Kutztown University
Kutztown, Pennsylvania

MAT 224: Foundations of Higher Mathematics

I. Three semester hours; three clock hours; required course for B.S. in Mathematics & B.S. in Secondary Education (Mathematics); also required for Mathematics minor.

II. Course Description:

MAT 224: Foundations of Higher Mathematics 3 s.h., 3 c.h.

This course is designed to prepare the student for the study of advanced mathematics. Topics include fundamentals of logic; proof strategies; the algebra of sets; relations; including equivalence relations; functions and their properties; countable sets and counting techniques; ordered and well-ordered sets. This course should be taken only after the student has taken at least two college-level mathematics courses. Prerequisite: C or better in MAT 171.

III. Course Objectives:

The student will:

A. Learn to use the proper mathematical techniques in analyzing information, extracting pertinent facts and drawing appropriate conclusions.

B. Examine the basic logic that underlies mathematical proofs

C. Write mathematical proofs using the following methods: direct proof; proof by contrapositive; proof by contradiction; proof by exhaustion; proof by induction.

D. Interpret and prove or disprove quantified mathematical statements.

E. Apply the abstract concepts of relations and functions in mathematical applications.

F. Recognize the foundational nature of set theory in mathematics and its development.

G. Apply the concepts and operations of sets in mathematical disciplines.

IV. Course Assessment

The course assessment will be a subset of tests, projects, presentations, quizzes, homework, team assignments and final exam.

V. Course Outline

A. Fundamentals of Mathematical Logic and Proof Strategies

1. Preliminaries
   a. The language of mathematics-defined and undefined terms
   b. Statements and truth values
   c. Definitions, axioms and theorems

2. Fundamentals of Logic
   a. The basic sentential connectives; negation and conjugation
   b. Truth tables, equivalent statements, tautologies, and contradictions
   c. Disjunctions
   d. Various properties of the above connectives

3. Conditionals and Biconditionals
   a. Conditionals and their relatives
   b. Biconditionals

4. Statement Formulas and Quantification
   a. Sets, statement formulas and quantifiers
   b. Combinations of the universal and existential quantifiers
   c. Negation of quantified statements

5. Strategies for Mathematical Proof
   a. Argumentation and valid consequences
   b. Rules of Inference
   c. Method of indirect proof
   d. Quantification in argumentation
   e. Fallacies commonly found in argumentation
   f. Inconsistent and/or unsatisfiable sets of premises
   g. Counterexamples

B. Set Theory and Related Concepts

1. Fundamentals of Classes and Sets
   a. Definitions of classes and sets
   b. The set N and proof by induction
   c. Algebra of sets
   d. Proofs of the properties of the algebra of sets

2. Relations
   a. Ordered pairs and cartesian products
   b. Relations, images and pre-images
   c. Properties of images and pre-images with respect to the algebra of sets
   d. Algebra of relations

3. Equivalence Relations and Partitions of Sets
   a. Equivalence relations and equivalence classes
   b. Partitions of a set
   c. Relationships between equivalence relations and partitions

4. Functions
   a. Functions, images and pre-images
   b. Properties of images and pre-images with respect to the algebra of sets
   c. Properties of functions-injectivity, surjectivity and bijectivity
   d. Algebra of functions

5. Families of Sets
   a. Indexed collection of sets
   b. Generalized algebra of sets
   c. Generalized cartesian products
   d. Applications

6. Countable Sets and Counting Techniques
   a. Infinite sets and equinumerous sets
   b. Countable sets and their properties
   c. Cantor's Theorem
   d. The existence of uncountable sets

7. Ordered Sets and Well-Ordered Sets
   a. Partial orderings
   b. Special elements of partially ordered sets
   c. Order isomorphism
   d. Well ordered sets
   e. Zorn's Lemma

VI. Instructional Resources

Barnier, W. and N. Feldman.. *Introduction to Advanced Mathematics*. 2nd ed. Upper Saddle River, NJ: Prentice Hall, 2000.

