and hold the mouse key down.
- ii. Drag the endpoint around to be certain that the segment is attached to the circle (the selected segment endpoint should "slide" around the circle).
- iii. If it's not and there are actually more than two points in the sketch:
 1. Choose **Undo** from the **Edit** menu until the segment disappears (you may have to select **Undo** more than once).
 2. Repeat parts of Step 4 to recreate the segment.
- iv. Select the other endpoint.
- v. Repeat steps ii and iii above.

5. Save your work by selecting **Save** from the **File** menu.

6. Click in the blank space to deselect all objects.

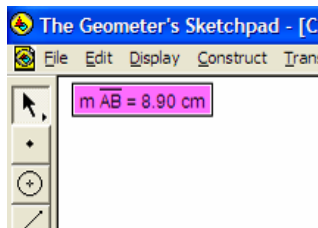
7. Measure the length of the chord.

- a. Select the segment by clicking anywhere on the segment except the endpoints.

The line will be highlighted pink.

- b. Select **Length** from the **Measure** menu.

Geometer's Sketchpad assigns names to each endpoint and puts the length measurement in the upper, left-hand corner of the sketch.



- c. Click anywhere in the blank space to deselect the objects.

8. Measure the length of the chord's arc.

- a. Click on point *A* then *B* then the circle to select the objects.

The points and circle will be highlighted in pink.

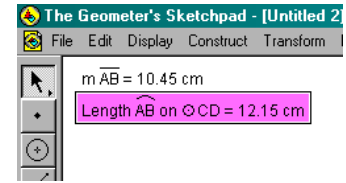
- b. Select **Arc on Circle** from the **Construct** menu.

Geometer Sketchpad Worksheet

An arc will be drawn from A to B.

- c. Select **Arc Length** from the **Measure** menu.

The measurement will appear in the upper left-hand corner of the sketch.



Notice Geometer's Sketchpad has assigned a label to the circle (in this example, C is the center and D is the point on the circle). Your labels might be different.

- d. Click anywhere in the blank space.
- e. Click on one of the chord's endpoints and drag it around the circle to see the arc length and chord length change accordingly.
9. Create a line that contains the diameter of the circle.

- a. Select the **Line** tool from the **Straightedge** palette by pressing and holding the mouse button over the current **Straightedge** tool.



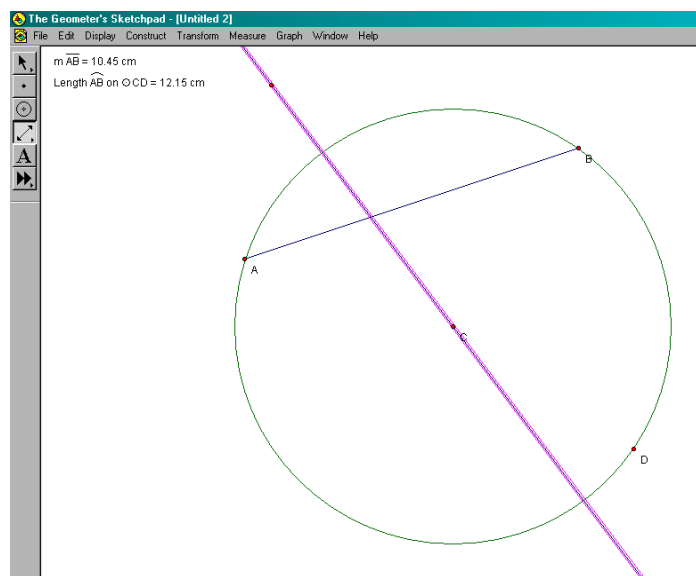
- b. Click once anywhere on the sketch outside of the circle.

This will create the first point on the line.

- c. Drag your mouse to create a line. As you drag your mouse, the line will follow the mouse.

- d. Make the second point on the line the center of the circle by clicking once on the origin.

