

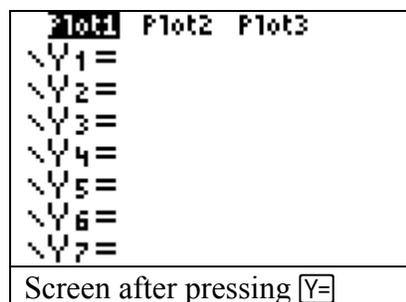
Learning how to Find Maxima and Minima Using the TI – 83Plus

In this activity you will learn how to use your TI – 83Plus graphing calculator to find the values for relative maxima and minima in polynomial curves. If you are unsure on how to graph functions, the practice problems should refresh your memory.

Recall, a relative maximum on a polynomial curve is a coordinate point that lies on the curve and exhibits the property of having lesser values of y on either horizontal side of itself. In the same light, a relative minimum on a polynomial curve is a coordinate point that lies on the curve and exhibits the property of having greater values of y on either horizontal side of itself. Also recall that an extrema is a maximum or a minimum. Now let us investigate the concept using the calculator.

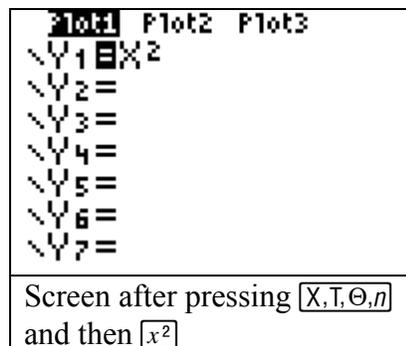
Investigative Practice (A look at maxima and minima by plotting parabolas)

Step 1: Press the $\boxed{Y=}$ key on your calculator. This should bring up a list of incomplete equations. They consist of a letter “y,” followed by a number in subscript, and then an equals sign. See the figure to the right for what your calculator screen should look like.



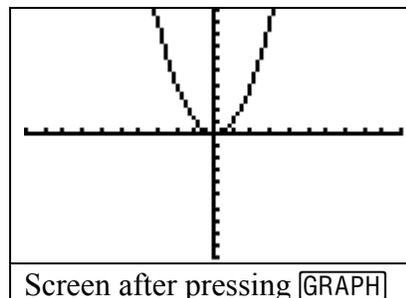
Summary: Press $\boxed{Y=}$

Step 2: In the first equation (labeled “y₁”), enter the following line: x^2 . The syntax involves using the following key sequence: the $\boxed{X,T,Θ,n}$ key followed by $\boxed{x^2}$. The $\boxed{x^2}$ key automatically places the 2 in superscript. See the picture for what the screen should look like. Please note that when input is entered onto a line, the equals sign becomes highlighted.



Summary: Press $\boxed{X,T,Θ,n}\boxed{x^2}$

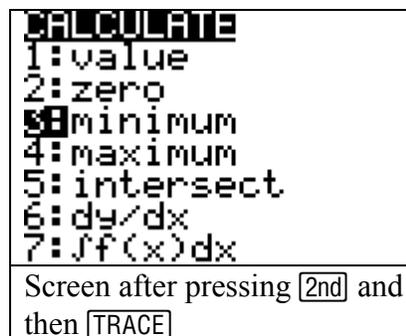
Step 3: Now press the $\boxed{\text{GRAPH}}$ button and a parabola should appear. If not, then press the $\boxed{\text{ZOOM}}$ key followed by $\boxed{6}$. This selects standard zoom features, which is a 10 by 10 two-dimensional Cartesian graph.



Summary: Press $\boxed{\text{GRAPH}}$ or $\boxed{\text{ZOOM}}\boxed{6}$

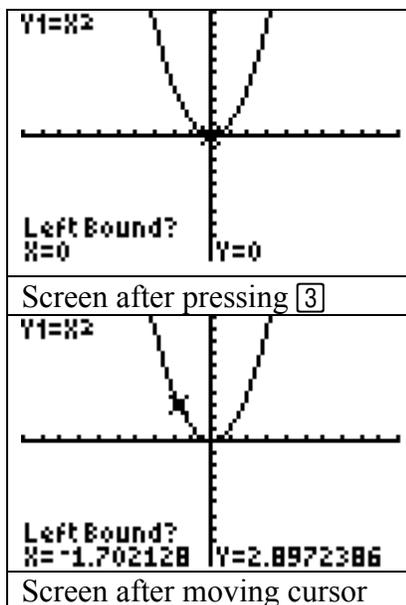
At what point do you think that the minimum will occur for this function? (,)

Step 4: Let us verify the guess by using the calculator. Press $\boxed{2\text{nd}}$ followed by $\boxed{\text{TRACE}}$. This will cause a menu to appear called "CALCULATE." Press $\boxed{3}$ to select "minimum." A picture of the menu is shown in the figure.