Biggs, N.L., E.K. Lloyd, , and R.J. Wilson, R.J. *Graph Theory 1736-1936*. Clarendon: Oxford, 1976.

Bittinger, M.L. *Logic Proof and Sets.* 2nd ed. New York, NY: West, 1983.

Bond, R. J. and W.J. Keane. *An Introduction to Abstract Mathematics*. Pacific Grove, CA: Brooks/Cole Pub. Co., 1999.

D'Angelo, J. P. and Douglas B. West. *Mathematical Thinking: Problem-Solving and Proofs*. 2nd ed. Upper Saddle River, NJ: Prentice Hall, 2000.

Dudley, J. *Elementary Number Theory.* San Francisco, CA: Freeman, 1978.

Dumas, Bob A. *Transition to Higher Mathematics: Structure and Proof*. New York, NY: McGraw-Hill, 2007.

Eisenberg, Murray. *The Mathematical Method: A Transition to Advanced Mathematics.* Englewood Cliffs, NJ: Prentice Hall, 1996.

Fisher, J.L. *Application Oriented Algebra.* New York, NY: T.Y. Crowell, 1977.

Foerster, P.A. *Algebra and Trigonometry: Functions and Applications.* Reading, MA: Addison-Wesley, 1980.

Gilbert, W.J. *Modern Algebra with Applications.* New York, NY: Wiley and Sons, 1976.

Gilbert, Will J., and Scott A. Vanstone. *Introduction to Mathematical Thinking: Algebra and Number Systems.* Englewood Cliffs, NJ: Prentice Hall, 2005.

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Hummel, K.E. *Introductory Concepts for Abstract Mathematics*. Boca Raton, FL: Chapman & Hall/CRC. 2000.

Kurtz, D.C. *Foundations of Abstract Mathematics*. New York: McGraw-Hill, Inc. 1992.

Lay, Steven R. *Analysis: With an Introduction to Proof.* 4th ed. Englewood Cliffs, NJ: Prentice Hall, 2004.

Lucas, J. F. *Introduction to Abstract Mathematics*. 2nd ed. New York, NY: Ardsley House Publishers, 1990.

Mendelson, E. *Boolean Algebra and Switching Circuits.* New York, NY: McGraw-Hill, 1970.

Niven, I. *Numbers: Rational and Irrational.* New York, NY: Random House, 1961.

O'Leary, Michael L. *The Structure of Proof: With Logic and Set Theory.* Englewood Cliffs, NJ: Prentice Hall, 2002.

Pinter, C. *A Book of Abstract Algebra.* New York, NY: McGraw-Hill, 1982.

Richmond, Bettina, and Thomas Richmond. *A Discrete Transition to Advanced Mathematics.* Pacific Grove, CA: Brooks/Cole Pub. Co., 2004.

Rodgers, Nancy. *Learning to Reason: An Introduction to Logic, Sets, and Relations.* Pacific Grove, CA: Brooks/Cole Pub. Co., 2000.

Rotman, Joseph J. *Journey into Mathematics: An Introduction to Proofs.* Englewood Cliffs, NJ: Prentice Hall, 1998.

Smith, D., M. Eggen, and R. Andre. *A Transition to Advanced Mathematics*. Monterey, CA: Brooks/Cole Pub. Co, 1983.

Solow, D. *How to Read and Do Proofs: An Introduction to Mathematical Thought Processes.* 4th ed. New York, NY: John Wiley and Sons, 2004.

Solow, D. *The Keys to Advanced Mathematics: Recurrent Themes in Abstract Reasoning*. Mansfield, Ohio: Books Unlimited, 1995.

Stein, Sherman K. *How the Other Half Thinks: Adventures in Mathematical Reasoning.* New York, NY: McGraw-Hill, 2003.

Sundstrom, Ted A. *Mathematical Reasoning: Writing and Proof.* Englewood Cliffs, NJ: Prentice Hall, 2003.

Wilson, R.J. *Introduction to Graph Theory*. 2nd ed. New York, NY: Academic Press, 1979.

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