When your cursor is over the origin, the point will be highlighted in turquoise.



- e. Click the **Selection Arrow**.

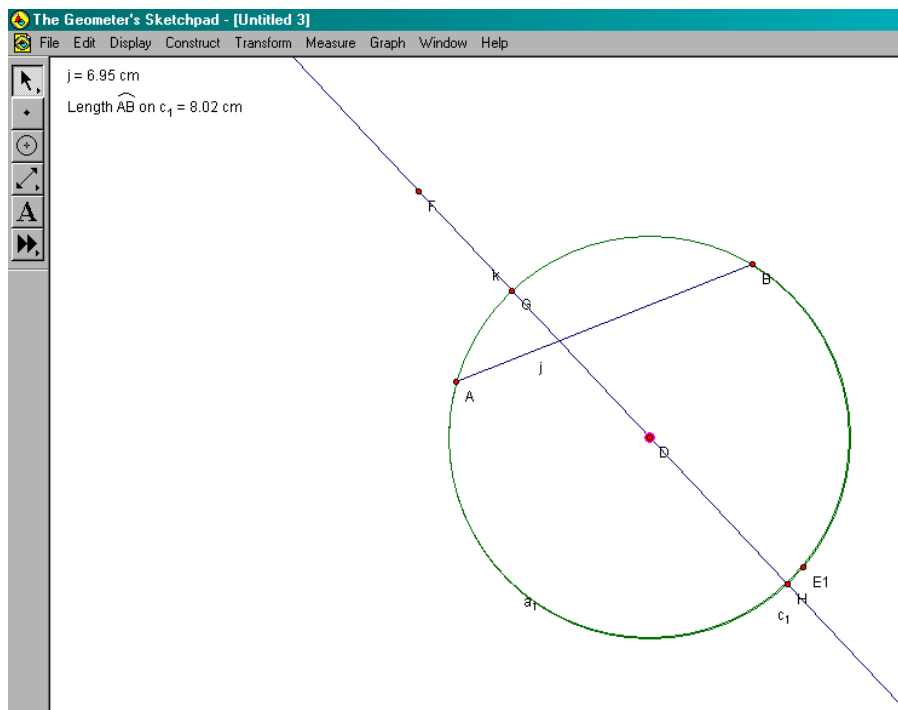
Geometer Sketchpad Worksheet

- f. Click anywhere in the blank space.
- g. Click on either of the line's two points and drag it around. Notice how moving the point "attached" to the center of the circle moves the circle, but moving the other point moves just the line around.

The line's point outside the circle will be used to move the diameter around.

- h. Click anywhere in the blank space.
10. Create the line's points of intersection with the circle.
- a. Click anywhere in the blank space of the sketch.
 - b. Click anywhere on the line and circle, except on a point on the object, to select them.
 - c. Select **Intersections** on the **Construct** menu. New points will appear at the intersection of the circle and line.
 - d. Select **Select All** from the **Edit** menu.
 - e. Select **Show Labels** from the **Display** menu.

All points, lines, and segments have labels. You can move a label by positioning your cursor over the label until the cursor turns into a hand with a pointed finger, then click on the label and drag it to another location. Your labels might be different from this worksheet's labels.



- f. Click anywhere in the blank space.

11. Create the diameter.

- a. Select the **Segment** tool from the **Straightedge** palette by pressing and holding the mouse button over the current **Straightedge** tool.



- b. Position the mouse over one of the intersection points for the line (G or H) and the circle (both the line and circle will become turquoise), and click once.

This will create the first point on the segment.

- c. Drag your mouse your mouse along the line to the other point of intersection for the line and the circle, and click once.

This is the diameter of the circle.

- d. Click on the **Selection Arrow**.