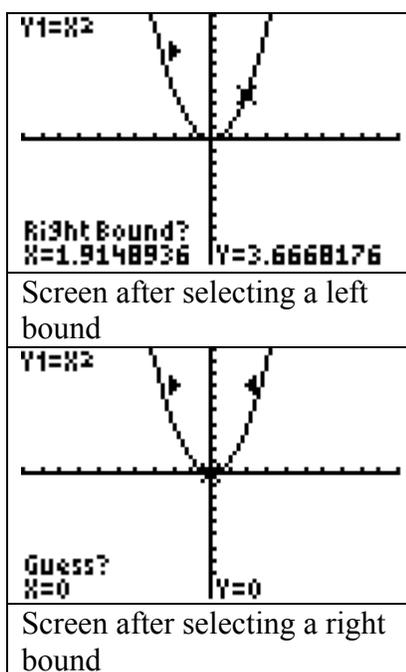
Summary: Press $\boxed{2\text{nd}}\boxed{\text{TRACE}}\boxed{3}$

Step 5: After pressing $\boxed{3}$, the graph will reappear with a blinking cursor on the curve at $x = 0$. You must select a left boundary for the minimum, which is to be deliberately placed to the left of the visibly apparent minimum point. Hold the $\boxed{\leftarrow}$ key to move the cursor left. This is done to contain the minimum, which we expect to be around $x = 0$. Then press $\boxed{\text{ENTER}}$ to select a left bound.



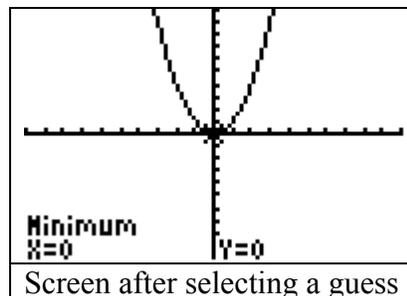
Summary: Hold $\boxed{\leftarrow}$ and press $\boxed{\text{ENTER}}$

Step 6: Upon selecting a left bound, a small arrow will appear above the curve indicating the position of your selection. Now the calculator needs you to select a right bound. Move the cursor right beyond the expected minimum by holding the $\boxed{\rightarrow}$ key. Then press $\boxed{\text{ENTER}}$ and another arrow marker will appear. Then you will be asked to make a guess. Move the cursor as close to the minimum as possible. Then press the $\boxed{\text{ENTER}}$ key and the calculator will find the minimum. (See next page)



Summary: Hold $\boxed{\rightarrow}$ and press $\boxed{\text{ENTER}}$ twice

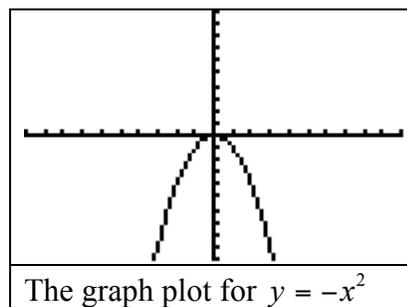
Objective Complete: The calculator may take a few seconds to find an answer. When done, the cursor is at the minimum and the coordinates at the bottom of the screen indicate that it is in fact at (0,0).



Cleanup Step: Press $\boxed{Y=}$ and then $\boxed{\text{CLEAR}}$ to remove the equation for the previous plot. Then press $\boxed{2\text{nd}}$ and then $\boxed{\text{MODE}}$ to exit the screen.

Now let us investigate how to find a maximum using the function $y = -x^2$.

Steps 1, 2, 3: Perform the same steps as in the previous example using $y = -x^2$. This will plot the function on the graph. In Step 2, make sure to use the $\boxed{(-)}$ key and not the $\boxed{-}$ key when inserting the negative sign. Failing to follow this will result in an error message. The resulting graph is shown on the right.



