Kutztown University
Kutztown, Pennsylvania

MAT 550: Foundations of Geometry

I. Three semester hours; three clock hours; this is an elective course in the M.Ed. or M.A. degree programs in mathematics.

II. Catalog Description:

MAT 550: Foundations of Geometry 3 s.h.

Foundational aspects of geometry; postulational systems and their properties; Euclidean geometry from both the metric and the synthetic viewpoints; finite geometries; non-Euclidean geometries; geometric transformations.

III. Course Objectives: In this course the student will:

A. Study geometry as a discipline in which there are many inter-related postulational systems, and to consider Euclidean geometry and its relationship to the general framework of geometry, showing that Euclidean geometry is really a special case of more general geometries.

B. Examine Euclidean geometry from a rigorious postulational viewpoint.

C. Study non-Euclidean geometries, their relationship to Euclidean geometry, and their place within the general geometrical framework.

D. Develop an understanding and appreciation of:

1. the role played in mathematics by postulational systems.
2. the nature and method of deductive proof.
3. the structural aspect of geometry.

E. Trace in part the historical development of geometry.

IV. Course Outline:

A. The Geometry of Euclid

1. Postulates and definitions
2. Development of theorems
3. "Flaws" of Euclid's geometry

B. Historical Development of Geometry

1. Early "earth measure"
2. Greek geometry
3. The parallel postulate
   a. Attempts to prove it
   b. Equivalent statements
4. Saccheri
5. Gauss, Bolyai and Lobachevski
6. Riemann
7. Subsequent developments in geometry

C. Postulational Systems

1. Elements
   a. Undefined terms
   b. Definitions
   c. Postulates
   d. Theorems
   e. Rules of logic
2. Characteristics of a postulational system
   a. Consistency
   b. Completeness
   c. Independence
   d. Categoricalness
3. Concepts of symbolic logic
4. Finite geometries

D. Intuitive Treatment of Three Geometries

1. Euclidean model for hyperbolic geometry
2. Euclidean model for elliptic geometry
3. Comparison of Euclidean, hyperbolic and elliptic geometries

E. Neutral Geometry

1. Postulates
2. Saccheri quadrilaterals
3. Theorems provable without a postulate equivalent to Euclid's parallel postulate
4. Angle-sums in a triangle

F. The Parallel Postulate and Euclidean Geometry

1. Uniqueness of parallels
2. Similarities between triangles

3. The Pythagorean Theorem
4. An area function
5. Perpendicular lines and planes in space
6. Circles and spheres
7. Rigid motion

G. Synthetic and Metric Treatments of Euclidean Geometry

1. Hilbert's treatment
   a. Examination of postulates and comparison with those of Euclid
   b. Development of geometry from Hilbert's postulates
2. Geometry based on the real numbers
   a. Algebra of the real numbers
   b. Incidence geometry
   c. Distance and congruence
   d. Separation properties
   e. Angle measure
   f. Congruences between triangles

H. Hyperbolic Geometry

1. Postulate on parallelism
2. Angle-sums in a triangle
3. Defect of a triangle
4. Collapse of similarity theory
5. Impossibility of Euclidean area theory in hyperbolic geometry
6. Uniqueness of hyperbolic area theory

I. Elliptic Geometry

1. Riemann's parallel postulate
2. Lines as closed figures
3. Another look at the model for elliptic geometry
4. Difficulties encountered in a formal treatment of elliptic geometry

J. Logical Consistency and Empirical Validity of Euclidean and Non-Euclidean Geometries

1. Non-Euclidean geometries
2. Euclidean geometry
3. Summarization and categorization of various geometries and their place in the geometric framework

K. Constructions with Straightedge and Compasses

1. Unmarked rulers and collapsible compasses

2. Algebra with straightedge and compasses
3. Impossible construction problems and antiquity

L. Elementary Transformations

1. Transformation theory
2. Fundamental point transformations of the plane
3. Isometries and similarities
4. Inversion and its application
5. Poles and polars
6. Space transformations

V. References:

Berger, Marcel. *Geometry I and Geometry II*. Secaucus, NJ: Springer-Verlag (1987).

Coxeter, H.S.M. *Introduction to Geometry*. New York, NY: John Wiley & Sons (1961).

Eves, H. *A Survey of Geometry* (revised edition). Boston, MA: Allyn and Bacon (1972).

Eves, H. *Foundationsl and Fundamental Concepts of Mathematics* (3rd edition). Boston: PWS-Kent (1990).

Greenberg, M.J. *Euclidean and Non-Euclidean Geometries* (2nd edition). San Francisco, CA: W. H. Freeman and Co. (1974).

Hall, D.W. and Szabo, S. *Plane Geometry, An Approach Through Isometries*. Englewood Cliffs, NJ: Prentice Hall (1971).

Heath, T.L. *The Thirteen Books of Euclid's Elements*. New York, NY: Cambridge University Press (1926).

Moise, E. *Elementary Geometry from an Advanced Standpoint*. Reading, MA: Addison-Wesley Publishing Co. (1963).

Perry, E. *Geometry: Axiomatic Development with Problem Solving*. New York: Marcel Dekker (1992).

Ryan, P.J. *Euclidean and Non-Euclidean Geometry: An Analytic Approach*. New York, NY: Cambridge University Press (1986).

Smart, J.R. *Modern Geometries* (3rd edition). Monterey, CA: Brooks/Cole Publishing Co. (1988).

Solow, Daniel. *How to Read and Do Proofs*. New York: John Wiley and Sons (1982).

Tuller, A. *A Modern Introduction to Geometries*. Princeton, NJ: D. Van Nostrand Co. (1967).

Wilder, R.L. *Introduction to the Foundations of Mathematics*. New York, NY: John Wiley & Sons (1967).

Wilf, H.S., Editor. *Geometry*. American Mathematical Monthly (Special Geometry Issue). Oct., 1990.

Yaglom, I.M. *Geometric Transformations*. (Tr. from the Russian by A. Shields). New York, NY: Random House (1962).

1995