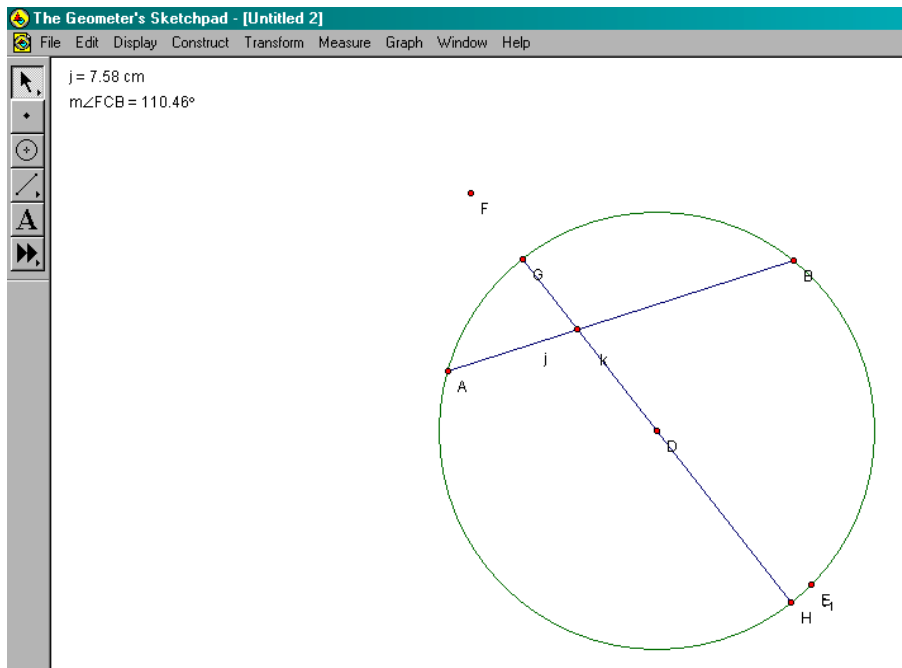
- e. Click anywhere in the blank space.

- f. Click anywhere on the line outside of the circle except for the point.

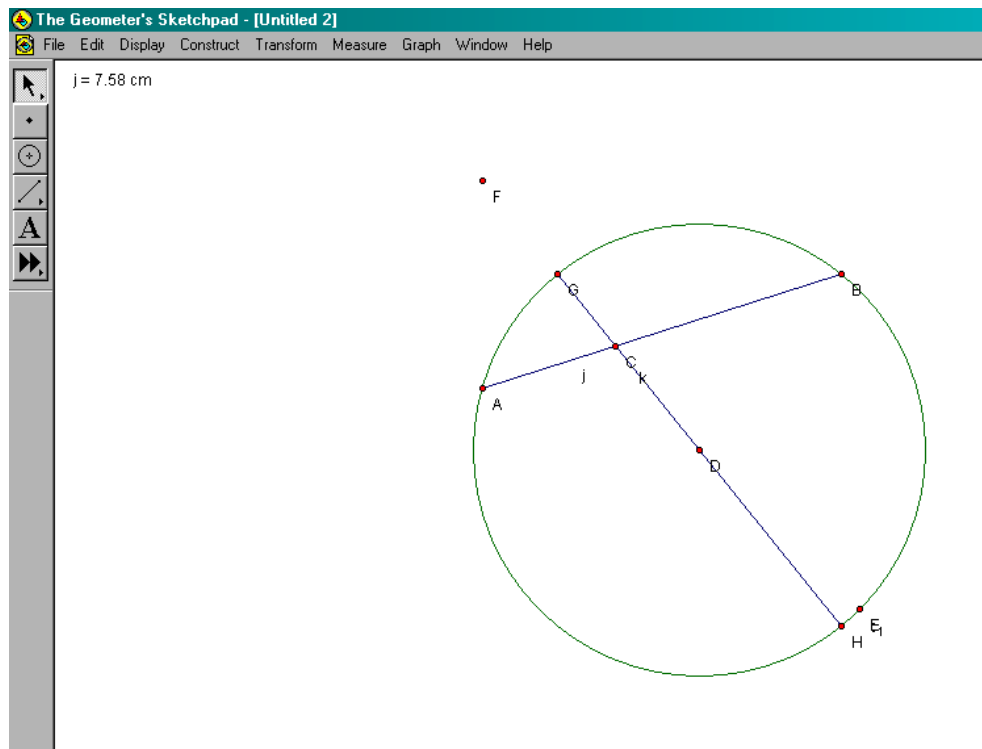
The line will be highlighted pink.