Summary: Press $\boxed{Y=}$ $\boxed{(-)}$ $\boxed{X,T,\theta,n}$ $\boxed{x^2}$ $\boxed{\text{GRAPH}}$

At what point do you think that the maximum will occur for this function? (,)

Step 4: This step differs from the previous example because here we are attempting to find a maximum, not a minimum. Press $\boxed{2\text{nd}}$ and then $\boxed{\text{TRACE}}$ to pull up the "CALCULATE" menu. This time, press $\boxed{4}$ to select "maximum"

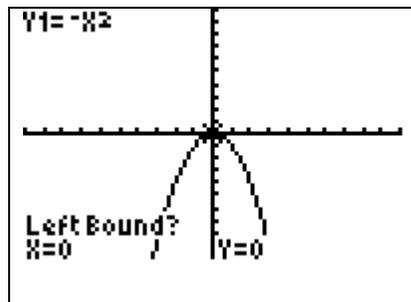
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CALCULATE
1:value
2:zero
3:minimum
4:maximum
5:intersect
6:dy/dx
7:∫f(x)dx
    
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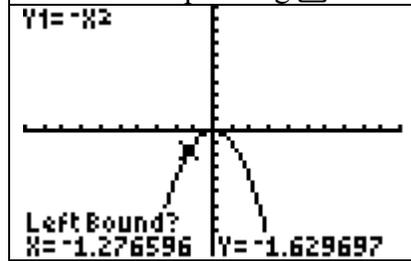
Summary: Press $\boxed{2\text{nd}}$ $\boxed{\text{TRACE}}$ $\boxed{4}$

Screen after pressing $\boxed{2\text{nd}}$ and then $\boxed{\text{TRACE}}$

Step 5: After pressing $\boxed{3}$, the graph will reappear with a blinking cursor on the curve at $x = 0$. You must select a left boundary for the maximum, which is to be deliberately placed to the left of the visibly apparent maximum point. Hold the $\boxed{\leftarrow}$ key to move the cursor left. This is done to contain the maximum, which we expect to be around $x = 0$. Then press $\boxed{\text{ENTER}}$ to select a left bound.



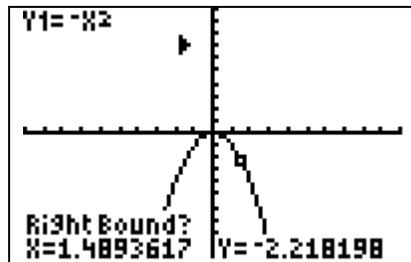
Screen after pressing $\boxed{3}$



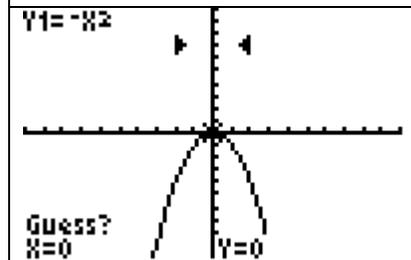
Screen after moving cursor

Summary: Hold $\boxed{\leftarrow}$ and press $\boxed{\text{ENTER}}$

Step 6: Upon selecting a left bound, a small arrow will appear above the curve indicating the position of your selection. Now the calculator needs you to select a right bound. Move the cursor right beyond the expected maximum by holding the $\boxed{\rightarrow}$ key. Then press $\boxed{\text{ENTER}}$ and another arrow marker will appear. Then you will be asked to make a guess. Move the cursor as close to the maximum as possible. Then press the $\boxed{\text{ENTER}}$ key and the calculator will find the maximum.



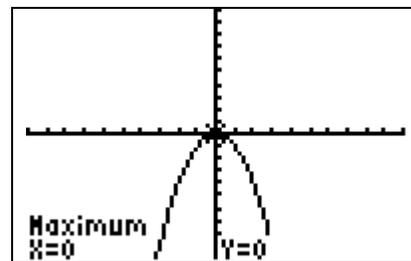
Screen after selecting a left bound



Screen after selecting a right bound

Summary: Hold $\boxed{\rightarrow}$ and press $\boxed{\text{ENTER}}$ twice

Objective Complete: The calculator may take a few seconds to find an answer. When done, the cursor is at the maximum and the coordinates at the bottom of the screen indicate that it in fact was at $(0,0)$. Now redo the “Cleanup Step” to clear all data.



