

## Vertical Motion in Free Fall

Name: \_\_\_\_\_ Date: \_\_\_\_\_

**Problem<sup>1</sup>:** You throw a ball straight up from the roof of a tall building. The ball leaves your hand at a point even with the roof railing with an upward speed of 15.0 m/s. On its way back down, the ball just misses the railing and falls to the ground adjacent to the building. At the location of the building, the acceleration due to gravity is  $g = -9.80 \text{ m/s}^2$ . The ball is considered to be in free fall from the moment it leaves your hand, so its vertical position and velocity are governed by the following equations:

$$\text{Vertical Position at time } t, y_t = y_0 + v_0t + \frac{1}{2}gt^2$$

$$\text{Vertical Velocity at time } t, v_t = v_0 + gt$$

If the point where the ball leaves your hand is considered to be the origin ( $y_0 = 0$ ) and the upward direction is taken to be positive, find:

- the position and velocity of the ball 1.00 s and 4.00 s after leaving your hand,
- the maximum height reached, the time at which it is reached, and the velocity of the ball at that height, and
- the time and velocity of the ball when it passes the railing on its way down.

**Approach:** A Texas Instruments TI-83 Plus graphing calculator can help you find the answers to this problem.

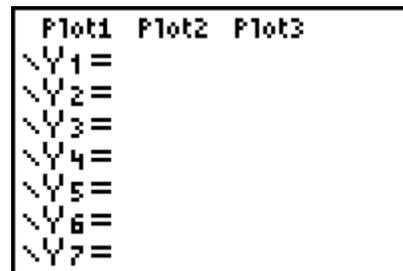
- First, let's rewrite our equations for position and velocity using the given information.
- The initial position,  $y_0 = 0 \text{ m}$ , the initial velocity,  $v_0 = 15.0 \text{ m/s}$ , and the acceleration due to gravity,  $g = -9.80 \text{ m/s}^2$ , therefore the equations can be rewritten as:

a)  $y_t = 15.0t - \frac{9.80}{2}t^2 = 15.0t - 4.90t^2$

b)  $v_t = 15.0 - 9.80t$

- Now that we've written our equations, let's enter them into the graphing calculator.

- a) Press  $\boxed{Y=}$  to display the **Y=** editor, which is where functions are defined. *A blinking cursor appears to the right of  $Y_1=$ .*



<sup>1</sup> Problem borrowed from Young & Freedman (2004) and adapted for the graphing calculator.

- b) Press  $15$   $[X,T,\theta,n]$   $-$   $4.9$   $[X,T,\theta,n]$   $^$   $2$   $[ENTER]$  to define the position function as  $Y_1$  in terms of  $X$ . A blinking cursor appears to the right of  $Y_2=$  and the  $=$  sign after the  $Y_1$  remains highlighted to indicate that  $Y_1$  is selected.

```

Plot1 Plot2 Plot3
Y1=15X-4.9X^2
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=

```

- c) Press  $15$   $[=]$   $9.8$   $[X,T,\theta,n]$   $[ENTER]$  to define the velocity function as  $Y_2$  in terms of  $X$ . The blinking cursor moves to the right of  $Y_3=$  and the  $=$  signs after both  $Y_1$  and  $Y_2$  remain highlighted to indicate that both are selected.

```

Plot1 Plot2 Plot3
Y1=15X-4.9X^2
Y2=15-9.8X
Y3=
Y4=
Y5=
Y6=
Y7=

```

- 4) We will use the Table feature to find the positions and velocities at the desired times.

- a) Press  $[2nd]$   $[TBLSET]$  (above  $[WINDOW]$ ) to display the TABLE SETUP menu.
- b) Press  $[ENTER]$  to accept  $TblStart=0$ .
- c) Press  $1$   $[ENTER]$  to define the table increment  $\Delta Tbl=1$ .

```

TABLE SETUP
TblStart=0
ΔTbl=1
Indpnt: Auto Ask
Depend: Auto Ask

```

- d) Leave **Indpnt:Auto** and **Depend:Auto** so that the table will be generated automatically.
- e) Press  $[2nd]$   $[TABLE]$  (above  $[GRAPH]$ ) to display the table.
- f) Using the table, record the position and velocity at 1.00 sec below:

X	Y1	Y2
0	0	15
1	10.1	5.2
2	10.4	-4.6
3	.9	-14.4
4	-18.4	-24.2
5	-47.5	-34
6	-86.4	-43.8

X=0

- i) Position 10.1 m ,
- ii) Velocity 5.2 m/s .
- g) Record the position and velocity at 4.00 sec below:
- i) Position -18.4 m ,
- ii) Velocity -24.2 m/s .
- h) Interpret the fact that the position and velocity are both negative at 4 seconds.

The negative position means the ball is lower than the railing.

The negative velocity means the ball is falling, not rising.