- g. Select **Hide Line** from the **Display** menu.

The line will become invisible, but you will still see the diameter and the line's point outside the circle. The diameter is linked to the line and the circle. You can only move the diameter by dragging the line's point outside the circle.



- h. Practice rotating the diameter around by holding the mouse on the line's point outside the circle (F in this example).
12. Determine the angle formed by the chord and the line.
- a. Create a point of intersection for the chord and line.
 - i. Select the chord and the diameter by clicking on them one at a time. Do not click on a point because that will not select the whole diameter or chord. *They will be highlighted in pink.*
 - ii. Select Intersection from the Construct menu. *A point will appear at the intersection of the chord and the diameter. The diameter and chord will be deselected.*
 - b. Measure the angle.
 - i. Name the remaining points.
 - 1. If point C , the point of intersection for the chord and diameter, is not highlighted, click on point C to select the point.
 - 2. Select Show Labels from the Display menu.



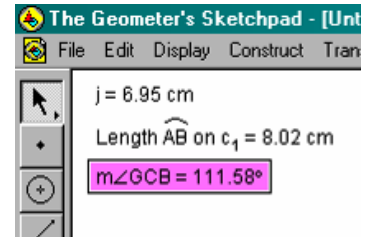
- 3. Click in the blank space on the sketch to deselect highlighted objects.

Geometer Sketchpad Worksheet

- ii. Select three points to determine the angle of intersection for the chord and the line. For example, referring to the sketch above, select points (in order) G , C , and then B .

The vertex of the angle must be the second point selected.

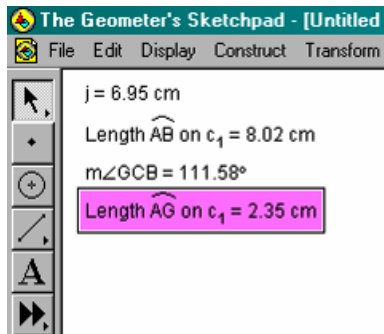
- iii. Select **Angle** from the **Measure** menu.
The angle measurement will appear in the upper left-hand corner below the segment length measurement.



Note: if the Angle option is grayed out, deselect highlighted objects by clicking in blank space, and start over with the previous step.

- c. Save your work by selecting **Save** from the **File** menu.
13. Measure the two arcs created by the intersection of the diameter, circle, and chord.
- a. Select points A and G and the circle.
 - b. Select **Arc Length** from the **Measure** menu.

Arc AG 's measurement will appear below the other measurements.

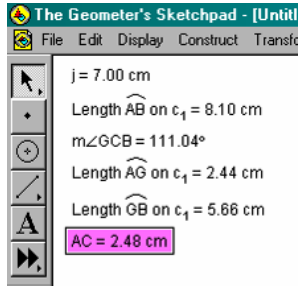


- c. Click in the blank space.
 - d. Select points G and B and the circle.
 - e. Select **Arc Length** from the **Measure** menu.
 - f. Click in the blank space.
14. Measure the two segments created by the intersection of the line and chord.
- a. Click on the leftmost endpoint of the chord (A in this example) and the point at the intersection of the chord and diameter (C in this example).

Geometer Sketchpad Worksheet

- b. Select **Distance** from the **Measure** menu.