Screen after selecting a guess

Summary: Cleanup by pressing $\boxed{\text{Y=}} \boxed{\text{CLEAR}} \boxed{2\text{nd}} \boxed{\text{MODE}}$

Associated Practice Problems

Instructions: This segment of the assignment is designed for you to gain practice in graphing various polynomial functions and finding their relative maxima and minima. Yes, some functions have multiple maxima and minima. The problems will progress from simple. In this investigation, answer all of the questions to the best of your ability.

The Third Order Polynomial

Step 1: Plot the function $y = \frac{1}{6}x^3 - x^2 + x$ by inputting the equation in the $\boxed{Y=}$ menu. There is no button for the cubic function, so you must replace it with a $\boxed{\wedge}$ $\boxed{3}$. The carat indicates an exponential power. The final image of the input is shown in the figure. Notice that the line carries over if it is too long. This does not affect the calculator output. Then press $\boxed{\text{GRAPH}}$ to get the plot.

Summary: Press the following keys:

$\boxed{Y=}$ $\boxed{(}$ $\boxed{1}$ $\boxed{\div}$ $\boxed{6}$ $\boxed{)}$ $\boxed{\wedge}$ $\boxed{3}$ $\boxed{-}$ $\boxed{X,T,\theta,n}$ $\boxed{x^2}$ $\boxed{+}$ $\boxed{X,T,\theta,n}$ \boxed{x} $\boxed{)}$ and then $\boxed{\text{GRAPH}}$

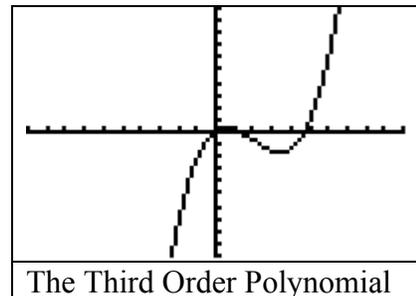
<p>The $\boxed{Y=}$ menu for the function</p> $y = \frac{1}{6}x^3 - x^2 + x$

How many relative extrema are there in this function? _____
 How many maxima do you see in the graph of this function? _____ Minima? _____

Step 2: Use the calculator to find the approximate values for these maxima and minima. Write in the form of ordered pairs. Round the coordinates to the nearest tenth.
 Maxima:

Minima:

Summary: See steps 4, 5, and 6 in the examples



The Fourth Order Polynomial

Step 1: Plot the function $y = \frac{1}{32}x^4 - \frac{1}{2}x^2$. To plot only this function you must press $\boxed{Y=}$ and deactivate the previous function. In order to do this, move the cursor left over the equals sign and press $\boxed{\text{ENTER}}$. This will remove the highlighting from it. Then press $\boxed{\downarrow}$ to get to the "y₂" line and input the equation. Recall, you must use the carat. The completed screen is shown to the right.

Summary: Press $\boxed{Y=}$ $\boxed{\leftarrow}$ $\boxed{\text{ENTER}}$ $\boxed{\downarrow}$ and then

$\boxed{(}$ $\boxed{1}$ $\boxed{\div}$ $\boxed{32}$ $\boxed{)}$ $\boxed{X,T,\theta,n}$ $\boxed{\wedge}$ $\boxed{4}$ $\boxed{-}$ $\boxed{(}$ $\boxed{1}$ $\boxed{\div}$ $\boxed{2}$ $\boxed{)}$ $\boxed{X,T,\theta,n}$ $\boxed{x^2}$ $\boxed{)}$

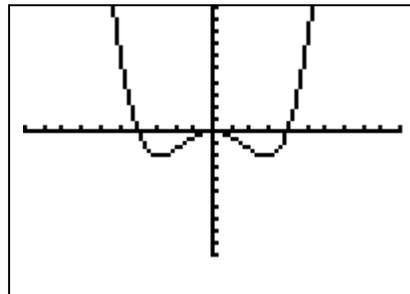
<p>The $\boxed{Y=}$ menu for the function</p> $y = \frac{1}{32}x^4 - \frac{1}{2}x^2$

How many relative extrema are there in this function? _____
 How many maxima do you see in the graph of this function? _____ Minima? _____