5) It will be easier to interpret these data in graph form. Before displaying a graph we should set the viewing window. We do this by setting a number of window variables.

a) Press **[WINDOW]** to display the window editor, where you can view and edit the values of the window variables. **Xmin**, **Xmax**, **Ymin**, and **Ymax** define the boundaries of the display. **Xscl** and **Yscl** define the distance between tick marks (or the scale) on the axes, and **Xres** controls the pixel resolution. We can use the values from the table (above) to set these variables appropriately.

```

WINDOW
Xmin=10
Xmax=10
Xscl=1
Ymin=-10
Ymax=10
Yscl=1
Xres=1
    
```

b) Press **0** to set **Xmin**.

c) Press **4** to set **Xmax**.

d) Press **1** to set **Xscl**.

e) Press **[(-)] 25** to set **Ymin**.

f) Enter **15** and **5** for **Ymax**, and **Yscl**, respectively.

g) Press **1** to set **Xres**.

```

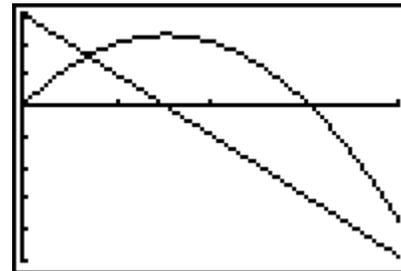
WINDOW
Xmin=0
Xmax=4
Xscl=1
Ymin=-25
Ymax=15
Yscl=5
Xres=1
    
```

6) With our functions defined and the display window set, we can now display and explore the graphs of position and velocity relative to time.

a) Press **[GRAPH]** to display the two selected functions in the viewing window.

b) By pressing **[2nd][TABLE]** and then **[GRAPH]**, you can toggle back and forth between the table and graph.

c) Which curve represents position ( $Y_1$ ) and which represents velocity ( $Y_2$ )? (Describe the curves by their shape.)

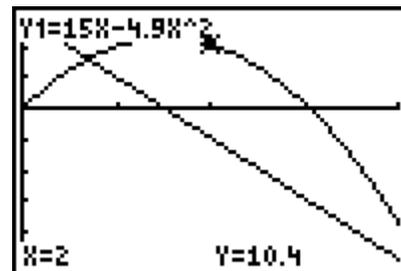


The graph of position looks like a parabola.

The graph of velocity is a straight line.

7) We will now use the Trace feature to move the cursor along the plots of these functions and display the (x, y) coordinates.

a) Press **[TRACE]** to begin the trace. The trace cursor appears on the first selected function in the Y= editor (in this case  $Y_1$ , or position), at the middle X value on the screen (in this case 2). The function equation is displayed in the top left corner of the screen and the coordinates are displayed at the bottom of the screen.



b) Press  $\leftarrow$  or  $\rightarrow$  to move along the curve to lesser or greater X values, respectively. Try to find the answers to the original question (b) in this fashion, and record your values here: Max Height 11.48 m Time 1.53 s.

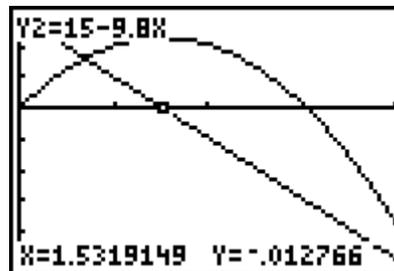
c) To find the velocity at the maximum height, we need to switch our trace to the velocity function.

d) Press  $\nabla$  to switch to the graph of  $Y_2$  (the velocity function).

e) Record the velocity of the ball at 1.53 s.

Max. Ht. Velocity: -.013 m/s.

f) Why is the velocity of the ball essentially zero at its highest point?



At its highest point the ball stops going up and starts going down.

Its velocity moves through zero as it switches from positive to

negative. When the velocity is zero it is at its maximum height.

8) We still need to find the time and velocity of the ball when it passes the railing on its way down for part (c). *Hint: the vertical position at that point in time will once again be zero.*

a) Use  $\triangleleft$  and  $\triangleright$  to switch between the graphs of the two functions.

b) Trace along the curves as above to find the time and velocity when the ball passes the railing on the way down and record below.

Time: 3.06 s, Velocity: -15.0 m/s.

c) How does the time at which the ball passes the railing,  $t_r$ , compare to the time the ball reaches its maximum height,  $t_m$ ? Why?

$t_r = 2 t_m$  because it takes as long for the ball to fall from its max.

height as it does for it to get there. The parabola is symmetric.

d) How does the speed of the ball as it passes the railing,  $v_r$ , compare to the initial speed of the ball,  $v_0$ ? What does this mean?

$v_r = -v_0$ .

The ball is falling at the same speed that it was initially rising.

### Bibliography

Young, H. D., & Freedman, R. A. (2004). *Sears and Zemansky's university physics: with modern physics* (11<sup>th</sup> ed.). San Francisco: Pearson Addison Wesley.