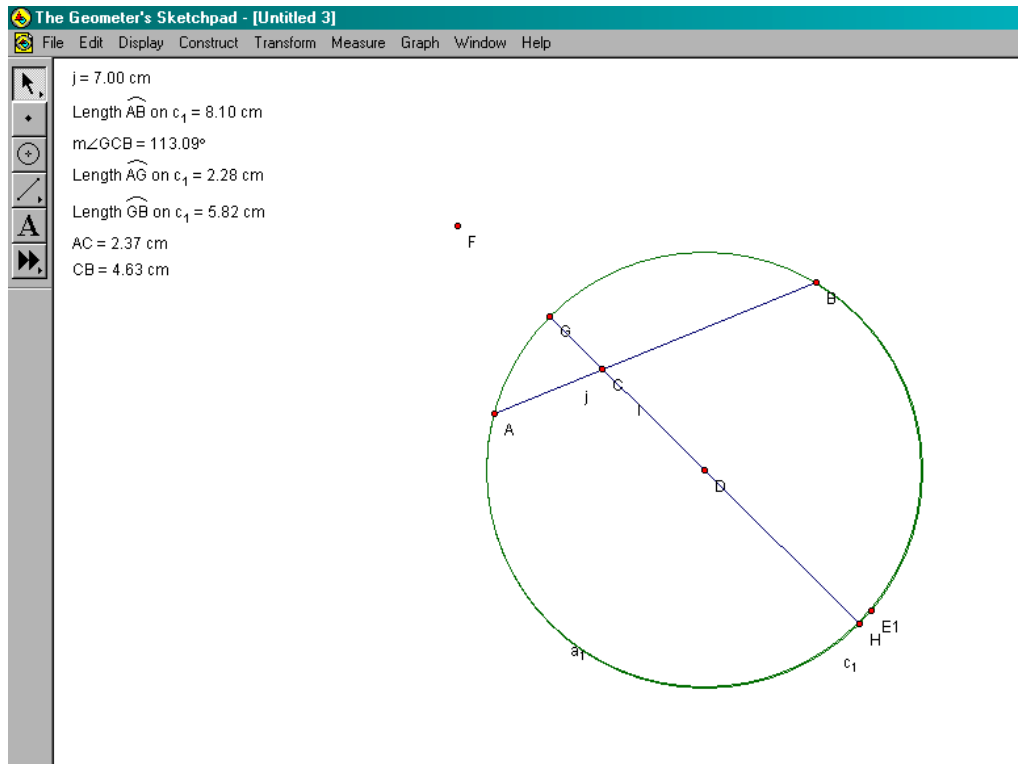
The measurement will appear below the others.



- c. Click on blank space.
- d. Click on points B and F .
- e. Select **Distance** from the **Measure** menu.

Save your work.

Your graph should look something like this:



Investigation: Chords of a circle

Name _____ Date: _____

Investigate:

15. Hold the mouse on the line's point outside the circle (point F in this example) and drag around the diameter. Which measurements are changing?

16. For those measurements that change, why are they changing?

17. What happens to the arc and segment measurements if you make the circle smaller and larger? (Click in the blank space to deselect everything, then click on point $E1$ — the point on the circle — to resize the circle.)

18. What happens to the angle measurement when you resize the circle? Why?

Geometer Sketchpad Worksheet

19. Drag the line containing the diameter so that it is perpendicular to the chord.
 - a. How do you know they are perpendicular?

 - b. Without actually measuring the angle, what would $m\angle GFA$ be? Why?

 - c. Write down the measurements for arcs AG and BG .

 - d. What is the ratio of the measures for arcs AG and arc AB ?

 - e. Write down the measurements for segments AC and BC .

 - f. What is the ratio of the measurements for segments AC and AB ?

 - g. Based on this exercise, how would you explain this relationship between a diameter, chord, and arcs?

20. Print out a copy of your sketch where the line and chord are perpendicular.
 - a. Select **Print** from the **File** menu.
 - b. Click the **OK** button on the pop-up.
 - c. Write your name at the top of the printout.