Step 2: Use the calculator to find the approximate values for these maxima and minima. Write in the form of ordered pairs. Round the coordinates to the nearest tenth.
 Maxima:

Minima:

Summary: See steps 4, 5, and 6 in the examples

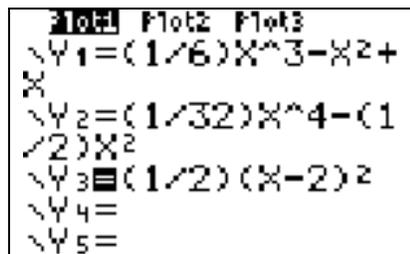


The Fourth Order Polynomial

Another Parabola (2nd Order Polynomial)

Plot the function $y = \frac{1}{2}(x - 2)^2$. Again you must deactivate the previous function. Move the cursor down to the “y₂” row and move it left to highlight the equals sign. Then press [ENTER] to deselect it. Now move the cursor down into the “y₃” row and enter the function. Finally, press [GRAPH] to plot the function.

Summary: Press [Y=] [↓] [↓] [ENTER] [↓] and then [1] [÷] [2] [)] [X,T,θ,n] [-] [2] [)] [x²]



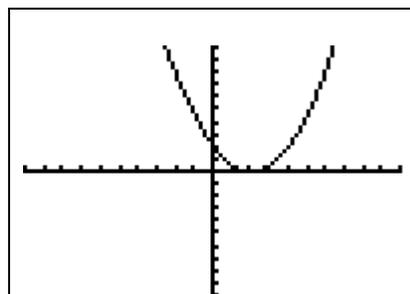
The [Y=] menu for the function $y = \frac{1}{2}(x - 2)^2$

How many relative extrema are there in this function? _____
 How many maxima do you see in the graph of this function? _____ Minima? _____

Step 2: Use the calculator to find the approximate values for these maxima and minima. Write in the form of ordered pairs. Round the coordinates to the nearest tenth.
 Maxima:

Minima:

Summary: See steps 4, 5, and 6 in the examples



The Second Order Polynomial

The Fifth Order Polynomial

Plot the function $y = \frac{1}{64}x^5 - \frac{1}{3}x^3 + x$. This is the most complicated polynomial that you will receive. In order to plot this function only, you must again deactivate the previous function. Do this by moving the cursor down to the “y₃” row, highlighting the equals sign, and pressing **ENTER**. Then move the cursor down into the “y₄” row and insert the function.

Summary: Press **Y=** **↓** **↓** **←** **ENTER** and then

(**1** **÷** **6** **4** **)** **X,T,θ,n** **^** **4** **-** **(** **1** **÷** **3** **)** **X,T,θ,n** **^** **3** **+** **X,T,θ,n**

```

Y1=(1/6)X^3-X^2+
X
Y2=(1/32)X^4-(1
/2)X^2
Y3=(1/2)(X-2)^2
Y4=(1/64)X^5-(1
/3)X^3+X
    
```

The **Y=** menu for the function

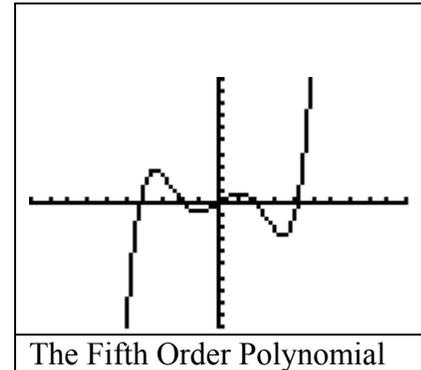
$$y = \frac{1}{64}x^5 - \frac{1}{3}x^3 + x$$

How many relative extrema are there in this function? _____

How many maxima do you see in the graph of this function? _____ Minima? _____

Step 2: Use the calculator to find the approximate values for these maxima and minima. Write in the form of ordered pairs. Round the coordinates to the nearest tenth. Maxima:

Minima:



Summary: See steps 4, 5, and 6 in the examples

Conclusive Questioning

Each of the Functions given had the maximum number of extrema possible. Fill in the table below and try to identify the relationship between function order and maximum number of extrema.

Order of Polynomial	Extrema (both max and min)
Second Order	
Third Order	
Fourth Order	
Fifth Order	

Conclusion About the Relationship Between Extrema and Order: _____
